

◎ 論 文

# Design and Experimental Evaluation of Sliding Mode Controller for Nonlinear Autonomous Underwater Vehicle

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비선형 무인잠수정을 위한 슬라이딩 모우드 조종기 설계 및 실험적 고찰

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**Key Words :** AUV(autonomous underwater vehicle 無人潛水艇), Sliding mode(슬라이딩 모우드), Sliding surface(슬라이딩 표면), Nonlinear feedback gain(비선형 피드백 이득)

## 抄 錄

비선형성 및 측정할 수 없는 외란에 영향을 받은 무인잠수정의 깊이 조종을 위한 슬라이딩 모우드 조종기를 설계하였다. 먼저, 선형화 된 운동방정식을 기초로 하여 슬라이딩 표면계수를 수치해석으로 최적화 시켰으며, 이 설계된 슬라이딩 표면을 비선형 운동방정식에 적용하여, 그 특성을 고찰하였다. 마지막으로, 용이하게 설계된 슬라이딩 모우드 조종기를 비선형 성과 외란을 갖는 NPS(Naval Postgraduate School)형태의 무인잠수정에 적용하여 얻어진 실험치의 동적 특성을 통해 슬라이딩 모우드의 강인성을 확인하였다.

## Abstract

A sliding mode controller for depth control of autonomous underwater vehicles(AUV), which have highly nonlinear terms, uncertainties and disturbances is designed. First, Sliding surface coefficients of the controller based on the linearized equation is optimized by numerical simulation. Second, the controller is applied to nonlinear equation and evaluated for system response. Finally, an experimental verification was attempted on the NPS prototype vehicle with nonlinear terms and disturbances.

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### 1. Introduction

There has been an increased interest recently on the need for autonomous underwater vehicles in both Navy and private industry. A variety of unclassified missions includes decoy, survey, reconnaissance, and ocean engineering work services. However, the dynamics of underwater vehicles are described by highly nonlinear systems with uncertain coefficients and disturbances that are difficult to measure. Those problems must be solved to make robotic underwater vehicle reality. We are pursuing a methodology called sliding control<sup>1)</sup> to satisfy these requirements. These techniques have been applied to other difficult nonlinear control system design problems, particularly the trajectory control high-speed robot<sup>1)</sup>.

The objective of this paper is to introduce a new robust tracking control scheme for AUV which have uncertainties in dynamics and enviromental uncertain disturbance.

Designed controller is estimated by using maneuvering parameters of AUV change with operating conditions through computer simulations and experiments.

### 2. The Dynamic Model of Underwater Vehicle

Motions of underwater vehicles are expressed in a body fixed reference frame such as Figure 1, because hydrodynamic forces and inertia properties are most readily computed in a body reference frame. The nonlinear model used for the verification of sliding mode control in this paper was derived from the original NSRDC 2510 document<sup>2)</sup>. The sliding mode controller design procedure begins with the expression of the equation of motion in linear time invariant state

space form. The highly nonlinear AUV system is

$$\frac{d}{dt} x(t) = M^{-1}f(x(t), u(t)) \dots\dots\dots (1)$$

$$y(t) = g(x(t)) \dots\dots\dots (2)$$

where  $x$  is the state vector,  $u$  is the control input vector, and  $y$  is the output vector. The nonlinear equations (1) can be linearized through a Taylor series expansion in the vicinity of a nominal point for small deviations and significantly simplified as in Larsen<sup>3)</sup> in order to design of sliding surface. The linearized and very simplified equations for the X-Z plane motion

$$\dot{x} = Ax + Bu \dots\dots\dots (3)$$

where  $x \in R^n$  is the state vectors,  $u \in R^m$  is the control inputs, and  $A$  and  $B$  are the known elements.

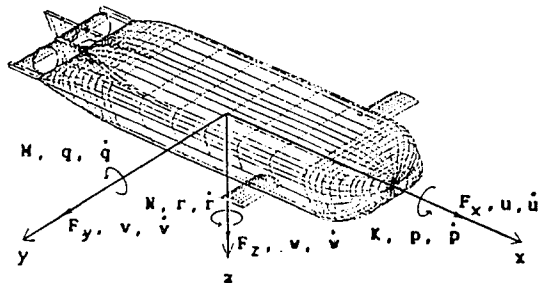


Figure 1. Body fixed coordinate reference frame

### 3. Sliding Surface Design

Sliding mode control offers the control designer new possibilities for improving the quality of the control in comparison with a fixed structure system. The basic idea is to design a controller structure which consists of a set of continuous subsystems together with suitable switching logic<sup>4)</sup>. The sliding surface has a very important property that determines desired dynamics of the

closed-loop system. The sliding surface should be designed so that system response restricted to  $\sigma(x)$  has a desired behavior, such as asymptotically stable state or tracking error. The sliding surface of the equation (3) has the following form<sup>1)</sup> :

$$\dot{\sigma}(x) = S^T \dot{x} = 0 \dots\dots\dots (4)$$

where S is the sliding surface coefficient(nxm). A necessary condition for the asymptotic stability of the system (3) is to have degenerating Liapunov function in the following form<sup>1)</sup> :

$$V(x) = 1/2[\sigma(x)]^2 \dots\dots\dots (5)$$

The asymptotic stability of the system (3) is guaranteed provided that  $\dot{V}(x)$  is negative definite. If  $\sigma(x, t)$  is a sliding surface by the definition above, it follow that

$$1/2 \frac{d}{dt} \sigma^2(x, t) \leq -\eta^2 | \sigma(x, t) | \dots (6)$$

where  $\eta$  is the sliding control gain. It will guarantee stability of the sliding mode motion. Using the method of equivalent control,<sup>6)</sup> and the closedloop dynamics of the linear model are

$$\dot{\sigma}(x)=0 = \frac{d\sigma}{dx} \frac{dx}{dt} = S^T \dot{x} = 0 \dots\dots\dots (7)$$

$$\dot{x} = [A - B(S^T B)^{-1} S^T A] x \dots\dots\dots (8)$$

Equation (6) gives the dynamics of the system on the sliding surface for  $t \geq t_0$  given  $\sigma(x)=0$ , but the S matrix is unknown. In order to determine S matrix, the equation (8) can be rearranged in the following form :

$$\dot{x} = [A \div BK_c] x \dots\dots\dots (9)$$

where  $K_c = (S^T B)^{-1} S^T A$ . The  $K_c$  matrix can be obtained from the pole placement<sup>7)</sup> for which we can select specifically desired closed-loop poles of the system equation (8) on the sliding surface. The left eigenvector of the  $[A - BK_c]$  matrix of equation (9) corresponding to a pole placed at the

origin are the sliding surface coefficients which the desired behavior system on the sliding surface.<sup>8)</sup>

### 4. Sliding Mode Control Law

Given the dynamic model, sliding surface definition, and the stability criteria, a suitable control law can be obtained. If  $u(t)$  could be chosen so as to keep the trajectory on  $\sigma(x, t)=0$ , we would have the sliding control law from the sliding condition equation (6) and sliding surface equation (4) that

$$\begin{aligned} \dot{\sigma}(x) &= S^T \dot{x} = -\eta_o^2 \text{sign}(\sigma) \\ S^T A + S^T B u &= -\phi_o^2 \text{sign}(\sigma) \\ u &= -(S^T B)^{-1} S^T A x - (S^T B)^{-1} \eta_o^2 \text{sign}(\sigma) \dots (10) \end{aligned}$$

where  $\eta_o^2$  is a arbitrary nonlinear feedback gain. The control law has two part.

$$\begin{aligned} u &= \bar{u} + \hat{u} \dots\dots\dots (11) \\ \text{where } \bar{u} &= -(S^T B)^{-1} \eta_o^2 \text{sign}(\sigma), \hat{u} = -(S^T B)^{-1} S^T A x \end{aligned}$$

where  $\hat{u}$  is linear feedback control law and  $\bar{u}$  is nonlinear feedback control law. The discontinuity of the nonlinear feedback control law results in a chattering type of control action that would be very undesirable for most systems and this chattering behavior has been one of the main reasons sliding control techniques have not been more widely applied. This problem will be solved by smoothing out the control law in a thin boundary layer around the sliding surface<sup>1)</sup> Large enough gain( $\eta^2$ ), sliding surface coefficients and boundary layer thickness( $\phi$ ) could be optimized in the computer simulation by using commercial package(Matrix-X) and fortran program to be minimized error and without chattering due to sign switching of the nonlinear part of the control law.<sup>9)</sup> The characteristic of dynamic response is shown in Figure 2.

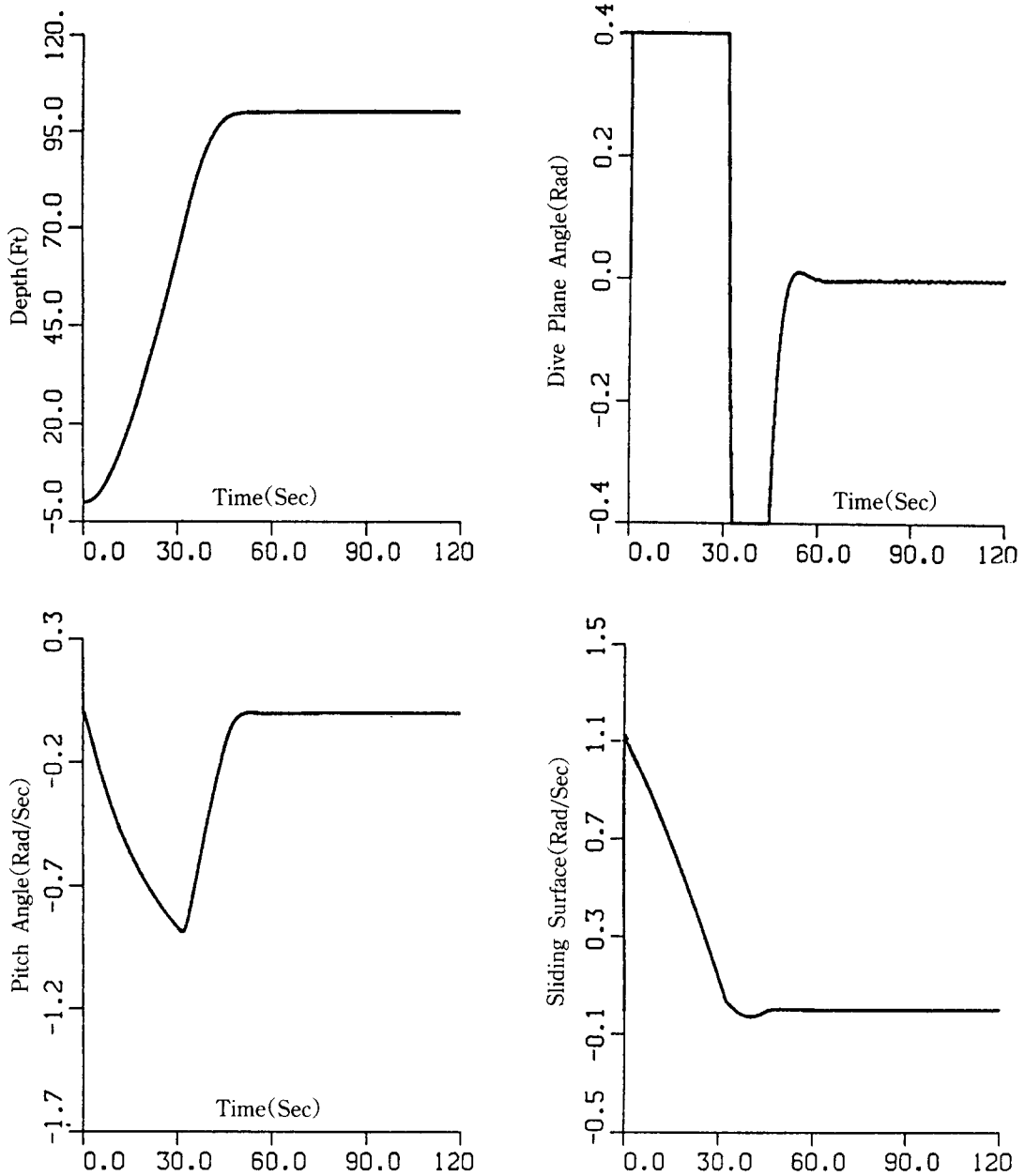


Figure 2. Dynamic response of system without chattering

## 5. Sliding Mode Control with Bounded Uncertainties

The appeal of sliding mode is based on its

ability to treat nonlinear, time varying and unmodeled systems in a straightforward manner.<sup>9)</sup> It is the purpose of this section to apply the linear sliding mode controller to the nonlinear model of the actual AUV. The determination of

sliding surface and control law gain for the actual AUV are based on the linearized nominal model. The mathematical form of the nonlinear model for the actual AUV is

$$\frac{dx}{dt} = Ax + \Delta f(x) + B(u) + d(x, t) \dots\dots (12)$$

where  $\Delta f(x)$  is the uncertainty of the nonlinear function  $f(x)$ ,  $B(x)$  is the nonlinear function associated with the control surface and actuator system, and  $d(x, t)$  is the uncertainty disturbance. The system matrix  $A$  is the estimate of  $f(x)$  and the magnitude of the uncertainty is bounded as

$$F \geq | S^T \Delta f(x) | \dots\dots\dots (13)$$

where  $F = [F_1 \ F_2 \ \dots \ F_n]^T$ . The individual bounds on any element of  $\Delta f(x)$  are estimated from some knowledge of the extremes of possibility of  $\Delta f(x)$ . Also, let  $B(x)$  be approximated by  $B$ , a constant, where the varying gain  $\beta$  is defined by  $B = \beta B(x)$ , and  $\beta$  is taken to be scalar, but bounded within the limits of  $\beta_{min} < \beta < \beta_{max}$ , and  $\beta_{nominal} = 1$ . Disturbance  $d(x, t)$  is unknown but is upper bounded by a known continuous function such that :

$$D \geq | S^T d(x, t) | \dots\dots\dots (14)$$

The true dynamics of the sliding surface with uncertainty are, however, given by

$$\begin{aligned} \dot{\sigma} &= S^T \dot{x} \\ &= S^T [Ax + \Delta f(x) + \beta Bu + d(x, t)] \dots\dots (15) \end{aligned}$$

Substituting equation (10) to the above equation for  $u$ , then the derivatives of  $\sigma$  is,

$$\begin{aligned} \dot{\sigma} &= (1 - \beta^{-1}) S^{-T} Ax + S^T \Delta f(x) + S^T d(x, t) \\ &- \beta^{-1} \eta^2 \text{sign}(\sigma) \dots\dots\dots (16) \end{aligned}$$

From the above, stability is guaranteed, if and only if

$$\dot{\sigma}(x) \geq \eta^2 \text{sing}(\sigma) \dots\dots\dots (17)$$

0 where  $\eta^2$  is the nonlinear feedback gain without an uncertainty. If the control matrix  $B$  is exactly known ( $\beta = 1$ ), then control law

$$u_b = -[S^T B]^{-1} S^T Ax - [S^T A]^{-1} \eta^2 \text{sign}(\sigma) \dots\dots\dots (18)$$

where  $\eta^2 \geq | \eta_0^2 + F(x) + D(x, t) |$ , can guarantee stability and perfect tracking for the nonlinear system with constant control matrix of the AUV. In case where the control system  $B(x)$  is uncertain, the following change ( $\eta^2$ ) must be made :

$$u = \hat{u} + \bar{u} \dots\dots\dots (19)$$

where  $\hat{u} = -[S^T B]^{-1} S^T Ax$ ,  $\bar{u} = -[S^T B]^{-1} \eta^2 \text{sign}(\sigma)$  and  $\eta^2 \geq \beta_{max} | \eta_0^2 + F(x) + D(x) | + | (\beta_{max} - 1) | | S^T Ax |$ . Since the nonlinear term of the  $B(x)$  and  $\Delta f(x)$  is uncertain in most cases, it is assumed to be zero in equation(12) and  $\eta^2$  is increased depending on their assumed bounds to guarantee stable sliding mode control.<sup>9</sup> The actual control law used in subsequent simulation has in fact form of equation (19) with sufficiently large  $\eta^2$  to accommodate the uncertainty in  $\Delta f(x)$  and  $B(x)$ . The sliding mode switching control law equation (19) guarantees that equation (13) is satisfied even in the presence of parameter variations and unmodeled dynamics provided  $\eta^2$  is large enough. The dive plane angle chattering due to modeling error and disturbances can be eliminated by defining a boundary layer thickness ( $\phi$ ) about  $\sigma = 0$ . The smooth sliding mode control law of the actual AUV is then

$$u = -[S^T B]^{-1} (S^T Ax + \eta^2 \text{satsign}(\sigma/\phi)) \dots\dots (20)$$

$$\text{where } \eta^2 \geq \beta_{max} | \eta_0^2 + F(x) + D(x) | + | (\beta_{max} - 1) | | S^T Ax | .$$

The dynamic response and dive plane angle obtained from nonlinear model simulation at 500 rpm are shown in Figure 3. Although the sliding

surface and control law gain based on the nominal linear system were applied to the

nonlinear system, the response of the system is satisfactory as expected. The designed variable

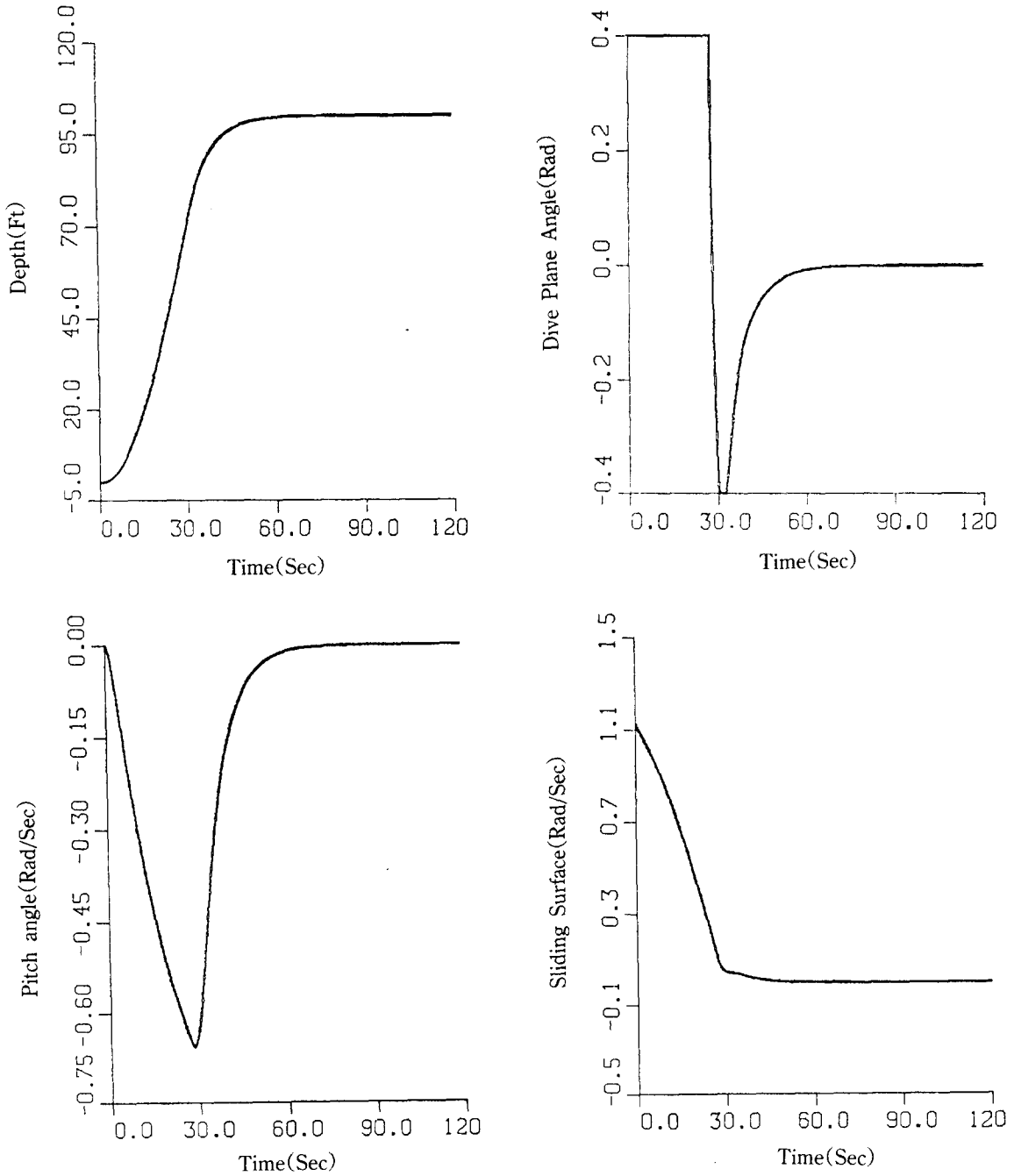


Figure 3. Dynamic responses of fully nonlinear system

structure system based on the nominal linear equations dealt with the full nonlinear dynamics of the AUV as shown in Figure 3. The technique of sliding mode control can be handle the nonlinear system directly without linearzation, if sliding surface coefficients are properly chosen.<sup>6)</sup> This is especially important for highly maneuverable underwater vehicles that can move in all directions.

### 6. Experiment Result and Conclusion

An experimental verification was attempted on the NPS prototype vehicle with coefficients the same as the ones used in<sup>8)</sup> A discrete time sliding mode controller was designed based on 25 Hz sample rate with controller poles at [0.9, 0.91, 1], then control law is

$$\begin{aligned}
 u &= -0.9q - 1.789 \theta - 21.2976 \eta^2 \text{sign}(\sigma) \\
 \sigma &= -0.3386 q - 1.6888 \theta + (z - z_{com}) \dots \quad (21)
 \end{aligned}$$

where  $\eta^2=0.2$  and  $\phi=1.0$ . The experimental results are shown in Figure 4, where the commanded depth was 5 Volts (1 ft corresponds to 3.1 Volts). The variable structure system was proven to be an attractive control system design method for autonomous underwater vehicles. The

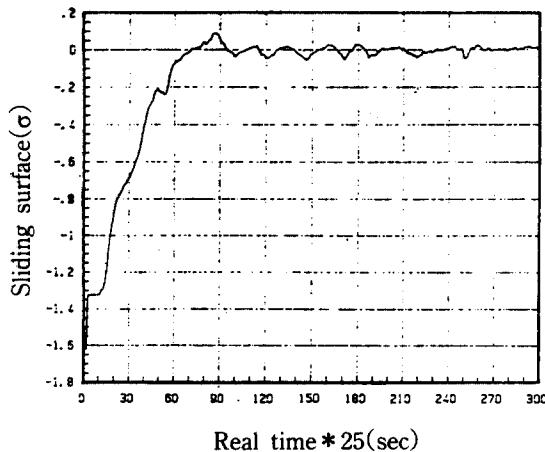
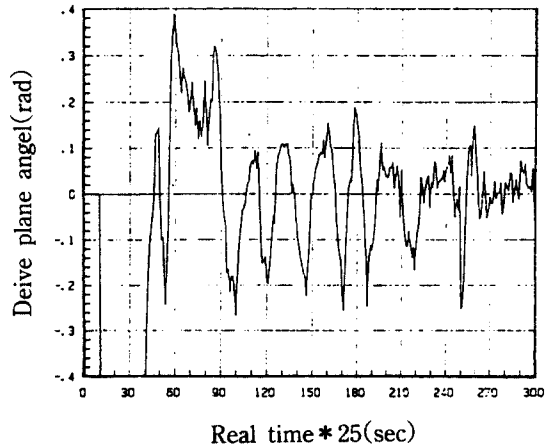
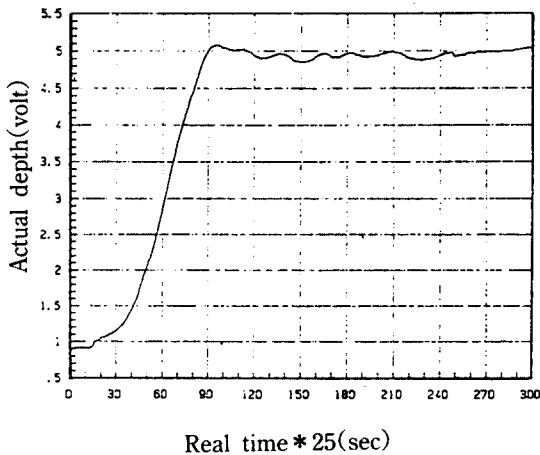


Figure 4. Experimental results of the discrete time sliding mode control

designed sliding mode controller based on this methodology dealt with the dynamic problems of the underwater vehicle with sufficient accuracy.

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