

# An Iterative Image Restoration

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## 화상의 반복 복원 처리

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### ABSTRACT

A local iterative image restoration method is introduced that processes with varying iteration numbers according to the local statistics. In general almost of the iterative method applies its algorithm to the whole image without considering the local pixel informations, which is not so effective for the processing time. Usually the edges or details have an important role in visual effect. So in this paper we process the edges or the details many times while in the flat region we just pass over or decrease iterations. This method shows good MSE (Mean Square Error) improvement, better subjective quality and reduced processing time.

### 要 約

본 논문은 일반반복 화상 처리가 화상 전체에 화상의 정보와 상관없이 일률적인 방법으로 적용함으로써 생기는 비 효율성을 고려하여 처리하는 화소의 주변 화상 정보를 이용하여 평면 부분과 윤곽 부분의 처리 회수를 달리하여 주므로서 시각적인 효과와 동시에 처리 시간을 단축하는 국부 반복 복원 방법을 제안하였다. 국부 반복 복원 방법은 일반 반복 복원 알고리즘을 적용하여 윤곽 부분을 집중 복원하고 평면 부분은 복원 없이 통과하는 방법으로, 여기에 사용된 일반 반복 알고리즘이 수렴하면 국부 반복 복원도 수렴하게 됨을 이용하였다. 각각 처리된 화상, MSE, 처리 시간 등을 비교하여 그 효율성을 확인 하였다.

### I. Introduction

In image restoration the ultimate goal is the recovery of the original scene from a distorted version. The distortion may be due to motion of the

camera with respect to the original scene, defocusing of the lens system, etc.

A number of techniques have been developed for the restoration of images.

It is well known, however, that the iterative image restoration in particular, has received considerable attention.<sup>(1)-(5)</sup>

Among the advantage of iterative solution methods we mention :

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- the possibility of including nonlinear constraints, reflecting certain deterministic a priori information about the original image,
- the truncation of the iterative process after a finite number of iterations in order to obtain an optimal result for the human visual system,
- the possibility of avoiding ill-conditioned solution.

However most of the existing iterative algorithms are applied equally to the whole area in the image. But every image has diverse dynamics as a random field which is nonstationary in mean and variance. But the pixels have a high correlation between neighbors. Furthermore, almost of the pixels are located in the large area of flat plateau, while the edges or detailed areas are occupied in the small portion of the image.

In fact, the flat region does not show so much blurrings as in the edges or details.

The edges or details corrupt badly by the blurring process. These areas are very important in the visual effects. But they lose their sharpness very easily.

The pixels in these areas need to be sharpened or restored with considerable amounts than those of the flat region.

In this paper, we develop a new simple efficient method for iterative restoration. The method is based on the different local statistics between flat regions of the image and the vicinity of edges or details. It varies the iterations according to the local statistics.

We will review some of the simple iteration algorithms. Then we present our local iteration method. Simulation results and performance comparison are presented at the end of this paper.

## II. Iterative Image Restoration

In many practical situations image degradations may be modeled by a linear blur. If we consider the image is formulated in the enough bright con-

dition, the additive noise is considerably reduced and may be omitted.

The blurred image can then be described by the following algebraic model :

$$y = H x, \quad (1)$$

where the linear blurring operator  $H$  is known or can be identified.

The original and blurred image are denoted by  $x$  and  $y$  respectively.

Image restoration concentrates on removing the degradations caused by the blur to obtain an improved image  $\hat{x}$  which is an acceptable approximation to the original image.

This can be achieved by an inverse processing like

$$\hat{x} = H^{-1}y. \quad (2)$$

Since the inverse problem formulated in eq.(2) is ill-posed, it usually is difficult to get images restored direct from inverse filters. Then we need the iterative algorithm to get over this shortcoming.

The relation between the original image  $x$  and the observed image  $y$  may be written as

$$\lambda y = \lambda H x \quad (3)$$

where  $\lambda$  is a constant.

Restoring  $x$  can be achieved by finding an inverse operator  $H^{-1}$  and by applying to the observed image  $y$  as following,

$$x = (\lambda H)^{-1} \lambda y. \quad (4)$$

A common method for solving equations iteratively of this form is to write it in the form of a fixed point equation :<sup>(6)</sup>

$$x = F(x) \quad (5)$$

When  $F$  is a continuous linear transformation of a Hilbert space  $X$  into itself, if  $\|T\| < 1$ , then  $(I - T)^{-1}$  exists and may be evaluated using the series expansion

$$(I - T)^{-1} = I + T + T^2 + T^3 + \dots = \sum_{n=0}^{\infty} T^n \quad (6)$$

If we define the operator  $T$  by

$$T = (I - \lambda H) \quad (7)$$

then

$$(\lambda H)^{-1} = \sum_{n=0}^{\infty} (I - \lambda H)^n = \sum_{n=0}^{\infty} T^n \quad (8)$$

Applying eq.(8) to eq.(4), we obtain the following expression :

$$x = \sum_{n=0}^{\infty} (I - \lambda H)^n \lambda y \quad (9)$$

A recursive solution to the inverse problem may be derived from eq.(8) if we define  $x_k$  to be the approximation that is based on  $k$  terms of this expression, i.e.,

$$x_k = \sum_{n=0}^{k-1} (I - \lambda H)^n \lambda y = \sum_{n=0}^{k-1} T^n \lambda y \quad (10)$$

From this, the relationship between  $x_{k+1}$  and  $x_k$  may be written as

$$x_{k+1} = (I - \lambda H)x_k + \lambda y = x_k + \lambda(y - Hx_k) \quad (11)$$

Incorporating the acceleration parameter  $\alpha$ , eq.(11) may be written as

$$x_{k+1} = x_k + \alpha(y - Hx_k) \quad (12)$$

Also, the acceleration parameter  $\alpha$  may be used as constraints.<sup>(7)</sup>

Eq.(12) can be expressed in the frequency domain, then it is written as

$$X^{(k)}(u, v) = \frac{1 - \{1 - H(\mu, v)\} \{1 - \alpha H(\mu, v)\}^k}{H(\mu, v)} Y(\mu, v) \quad (13)$$

Eq.(13) converges if the condition

$$|1 - \alpha H(u, v)| < 1 \quad (14)$$

is satisfied, which results in

$$X^{(k)}(u, v) = H^{-1}(u, v) Y(u, v) \quad (15)$$

Eq.(15) shows equivalent inverse algorithm.

To satisfy eq.(14),  $\alpha$  must be in the range

$$0 < \alpha < 2. \quad (16)$$

Eq.(12) is a general form of any iterative method. For example, Jacobi method is

$$x_k = x_{k-1} + D^{-1}(y - H x_{k-1}) \quad (17)$$

where  $D$  denotes the diagonal matrix of  $H$ .

Eq.(17) is a form of dialect for eq.(12).

### III. Local Iterative algorithm.

Blurring process in the image prevails in the area of edges than in the flat plateau. The edges or detailed areas have an important role in the visual information. Restoration process should be effective in the edges. The degradation in the flat region is not so severe, besides it makes very slight changes occur in the restoration process. On the other hand, it takes up so much time to process the less important flat regions, which occupies so much portions in the whole images.

In this paper, we use the local statistics to differentiate detailed regions from the flat ones. In the detailed regions, we process many iterations, but we jump over the flat region, which saves so much processing time.

2-D window of Fig.1 is used to calculate the local statistics.

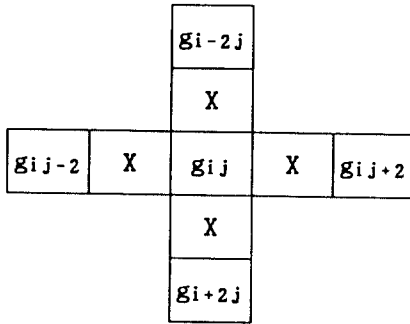


Fig. 1 Window for the local statistics

The local mean is

$$M = \left( \sum_{\substack{m=-1 \\ |m| \neq |n|}}^1 \sum_{\substack{n=-1 \\ |n| \neq |m|}}^1 g_{i+2m, j+2n} + g_{ij} \right) / 5 \quad (18)$$

The local variance is

$$V = \left( \sum_{\substack{m=-1 \\ |m| \neq |n|}}^1 \sum_{\substack{n=-1 \\ |n| \neq |m|}}^1 (g_{i+2m, j+2n} - M)^2 + (g_{ij} - M)^2 \right) / 5 \quad (19)$$

The iteration number  $k$  is

$$k = \begin{cases} 0 & \text{if } V \leq 30 \\ \text{int}(K \log V) & \text{if } V > 30 \end{cases} \quad (20)$$

where we considered the pixel is in the flat regions if its local variance is below 30, and  $\text{int}(\ )$  is a function to make its argument to be an integer.

The iteration algorithm in the local iteration method may be any one that converges, for it varies iteration number  $k$  only according to the local statistics without changing any algorithms. As a result, whenever the main iteration algorithm converges this local method converges also.

#### IV. Experiments

To justify the proposed method, we applied 5 X 5 Gaussian PSF[point spread function] of Fig.2 to make original Lena image of Fig.3 blurred.

Fig.4 displays the blurred image.

0.00854	0.02231	0.03072	0.02231	0.00854
0.02231	0.05826	0.08023	0.05826	0.02231
0.03072	0.08023	0.11049	0.08023	0.03072
0.02231	0.05826	0.08023	0.05826	0.02231
0.00854	0.02231	0.03072	0.02231	0.00854

Fig. 2 Point Spread Function



Fig. 3 Original Lena Image



Fig. 4 Blurred Lena Image

The acceleration parameter  $\alpha=1$  was used in each method and the constant  $K=1$  are used in the local iteration eq.(20).

Each iteration results are obtained in Fig.5. The results of the local iteration is displayed in Fig.6. MSE(Mean Square Error) for each method is compared in Table 1 and Fig. 8.

In the local iterative method it was made to iterate up to 6 times according to the statistics,

in order to make comparison with the same iteration for the general method.



k=1	k=2
k=3	k=4
k=5	k=6

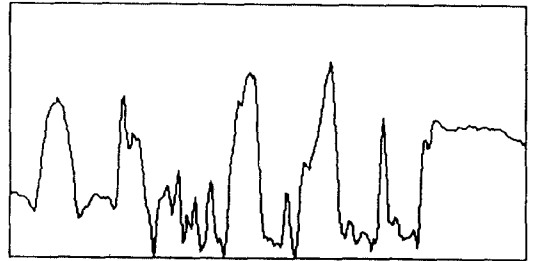
Fig. 5 Restored image for General Iteration



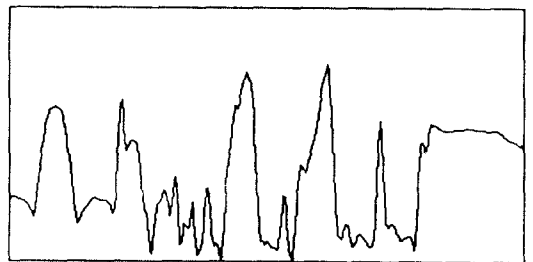
Fig. 6 Image for Local Iterative Restoration

Table. 1 MSE for each method

Itr.	General	Proposed
0	124.438039	124.438039
1	84.115362	58.812731
2	70.505232	
3	63.560384	
4	59.364478	
5	56.550439	
6	54.617825	



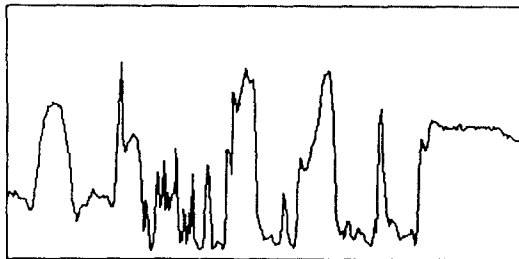
(c)General 6 iteration



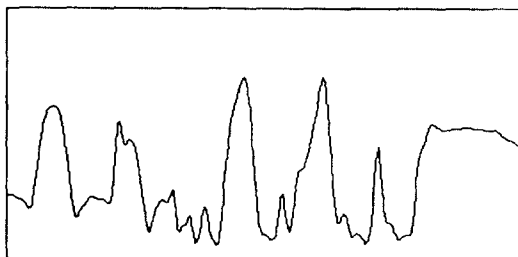
(d)Local iteration

Fig. 7 One Profile Line for each method

We processed one horizontal profile line made of 255 pixels, to compare with each method, as Fig.7.



(a)Original



(b)Blurred

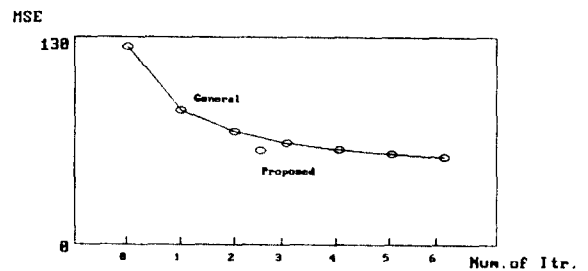


Fig. 8 MSE for each method

In Table 1 and Fig.8 restored image by the local iteration shows good MSE, almost equal to that of 4-iterated restoration.

Fig.6 and Fig.7 show the restored image by local method is comparative to that of 6-iterated restoration.

Each processing time is summarized in Table 2.

Table. 2 Processing Time

General (1 iteration)	526 sec
Proposed (Total Processing)	1369 sec

The processing time of proposed method is 1369 sec which is compared to that of 2 or 3 iteration.

As a whole, one can see that local iterative method shows good MSE and better subjective results with much reduced processing time.

### V. Conclusion

Almost of the iterative restoration apply their algorithm equally to all over the area without considering its local statistics.

In this paper the local statistics decide the iteration number for each pixel according to the neighbors. In this method the edges or detailed areas iterate many times to make further restoration effect. In the flat area which possesses almost of the portion in image it passes without restoration, to spare much processing time.

This algorithm shows good MSE improvements, better subjective results and reduced processing time is another considerable improvements.

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