Technical Paper

Transactions of the Society of Naval Architects of Korea
Vol. 29, No.4, November 1992
大韓造船學會論文集
第 29 卷 第 4號 1992 年 11 月

The Development of Analysis Techniques of Extreme Tensions in a Snapping Cable

—Parameter Studies—

by

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스내핑 케이블의 극단장력의 해석기법 개발

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Abstract

In this paper, extreme tensions in a snapping cable are studied and systematic parameter studies are made in the selected cable using the clipping-off model. The anticipation of incipient clipping frequencies of a cable are of use in giving an indication of the behavior of cables for marine applications in which large dynamic tension build-up in rough seas may cause the total tension to become negative.

요 약

이 논문에서는 스내핑 케이블(Snapping Cable)의 극단 인장력(Extreme Tension)이 연구되어지고 해석기법으로 클리핑-오프 모델(Clipping-off Model)을 이용하여 매개변수 연구(Parameter Study)가 수행되었다. 이 매개변수 연구에 의한 주어진 케이블의 초기 클리핑 발생 주파수 예측은 거친 해상에서 큰 동인장력(Large Dynamic Tension)이 발생되어 전체 인장력(Total tension)이 음의 값이 되는 스내핑현상이 일어날 수 있는 케이블의 동적 거동을 예견하기 위한 유용한 지침으로서 사용될 수 있을 것이다.

발 표 : 1992년도 대한조선학회 춘계연구발표회('92. 4. 18.)
Manuscript received : May 19, 1992, revised manuscript received : July 20, 1992.
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1. Introduction

The result and computer program development of the cables used in marine operations such as towing and mooring have been made in the last two decades. The problem of large dynamic tension amplication in rough seas is particularly severe for snapping cables, thus necessitating a detailed analysis. Goeller and Laura(1970), in a study of the dynamic response of stranded steel cables, found that maximum cable forces nine times the static payload weight in water were developed in snap condition [1]. Kirk and Jane(1976), in their numerical simulations of anchor chain dynamics of tension-leg single buoy mooring system, found that the change in chain length was of the order of 1.7m(the unstrained chain length=152m) which corresponds to an increase in tension of 6.2 MN [2]. Also, Fylling and Wold(1979) presented the comparison of cable dynamics in their paper which included the cases when the dynamics force exceeded the static force[4]. Shuara et al.(1981), found that the dynamic tensions of oscillating chain become maximum in the snap condition [5]. Milgram et al. (1988), developed simplified models for the horizontal towline dynamics and showed the experimental and theoretical results of the dynamic towline tension [9]. Shin (1991) made an theoretical analysis of extreme tensions in a horizontal snaping cable and compared with experimental results [4][10].

This paper presents a brief description of simplified cable dynamic equations with a clipping-off model as an analysis technique. In order to grasp full meaning of numerical results and their comparison with experimental ones, systematic parameter studies of the horizontal cable which was already used by Fylling(1979) [4] are made.

2. Description of Simplified Cable Dynamic Equations

It is assumed that the dynamic tension is almost uniformly distributed along the cable for frequencies which are in the range of water wave frequencies, i.e. a quasi-static stretch assumption is employed. Also, the axial motions of the cable much smaller then the transverse motions when a horizontal cable is forced to move at the excitation of the first few natural frequencies imposed on the end of the cable[7][8].

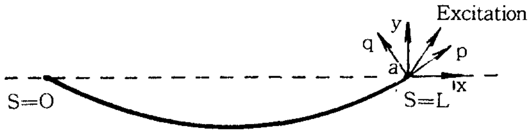

Fig.1 Excitation and lagrangian coordinates

Then a relatively simple mathematical model for a horizontal cable is constructed as follows [10].

$$M\frac{\partial^2 q}{\partial t^2} = (T_0+T_1)\left(\frac{\partial^2 q}{\partial s^2} + \frac{d\phi_0}{ds}\right) + F_q - T_0\frac{d\phi_0}{ds} \quad (1)$$

with

$$T_1 = \frac{EA}{L}\left[p(L) - \int_0^L q\frac{d\phi}{ds}\,ds + \frac{1}{2}\int_0^L\left(\frac{\partial q}{\partial s}\right)^2 ds\right] \quad (2)$$

$$F_q = -\frac{1}{2}\rho\, C_D\, D\frac{\partial q}{\partial t}\left|\frac{\partial q}{\partial t}\right|$$

where

- T_0 static effective tension
- T_1 dynamic effective tension
- p tangential displacement based on the static configuration
- q normal displacement based on the static configuration
- s Lagrangian coordinate
- ϕ_0 static angle
- ϕ_1 dynamic angle
- m mass per unit length
  added mass per unit length($M=m+m_a$)
- E Young's modulus
- A cable section area

F_q normal component of the fluid drag force
ρ mass density of water
p(L) tangential displacement imposed on the end of the cable
C_D drag coefficient
D cable diameter

Large dynamic tension build-up in rough seas may cause the total tension to become negative in certain parts of the cable. This cannot be sustained by a cable due to their low bending stiffness. The appearance of even a small negative overall tension sets in action a buckling mechanism very quickly. On top of the formation of a buckling mode, there occurs a free-falling of the cable opposed only by the action of the drag force, unlike a string with zero static curvature.

In order to get a model of a slack and then snapping cable, we assume that the buckling mechanism keeps the tension at near zero levels until a positive value is regained, while its dynamic behavior is governed by the balance of inertia and drag forces as soon as the total tension in an element of the cable reaches a negative value. Then the governing equations are reformulated as [10]:

$$M\frac{\partial^2 q}{\partial t^2} = (T_0+T_1)\left(\frac{\partial \phi_1}{\partial s} + \frac{d\phi_0}{ds}\right) + \phi_1\frac{d}{ds}(T_0+T_1) - T_0\frac{d\phi_0}{ds} + F_q \quad (3)$$

$$\Rightarrow M\frac{\partial^2 q}{\partial t^2} = -T_0\frac{d\phi_0}{ds} + F_q \text{ as } T_0+T_1 \to 0$$

3. Numerical Applications

For the numerical scheme, the responses are expanded in terms of Chebyshev polynomials[3]; a collocation method spatially and Newmark's mathod for the time integration[6]

3.1 Spectral Method Using Chebyshev Polynomials

The equation $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2 y = 0$ is satisfied by the nth Chebyshev Polynomial, $y = T_n(x)$, when n is a positive integer or zero.

Then it is conventional to take $T_0(x)=1$ and the following results.

$$T_0(x)=1,\ T_1(x)=x,\ T_2(x)=2x^2-1,\ T_3(x)=4x^3-3x,\cdots,\ T_n(x)=\cos(n\cos^{-1}x) \quad (4)$$

If it is assumed the required solution of equation (3) exists, variables and their derivatives with respect to Lagrangian Coordinates of the Chebyshev Polynomials (4) and the expansion coefficients can be determined.

$$T_1(x)=\sum_{n=0}^{\infty} a_n T_n(x)$$
$$q(x)=\sum_{n=0}^{\infty} q_n T_n(x) \quad (5)$$
$$\phi_1(x)=\sum_{n=0} b_n T_n(x)$$

To determine the coefficients, the expansion series (5) are introduced into equation(3).

The collocation method can be considered as one of the cases of the general criterion that weighted average of the residual should vanish. Then, for acceleration forms of q_n Newmark's Method is employed and for the nonlinear terms like fluid drag the iterative numerical technique is used.

The principal parameters of the horizontal cable used in the experiment of the Ship Research Institute of Norway are found in Table 1 [4].

Table 1 Cable used in the experiment of the Ship Research Institude of Norway [4]

T_0=88N	M=0.666kg/m
W=5.05N/m	EA=7,854,000N
L=10.9774m	D=0.01m
Cd=1.5	

In order to provide further insight the numerical results have been plotted for a range of parameter values. The parameters employed are the catenary stiffness $\varepsilon=T_0/WL$, the excitation amplitude ratio to cable diameter $\kappa=A/D$.(T_0=68N, 78N, 88N, 98N, 108N, A=2.5D, 5D, 7.5D, 10D, 12.5D, 15D, 17.5D and 20D)

4. Numerical Results and Parameter Studies

In Figures (2) to (6), extreme tensions predicted using equations (1) to (3) are plotted as a function of system parameter κ=A/D and excitation frequency ω when the static tensions are 68N, 78N, 88N, 98N and 108N, respectively.

Extreme tensions shown in Fig.4 were already compared with experimental results from the Ship Reseach Institute of Norway [4] by Shin (1991) [10].

In Figure (7), the incipient clipping frequencies of horizontal excitation at which the total tension become negative and the slack-and snapping begins are plotted as a function of system parameters, the catenary stiffness $\varepsilon = T_0/WL$ and the excitation amplitude ratio to the cable diameter κ=A/D. Also Fig.7 shows that the incipient clipping frequencies of horizontal excitation decrease as the catenary stiffness ε and the ratio of excitation amplitude to cable diameter κ increase. Especially it is found that, in the small excitation amplitudes(for example,

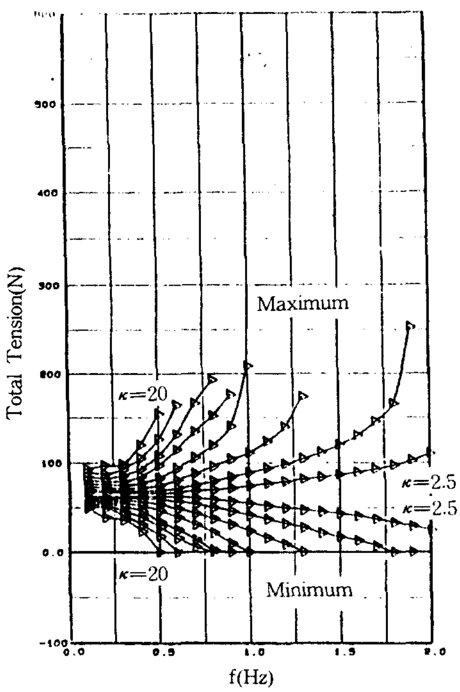

Fig.2 Extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, 12.5D, 17.5D, 20D, D=0.01m, static tension=68N

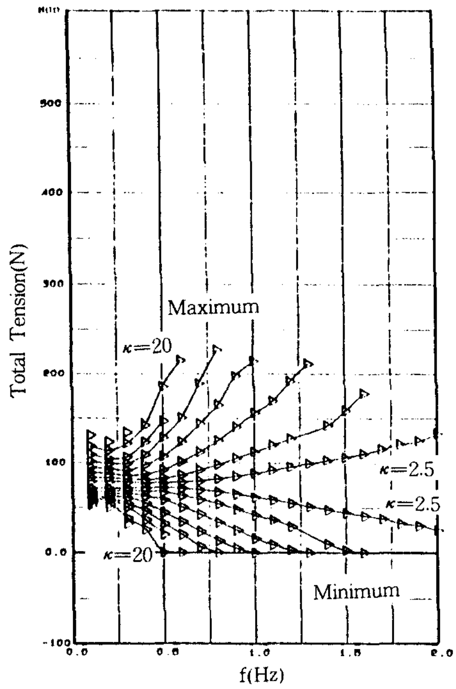

Fig.3 Extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, 12.5D, 17.5D, 20D, D=0.01m, static tension=78N

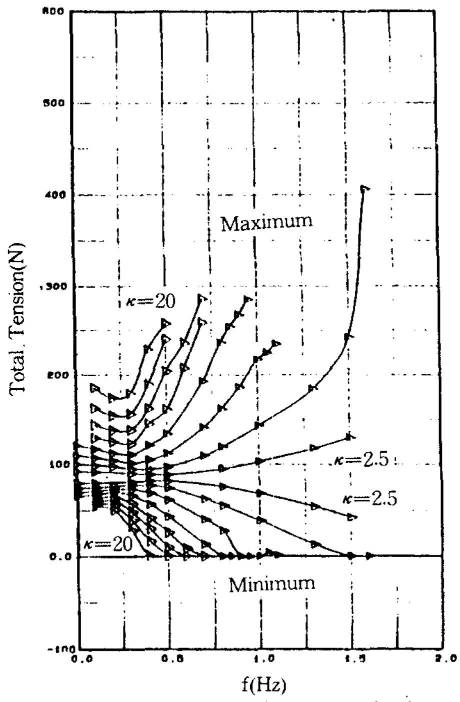

Fig.4 Extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, 12.5D, 17.5D, 20D, D=0.01m, static tension=88N

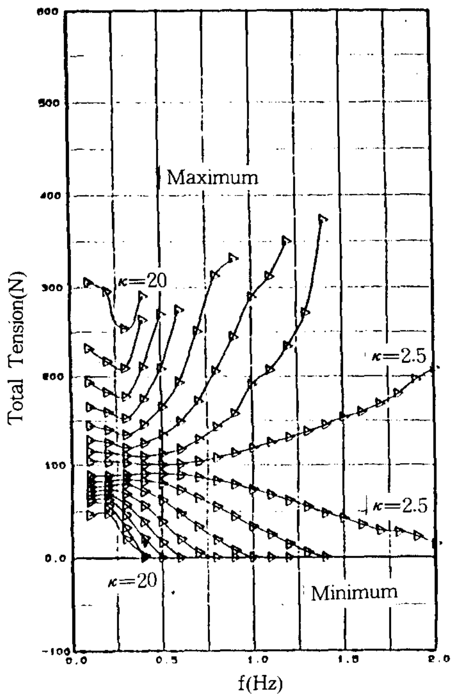

Fig.5 Extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, 12.5D, 17.5D, 20D, D=0.01m, static tension=98N

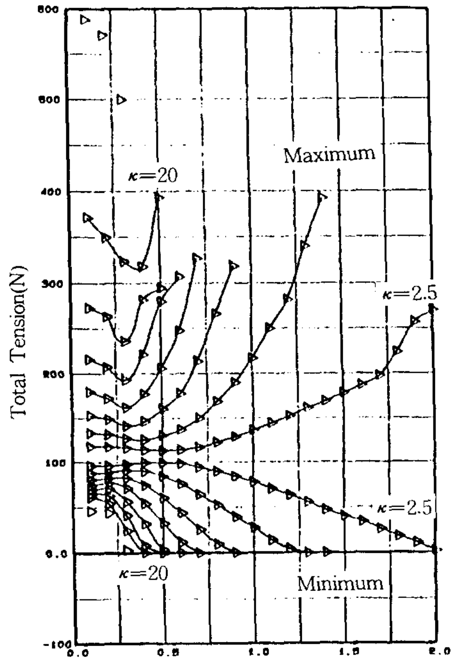

Fig.6 Extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, 12.5D, 17.5D, 20D, D=0.01m, static tension=108N

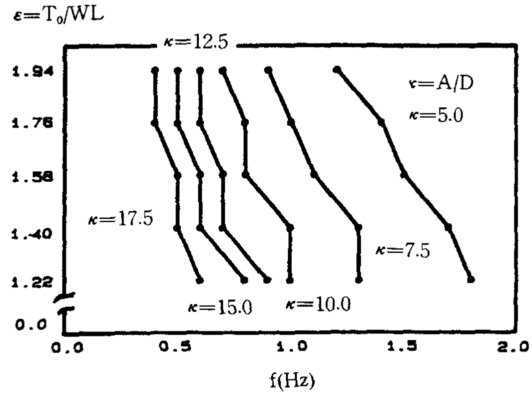

Fig.7 Incipient clipping frequencies of horizontal excitation imposed on the end of the horizontal cable[4]

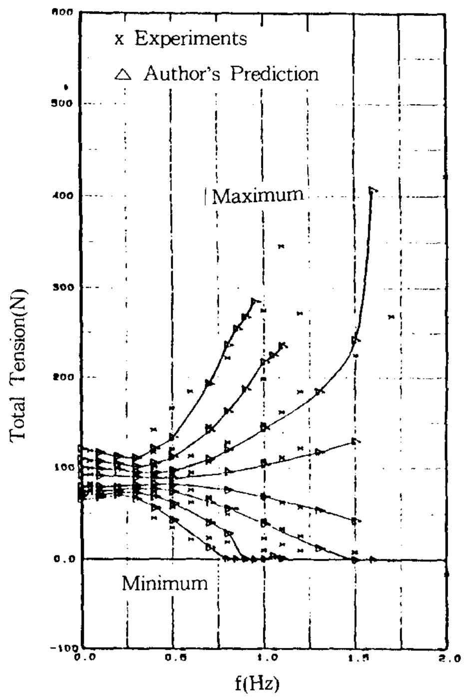

Fig.8 Comparison of extreme tensions : horizontal excitation amplitude=2.5D, 5D, 7.5D, 10D, D=0.01m, static tension=88N[4],[10]

κ=5.0)), the incipient clipping frequencies decrease rapidly as ϵ increases.

Considerable information can be gleaned from these plots. It is possible to predict the incipient clipping frequencies of the horizontal snapping cable. They are conservative and smaller than the experimental incipicent frequencies of the horizontal cable due to the delayed onset of zero tension from the bending stiffness effect.

In the region over the incipient clipping frequencies numerical difficulties may arise in the form of high frequency oscillations, that eventually lead to divergence. Therefore smaller time steps must be needed to ensure numerical accuracy for high excitation frequency and after clipping-off sets in.

5. Conclusions

It is possible to predict conservatively incipient clipping frequencies by employing the clipping-off model suggested as an analysis technique for the horizontal snapping cables in this paper.

From the trend of rapid decrease of incipient clipping frequencies with the catenary stiffness ε increase in small amplitudes of excitation, it is very important to keep the prescribed static tension in marine operations like towing.

Also the anticipation of incipient clipping frequencies as a function of the elastic stiffness EA/L (which is another important parameter in cable dynamics) is needed in order to make full descriptions of the frequency regions where the cable snapping phenomena occur.

6. Acknowledgement

This Research was supported by the Korea Science & Engineering Foundation (KOSEF) under contract No. 901-0902-001-2.

The author would thank Mr. K.W. Park(Research Center for Ocean Industrial Development, National Fisheries University of Pusan) for his help in plotting the numerical results.

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