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⦿ 論　文

Dynamic Interaction of Waves with a Moored Structure

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계류된 구조물에 작용하는 파도의 동적작용에 대하여

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Key Words : Diffraction Theory(回折理論), Sway Motion(左右動搖), Heave Motion(上下動搖), Roll Motion(橫動搖), Representative Structure Length

Abstract

This paper presents the method of numerical analysis concerned with the hydropdynamic forces and moments of the floating bodies exerted by waves. The analytic methods of hydrodynamic wave forces and moments for large volume structures are generally classified into four categories ; the strip method, the boundary element method, the finite element method, and the potential matching method. In the case of the comparatively large structures, diffraction theory can be applied. However, there are no application limits of diffraction theory which have been known concerning with the analytic method of the rectangular structures.

In this paper, the two-dimensional B.E.M. is treated for a moored small rectangular structure in order to evaluate applicability of diffraction theory. Numerical calculation is carried out for the structure. The results are compared with some other ones for verification. The result shows that diffraction theory is applicable to structures smaller than 0.15 in the ratio of the representative structure length d to wave length L for rectangular ones.

1. Introduction

Morison equation which is a semi-intuitive formula, but includes both the drag and the inertia forces, is useful to calculate the hydrodynamic wave forces and moments for the structures of small volume. However, it is well known that diffraction effects are remarkable, when $H/d<1.0$ and $d/L>0.15$, in which d is the representative length of the structure, H wave height and L wave length, and then diffraction theory has to be

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applied to calculate the hydropdynamic wave forces and moments for the structures of the large volume.[1]

A number of studies have been conducted to compute the hydrodynamic forces and moments acting on the various types of the floating bodies. The results have shown the effects of their shapes, water depth, and wave characteristics related to the dynamic response on the floating bodies and hydrodynamic forces acting on them, and then the accuracy of the computational techniques has been evaluated. Among them ; 1) The strip method[2),3)] has been usually applied to the field of naval achitecture engineering, and then many results have been obtained for the case of slender floating bodies like a ship. On the other hand, it is clear that there occur some cases not applicable to bodies such as complex-shaped structures. 2) In the boundary element method,[1,4,5)] it is convenient to treat the unbounded domains and simple in mesh making, but it requires the constant depth and fails for irregular frequencies. 3) Although the finite element method[5,7)] is capable of treating arbitary depth and bodies, there is a difficult problem to put infinite fluid domains into finite structure boundary domains. 4) The potential matching method[8,9)] is simple one as a computational technique, but it is difficult to obtain the solutions for the case of the fluid field unable to separate into recatangular domains.

In this paper, the problems related to the dynamic response of a moored small body are calculated by using the boundary element method in order to solve the hydrodynamic problems which have been difficult to be solved by the other traditional methods because of the complex geometries of the bodies or boundary conditions. In order to examine the applicability of the method, the numerical solutions obtained by this method are compared with those obtained by other methods. Finally, the aplicability of diffraction theory is evaluated for the moored small bodies as the first stage.

2. The equations of motion of a moored body

Fig. 1 shows the sectional view of a floating body and the co-ordinate system in which a right handed co-ordinate system(x, y, z) fixed with respect to the mean position of the body is used, where x is the horizontal distance from the origin taken on the still water level, z the vertical distance upwards through the center of gravity of the body from the origin. In the case of two-dimensional analysis, the body has three degrees of freedom of motion. Introduced the translatory displacements in the x and z direction respectively, D_1 is the sway displacement and D_2 the heave one. Furthermore, taken the angular displacement of the rotation about y-axis, D_3 is the roll angle.

Assuming that the responses for a body which is symmetric with respect to the x-z plane are linear and harmonic, the linear frequency domain equations of motion in regular incident waves of small amplitudes can be written as follows :

$$(\vec{m}+\vec{M})\frac{d^2\vec{D}}{dt^2}+\vec{N}\frac{d\vec{D}}{dt}+(\vec{C}+\vec{K})\ \vec{D}=\vec{F}e^{-i\omega t} \quad (1)$$

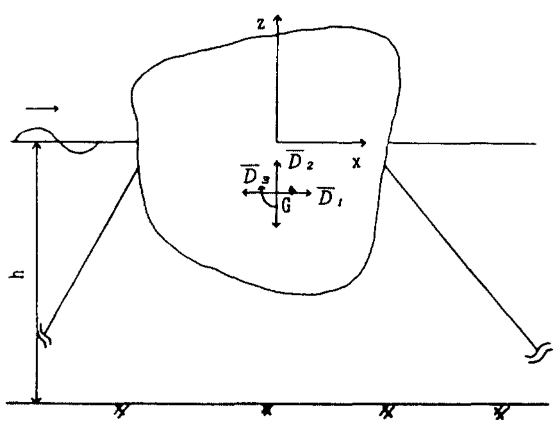

Fig. 1 Description of displacements

where, $\vec{m}=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$

: the masses or the moment of inertia of the body

$$\vec{M}=\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix},\ \vec{N}=\begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix}$$

: the added mass and damping coefficients, respectively

$$\vec{C}=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix},\ \vec{K}=\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

: the hydrostatic restoring coefficients and the spring factors, respectively

$$\vec{F}=\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

: the complex amplitudes of the wave exciting forces and moment

i : $\sqrt{-1}$

ω : the frequency of the waves and the same as the frequency of the response

Each component in Eq. (1) may be written as follows.

Sway :

$$(m+M_{11})\ddot{D}_1+N_{11}\dot{D}_1+K_{11}D_1+M_{13}\ddot{D}_3+N_{13}\dot{D}_3+K_{13}D_3=F_1e^{-iwt} \quad (2)$$

Heave :

$$(m+M_{22})\ddot{D}_2+N_{22}\dot{D}_2(\rho gA_0+K_{22})D_2=F_2e^{iwt} \quad (3)$$

Roll :

$$(I_{33}+M_{33})\ddot{D}_3+N_{33}\dot{D}_3+(\rho gV_0\overline{GM}_R+K_{33})D_3+M_{31}\ddot{D}_1+N_{31}\dot{D}_1+K_{31}D_1=F_3e^{-iwt} \quad (4)$$

where, ρ : density of water

g : gravitational acceleration

A_0 : width at the waterline

V_0 : immersed volume per unit length

$\overline{GM}_R$: metacentric height of the roll motion

In addition, the first index in M, N, K and I denotes the direction of the force(or moment) and the second one the mode of the motion. In the case of a body symmetric with respect to the x-z plane, it follows that $I_{kj}=M_{kj}=N_{kj}=C_{kj}=K_{kj}=0$ ($k \neq j$, except that $kj=13$, 31). The dots stand for time derivatives so that $\dot{D}_k$ and $\ddot{D}_k$ are velocity and acceleration terms, respectively.

3. The hydrodynamic boundary value problem and Green funtion solution

It is assumed that the fluid is two-dimensional inviscid and incompressible. The depth h is finite and constant, and the free surface is infinite in x direction. The motion of both the body and the fluid is assumed to be small so that the body boundary and free surface conditions can be linearized.

For the case of wave interaction with the restrained body, it is convenient to express the velocity potential as

$$\phi_E e^{-iwt}=(\phi_0+\phi_4)e^{-iwt} \quad (5)$$

where $\phi_0 e^{-iwt}$ denotes the incident wave potential and $\phi_4{}^{-iwt}$ the diffracted wave potential for the restrained body. The velocity potential for the small amplitude incident waves is given by

$$\phi_0 e^{-iwt}=\frac{gH\cosh k(h+z)}{2\omega\cosh kh}e^{i(kx\cos\alpha-wt)} \quad (6)$$

where H is the wave height, α the direction of incident waves, k is the wave number which is related to the frequency of the waves by the dispersion relationship

$$\frac{\omega^2}{g}=k\tanh kh \quad (7)$$

The total velocity potential associated with the incident wave and scattered wave as well as the motion of the body is periodic in time and in the z direction and then, may be expressed in the form

$$\phi=\phi_E e^{-i\omega t}+\sum_{l=1}^{3}\phi_l\,\dot{D}_l \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (8)$$

where $\phi_E e^{-i\omega t}$ is the wave exciting potential and ϕ_l is the contribution to the velocity potential from the l-th mode of motion.

All the velocity potentials must satisfy the Laplace equation and it can be shown that $\phi_j(j=0\sim 4)$ must satisfy

$$\Delta\phi_j=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (9)$$

in the fluid domain

The boundary conditions are given by

$$-\omega^2\phi_j+g\frac{\partial\phi_j}{\partial t}=0 \qquad \text{at } z=0 \ \cdots\cdots \ (10)$$

$$\frac{\partial\phi_j}{\partial z}=0 \qquad \text{at } z=-h \ \cdots \ (11)$$

Moreover, $\phi_l\dot{D}_l(l=1\sim 3)$ and $\phi_4 e^{-i\omega t}$ satisfy a radiation condition and the following body boundary conditions on the average position of the wetted surface of the body, respectively, that is ;

$$\frac{\partial\phi_j}{\partial n}=n_j \qquad j=1\sim 3 \quad \cdots\cdots\cdots\cdots \quad (12)$$

and

$$\frac{\partial\phi_4}{\partial n}=-\frac{\partial\phi_0}{\partial n} \qquad \text{on } C(x,\ z)=0 \ \cdots\cdots \ (13)$$

where $\vec{n}$ is in the direction of the outward normal to the surface of the body and $\frac{\partial}{\partial n}$ the normal gradient. In addition, $C(x,\ z)=0$ represents the immersed contour of the cross-section of the body. The quantities n_j, are defined as

$$n_1=n_x$$
$$n_2=n_z \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (14)$$
$$n_3=(z-z_0)n_x-(x-x_0)n_z$$

where n_x and n_z are x and z components of the unit normal vector on the immersed surface and x_0, z_0 the center co-ordinate about which the motion in roll is prescribed and the roll moment is solved, respectively. The condition which all of the potentials must approach outgoing regular water waves far from the axis of the body, is given by

$$\lim_{R\to\infty} R^{1/2}\left(\frac{\partial\phi_j}{\partial R}-ik\phi_j\right)=0 \qquad j=1\sim 4$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (15)$$

where R is the horizontal distance from the origin. It is possible to show that the solution of ϕ_j $(j=1\sim 4)$ may be written as

$$\phi_j=\int_C f_j(X,Z)\frac{\partial G(x,z\,;\,X,Z)}{\partial n}dc,\ j=1\sim 4$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (16)$$

where f_j denote the unknown source strength functions, dc a differential arc length along C and (X,Z) a point on C. In addition, a point(x,z) is generally taken in the fluid region. G is the Green's function(source potential of unit strength) for the problem. It can be written in two ways as follows[10] ;

$$G(x,\,;X,Z)=\frac{1}{2\pi}ln\frac{r}{h}+\frac{1}{2\pi}ln\frac{r'}{h}+2PV\int_0^\infty\frac{dk}{k}\times$$
$$\left\{\frac{cosk(x-X)}{k_0coshkh-ksinhkh}coshk(z+h)coshk(Z+h)-e^{-kh}\right\}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (17)^{11)}$$

and

$$G(x,z\,;X,Z)$$
$$=-\frac{i}{k}\frac{k^2-k_0^2}{(k^2+k_0^2)h+k_0}coshk(h+z)coshk(h+Z)$$
$$\times e^{ik|x-X|}-\sum_{n=1}\frac{1}{k_n}\frac{k_n^2+k_0^2}{(k_n^2+k_0^2)h-k_0}$$
$$\times cosk_n(h+z)cosk_n(h+Z)\times e^{-k_n|x-X|}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (18)$$

where,
$r=[(x-X)^2+(z-Z)^2]^{1/2}$, $r'=[(x=X)^2+(z+Z+2h)^2]^{1/2}$ and PV indicates a principal value integral in Eq. (17), and k_n is the solutions of the equation

$$k_n \tan k_n h + k_0 = 0 \quad \cdots\cdots (19)$$

and

$$k_0 = \frac{\omega^2}{g} \text{ (wave number in deep water)} \quad \cdots\cdots (20)$$

in Eq. (18).

The source densities f_j in Eq. (16) must be determined so as to satisfy the body boundary conditions (12) and (13), that is ;

$$\int_C f_j \frac{\partial G}{\partial n} dc = \begin{cases} n_j & j=1\sim3 \\ -\dfrac{\partial \phi_0}{\partial n} & j=4 \end{cases} \quad \cdots\cdots (21)$$

Consequently, the normalized potentials ϕ_j may be determined by substituting the solution of source densities f_j into Eq. (16).

By definition, M_{kj} and N_{kj} $(k,j=1\sim3)$ which are the added masses and the damping coefficients, respectively, can be expressed as follows ;

$$M_{kj} = -\rho \, Re[\int_C \phi_n n_k dc] \quad \cdots\cdots (22)$$

$$N_{kj} = -\rho\omega \, Im[\int_C \phi_n n_k dc] \quad \cdots\cdots (23)$$

where Re and Im mean the real and imaginary parts, respectively.

The wave exciting forces $F_k (k=1\sim3)$ are obtained from $\phi_E e^{-i\omega t}$ by using the linearized Bernoulli's equation to obtain the pressure and integrating this pressure properly over the body surface C. Finally, taking Eqs. (22), (23) and the wave exciting forces $F_k (k=1\sim3)$ mentioned above into account, the displacements D_1, D_2, D_3 can be solved by using Eqs. (2), (3) and (4), and then the velocity potential expressed by Eq. (8) is also determined. As the result, it is possible to find the pressure at any point on the body. This may be used as the dynamic load in a quasi-static structural response calculation. Further, the free surface elevation at any point, the fluid velocity and acceleration can also be readily obtained.

4. Numerical results and comparison with experimental data

In order to verify the validity of the boundary element method, comparisons with other authors' computational results[5,7] have been conducted for a free floating body. The computational results are those using the three-dimensional finited element method, the two-dimensional potential matching method and the two-dimensional B.E.M. for the same body, respectively.

Computational data used are given in Table 1. Figs. 2~4 represent the non-dimensional exciting forces and moment. The results of the B.E.M. presented show good agreement with those of the P.M.M., while those of the F.E.M. obtained by three-dimensional analysis show a little difference from the other two, although they have the same tendencies.

Hydrodynamic coefficients are presented in Figs. 5~10. These also show the same tendencies, especially in the results of the B.E.M. and the P.M.M. excluding $\bar{N}_3$, $\bar{N}_1$ and $\bar{N}_2$ show large differences among the F.E.M., the B.E.M., and the P.M.M. at small values of ξ_B. This indicates that the use of two-dimensional hydrodynamic coefficients would be predicted to result in exaggerated responses as the values of ξ_B decrease.

The non-dimensional motions of the free floating body are presented in Figs. 11~13. In the case of roll only, resonance of the B.E.M. appears at larger values of ξ_B than the others, although they generally have the same tendencies.

Table 1. Computational data

body & wave	unit	quantity
L×B×d	(m)	1×50×10
mass	(kg)	0.5×10^{6}
moment of inertia	(kg · m^4)	$I_{yy}=0.242\times10^{9}$
metacentric height	(m)	$GM_y=15.8$
center of gravity at undisturbed free surface	(m)	(0.0, 0.0, 0.0)
wave height	(m)	2.0
period	(sec)	10~30
wave direction	(°)	0
water depth	(m)	20

The non-dimensional coefficients are given in Table 2 in detail.

Table 2. Non−dimensional coefficients

$\xi_B=w^2/g(B/2)$	
$\bar{E}_j=E_j/\{\rho g\zeta_a(B/2)\}$	j=1~3
$\bar{M}_j=M_j/(\rho Bd)$	j=1~2
$\quad=M_j/\{\rho Bd(B/2)^2\}$	j=3
$\bar{N}_j=N_j/\{w\rho Bd\}$	j=1~2
$\quad=N_j/\{\omega\rho Bd(B/2)^2\}$	j=3
$\bar{D}_j=D_j/\zeta_a$	j=1~2
$\quad=(B/2)\ \theta/\zeta_a$	j=3
E_j : wave exciting forces or moments	
ζ_a : incident wave amplitude	
M_j : added mass and added moment of inertia coefficients	
N_j : damping coefficients	
θ : rotational angle about y axis	

A series of laboratory tests were carried out in a wave flume 24m long, 0.6m wide and 1.1m deep. The water depth was constantly maintained in 0.40m. The wave periods were varied from 1.07 sec to 2.07 sec and the wave heights from 4.02cm to 6.27cm. A 1/20 scale ratio model used in experiments was presented in Table 3. The movements of the body were recorded on the video tape through the glass side wall of the wave tank. The images on the display were converted into digital data by means of a video image analyzing system. The water surface elevations were also measured.

Table 3. A 1/10 model

body & wave	unit	quantity
L×B×d	(cm)	16.4×11×2.04
mass	(g)	14.28
moment of inertia	(g · m^4)	$I_{yy}\doteq79.364$
center of gravity at undisturbed free surface	(cm)	(0.0, 0.0, -0.28)
wave height	(cm)	4.02~6.27
period	(sec)	1.07~2.07
wave direction	(°)	0
water depth	(cm)	40
wave length	(cm)	162.95~384.04
B/wave length	non-dim.	0.03~0.08

Fig. 14 shows the non-dimensional sway motion. The experimental results show larger values than those of the computational results at short periods while smaller ones at long periods.

Figure 15 shows the non-dimensional heave motion. The experimental results are in good agreement with twice the computational results.

It seems that the discrepancy between the results in Figs. 14 and 15 is mainly due to the fact that the spring factors were determined as arbitrary constants.

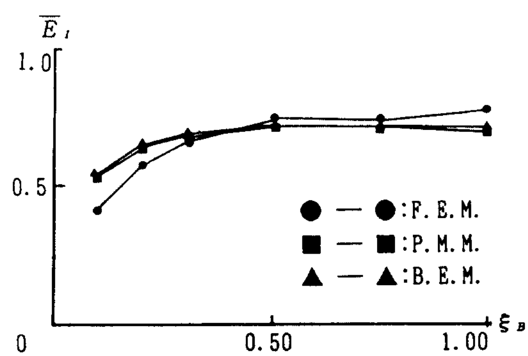

Fig. 2 Sway force coefficients

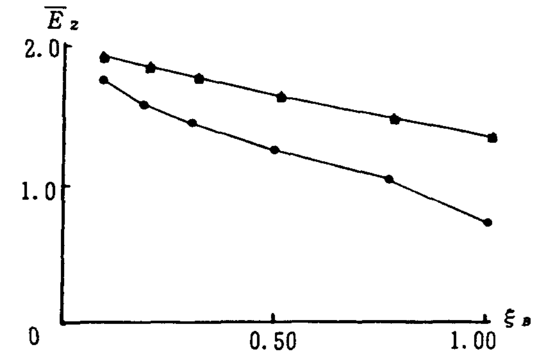

Fig. 3 Heave force coefficients

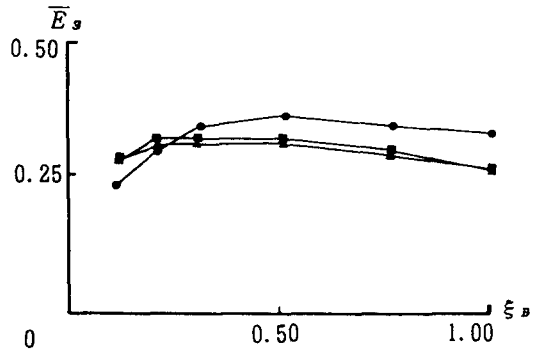

Fig. 4 Roll moment coefficients

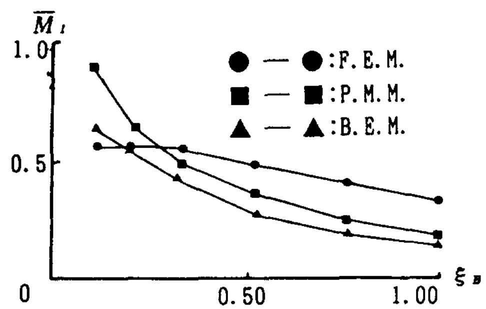

Fig. 5 Sway added mass coefficients

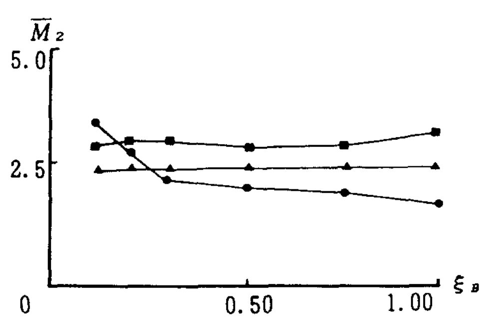

Fig. 6 Heave added mass coefficients

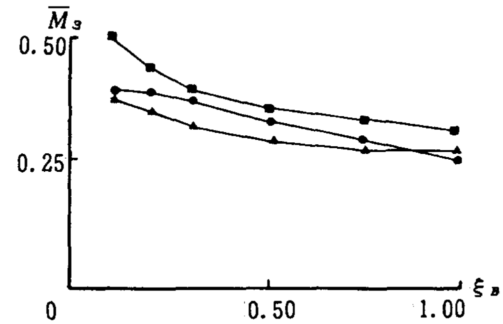

Fig. 7 Roll added mass coefficients

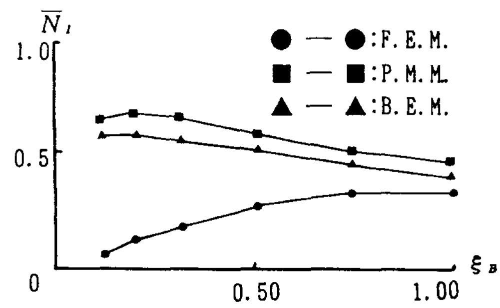

Fig. 8 Sway damping coefficients

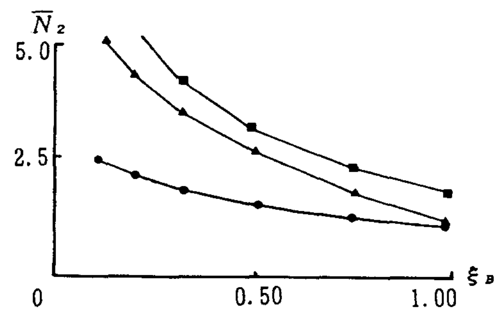

Fig. 9 Heave damping coefficients

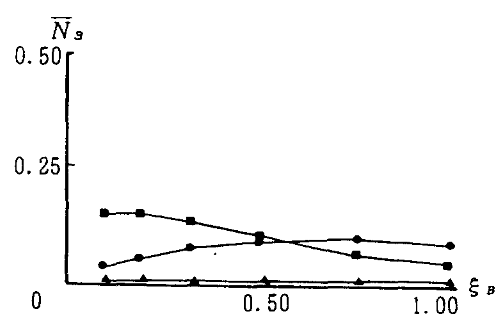

Fig. 10 Roll damping coefficients

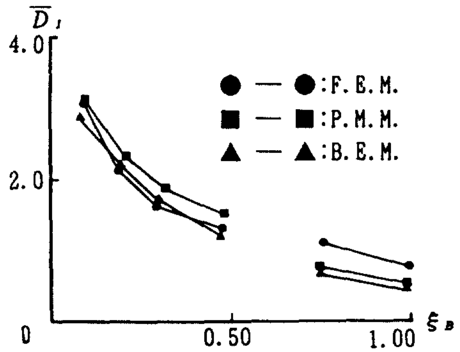

Fig. 11 Sway motions

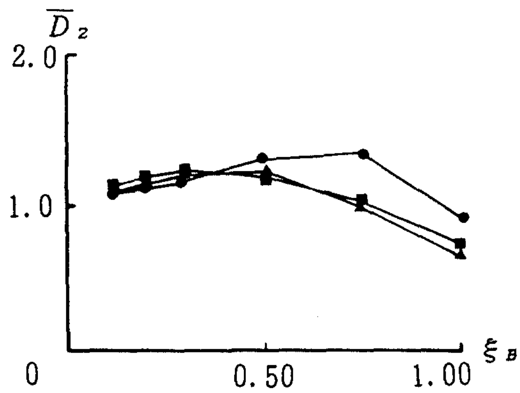

Fig. 12 Heave motions

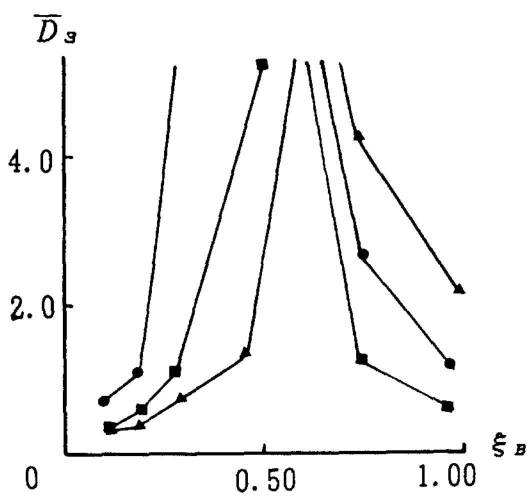

Fig. 13 Roll motions

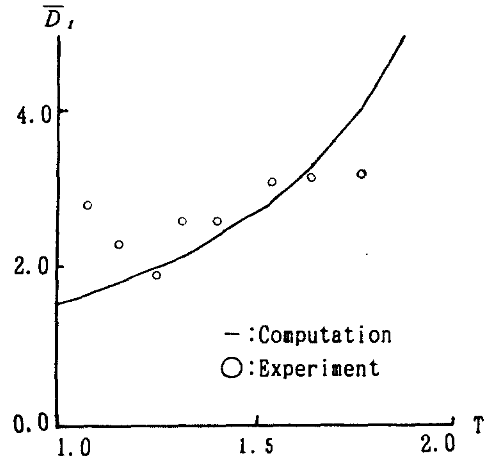

Fig. 14 Sway motion of a moored body

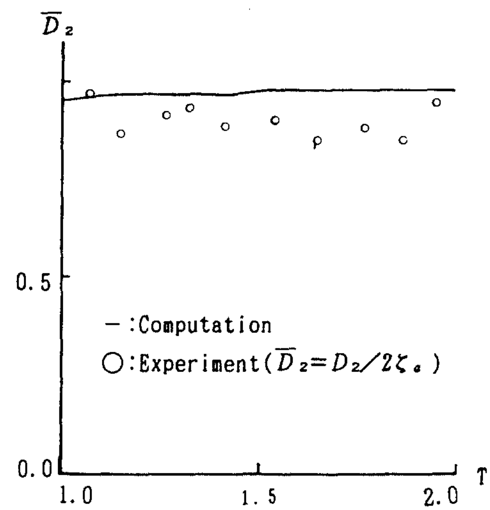

Fig. 15 Heavey motion of a moored body

5. Conclusions

The results for two-dimensional B.E.M. show that diffraction theory is applicable to structures smaller than 0.15 of d/L for rectangular ones.

Although both computational results and experimental ones generally have the same tendencies, a little diferences between them seem to result from using the the assumed values of the spring constants in computation.

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