Experimental Study on the Mix Design Method using the Fracture Energy and the other Parameters in Concrete.

콘크리트의 파괴에너지와 다른 재료 특성을 이용한 배합설계법에 관한 실험 연구

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요 약

콘크리트 압축강도가 설계의 규준이 될 경우 배합비를 결정하는 방법은 여러가지가 있으나, 파괴에너지 및 탄성계수와 같은 규준이 주어질 경우 배합비 결정에 적용하는 방법은 거의 없다. 이를 위하여 본 연구는 콘크리트 재료성질의 관계에 관한 배합설계도(Mix design diagram)를 제안 하였다. 이 방법은 시멘트량, 물－시멘트 비가 콘크리트의 압축강도, 탄성계수, 할렬인장강도, 파괴에너지 그리고 콘크리트 특성길이(Characteristic length)에 주는 영향을 실험에 의하여 규명하였다. 시편제작을 위하여 각기 다른 물－시멘트비와 워커빌리티를 갖는 6종류의 무근콘크리트 배합이 사용되었다.

Abstract

There are many methods for determining a concrete mix proportion when the compressive strength is the design criterion, however there is much less information available when other criteria, such as the fracture energy or the elastic modulus, are specified. For these cases, a new mix design diagram, which is easy to use, has been developed from wellestablished concrete relationships. The application of this method is demonstrated by an experimental program which shows the influence of cement content, water-to-cement ratio and aggregate-to-cement ratio on the compressive strength, modulus of elasticity, splitting tensile strength, fracture energy, and characteristic length of concrete. Six concrete mixtures with different water-to-cement ratio and workability were studied.

Keywords : Fracture energy, mix design diagram, water-to-cement ratio compressive strength, modulus of elasticity, characteristic length, workability, slump value.

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1. INTRODUCTION

Fracture mechanics of concrete is entering a more mature stage where it can provide significant insight on the design of reinforced concrete. It has been particularly useful in the analysis of special structures such as large dams, wide-span bridges, prestressed pressure vessels, and unreinforced pipes. For this purpose it was necessary to establish suitable and reliable experimental methods to determine the fracture mechanics properties of concrete. Hillerborg(1) presented the theoretical basis of a method to determine the fracture energy GF of concrete, which was later adopted by RILEM(2). While researchers have been successful in the development of new test procedures to measure fracture energy, strain softnening and toughness of concrete(3, 4, 5), comparatively little has been done in developing methods of mix proportioning to obtain a given fracture mechanics property. It is not adequate to arbitrarily change the concrete composition and measure its resulting properties.

In this work, a mix design diagram is introduced for the complete and fast prediction of the fresh and hardened concrete properties. The diagram is based on three basic and classical concrpts of mix design: "Abrams' las" for hardened concrete, "Lyse's law" for fresh concrete, and "Molinari's law" for cement content. The first relationship was originally developed by Abrams(6) and generalized by Powers(7) for the compressive strength as a function of the water-to-cement ratio. Recent research has shown some limitations of this relationship for high-strength concrete where the strength of the aggregate and the aggregate-cement paste transition zone are important parameters. The work described here is limited to normal-weight, normal-strength concrete. The second relationship is based on the work of Lyse (8) who showed that when using the same concrete materials it is possible to obtain concretes with same consistency by keeping constant the ratio between the volume of water to volume of compacted fresh concrete. Using this rule it is possible to obtain fresh concretes with the same consistency but very different mix proportions and consequently different mechanical properties. Finally, the third relationship was formulated by Molinari(9) who correlated the cement content with the aggregate-to-cement ratio. Once the trial mixes are performed, these three relationships are arranged in a graphical form which permits the determination of a mix proportion for a specified property.

To demonstrate the utility and the prediction capacity of the proposed methodology, experimental results from six different mix proportions using the same materials are presented. The following mechanical properties for the mixes were determined: compressive strength, splitting tensile strength, modulus of elasticity, fracture energy and characteristic length. This study points out that when studying the effect of the mix parameters on the properties of concrete cetain constraints should be used. For instance when varying the water-to-cement ratio, the workability of fresh concrete should be kept constant and vice versa. Under these conditions the mix parameters may have different effects on the concrete properties; for example a decrease in cement content or an increase in aggregate content leads to an increase in fracture energy for a constant water-to-cement ratio and leads to a decrease in the fracute energy for a constant workability.

2. MIX DESIGN DIAGRAM

The concrete mixtures were obtained by using the mix design which combines three relationships developed for the properties of fresh and hardened concrete in one graph, as shown in Fig. 1. The mix design uses the following correlation:

1 Abrams' law:

correlates the concrete compressive strength with the water-to-cement ratio(by weight) for a determined level of cement hydration. Since the water-to-cement ratio is the most important variable in concrete, this relationship can also be extended to other properties such as modulus of elasticity, permeability and fracture energy. For a given workability, the equation which best fits this behovior law ishy:

$$f_c = \frac{k_1}{k_2^{w/c}} \text{ where;}$$

f_c=compressive strength(MPa),

k_1, k_2=constants which depend on the materials used,

w/c=water/cement ratio, by weight(kg/kg).

2 Lyse's law :

correlates the water/cement ratio with the (fine+coarse aggregates)/cement ratio(by weight). The equation that best fits this behavior low is:

$m = k_3 \cdot w/c + k_4$ where;

m=aggregates/cement ratio, by weight (kg/kg),

k_3, k_4=constants which depend on the materials used on the a given workability

w/c=water-to-cement ratio, in mass (kg/kg).

#3, Molinari law :

correlates the cement content, C, and the aggregates to cement ratio, m. The equation that best this behavior law is:

$$C = \frac{1000}{k_5 \cdot m + k_6} \text{ where;}$$

C=cement content(kg/m^3)

k_5, k_6=constants which depend on the materials used.

m=aggregates/cement ratio(kg/kg).

Fig. 1 indicates that for a given water-to-cement ratio it is possible to have different mix proportions using the same materials and to obtain different properties of fresh and hardened concretes as exemplified in Table 1. The water-to-cement ratio is the most important variable in the concrete mix proportion and it can significantly change the properties of hardened concrete. To avoid its influence when studying the effect fo other variables, such as cement content and aggregate to cement ratio, the water-to-cement ratio should kept constant. When the water-to-cement ratio is kept constant, any change in the concrete mix proportion causes a change in the consistency of the fresh concrete, modifying its slump as shown in Fig. 1 and Table 1.

To analyze the influence of the water-to-cement ratio on the properties of concrete, it is necessary to fix other variables such as consistency of fresh concrete, cement content, or aggregate content, Producing concretes with the same cement content or the same aggregate content and different water-to-cement ratio means changing significantly the consistency of these concretes.

It is recommendable that the water-to-cemet ratio should be modified using the criterion of maintaining the same consistency of fresh concrete. Using this condition, the influence of water-to-cement ratio on the properties of hardened concrete can be studied without the influence of other variables. For a given consistency of fresh concrete, measured by the standard slump test, it is possible to have different concretes by changing the mix proprotions as indicated in Fig. 2 and exemplified Table 2.

For normal strength concrete, the aggregate is stronger than both the matrix and the transition zone, therefore the strength of these two weaker phases, which is controlled by the water-to-cement ratio and degree of hydration, is the limiting parameter to the strength of concrete, as indicated in Fig. 1. the elastic modulus and fracture energy, however, are greatly influenced by other mix parameters, such a volume of aggregate or cement content. This is reflected in Fig. 3 where instead of having only one curve in the first quadrant as in the case of Fig. 1, a series of curves associated to the existing volume fraction of the phases are necessary.

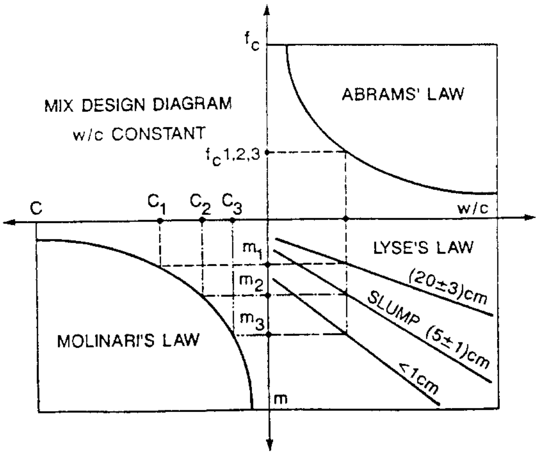

Fig. 1 Mix design diagram for a given w/ c ratio. Compressive strength as the design criterion.

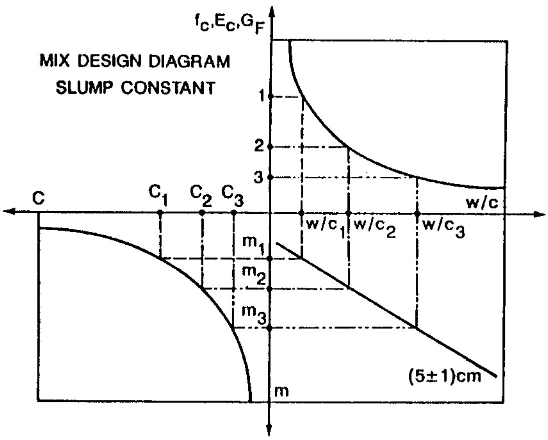

Fig. 2 Mix design diagram for a given consistency of fresh concrete. Compressive strength as the design criterion.

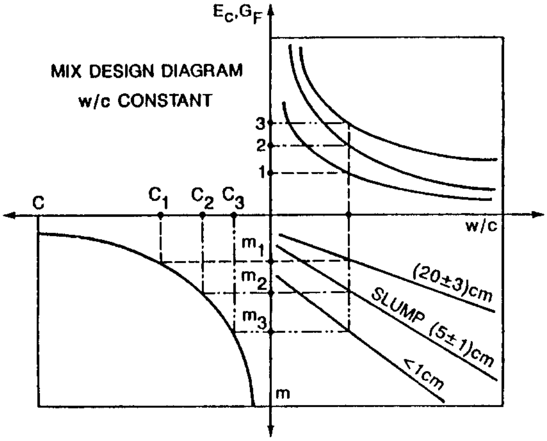

Fig. 3 Mix design diagram for given w/ c ratio. Modulus of elasticity or fracture energy as the design criteria.

Table 1. Example of mix proportions for a given water - to - cement ratio.

Water / Cement Ratio kg /kg	Consistence slump cm	Compressive Strength MPa	Aggregates / Cement Ratio kg /kg	Cement Content kg /m^3
w/c	fluid	f_c	low	high
w/c	plastic	f_c	medium	medium
w/c	dry	f_c	high	low

Table 2. Example of mix proportions for a given consistence of fresh concrete.

Consistence Slump cm	Water / Cement Ratio kg / kg	Compressive Strength MPa	Aggregates / Cement Ratio kg / kg	Cement Content kg / m^3
plastic	low	high	low	high
plastic	medium	medium	medium	medium
plastic	high	low	high	low

3. EXPERIMENTAL PROGRAM

3.1 Materials and mix proportions

ASTM Type Ⅰ-Ⅱ portland cement was used. A natural sand meeting ASTM C 33 was employed as the fine aggregate with maximum size of 2.4 mm, fineness modulus of 3.1, bulk specific gravity of 2640 kg / m^3, and water absorption of 1.45%

Natural siliceous river gravel meeting ASTM C 33 was used as the coarse aggregate with a maximum size of 9.5 mm, fineness modulus of 5.61, bulk specific gravity of 2680 kg / m^3, and water absorption of 0.34%.

For all the concrete mixtures the ratio (cement+fine aggregate) / (cement+fine aggregate+coarse aggregate) was kept constant at to 58%. The concrete mixtures are shown in Table 3.

Table 3. Mix proportions for fresh concrete

Mix Proportions	SI Units	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5	Mix 6
Cement	kg / m^3	478	398	329	282	306	484
Fine Aggregate	kg / m^3	765	866	929	976	1023	721
Coarse Aggregate	kg / m^3	902	920	914	911	961	875
Water	kg / m^3	196	199	198	198	153	212
Entrapped air	% vol.	0.09	0.76	0.63	0.51	0.57	0.65
m[(fine+coarse agg.).cem.]	kg / kg	3.49	4.49	5.59	6.69	6.50	3.30
Slump	cm	5±1	5±1	5±1	5±1	<1	20±3
Specific Weight	kg / m^3	2339	2365	2355	2355	2429	2307

3.2 Concrete beams and specimens

Following the RILEM recommendation(2), the concrete beams were cast with the following geometry:depth:(100±5)mm, width:(100±5)mm, length:(840±10), span : (800±5) mm.

ASTM standard test methods were followed for the other specimens that were cast at the same time of the beams. For each mix proportion five beams were cast for the fracture energy test, three cylinders for the compression test, two cylinders for the modulus of elasticity test, and three cylinders for the splitting tensile test. All cylinders had a 10-cm diameter and 20-cm height.

The specimens and beams were protected against evaporation. After demolding at 24 hours, all specimens and beams were stored in the fog-room at (20±2)℃ and R.H.>99%.

When the beams were 20-days old, they were notched at midspan with a length half the height. The beams and specimens stored in the fog-room conditions until less than 30 minutes before test. The beams were protected by a wet cloth against stress and crack formation.

3.3 Test methods

The modulus of elasticity was determined according to ASTM C 469, the compressive strength of according to ASTM C 39 and the splitting tensile strength according to ASTM C 496. These tests were performed in a 1335 KN BALDWIN Universal testing machine, with at 241 kPa per second rate of loading.

A special compressometer fabricated in the U.C Berkeley laboratory was used for reading the deformations in the modulus of elasticity test. Two linearly variable differential transformers(LVDTs) measured the deformations between the upper and lower rings, which were rigidly fastened to the cylinder specimens. The digital data were acquired using an IBM-PC/AT and a Keithley System 500 data acquisition system. The average longitudinal deformations of the specimens were measured using two LVDTs in each position. The LVDTs were conditioned and the signal was amplified using a Daytronic 300D Amplifier unit. The load was taken from the BALWIN Universal Testing Machine load cell and conditioned using a Daytronic 300D amplifier unit. The Data Acquisition Software (DAS) program was used to acquire, reduce, and plot the stress-strain data. The plot was displayed on an IBM enhanced graphic terminal, and printed out on an IBM printer. The data were sampled at 1 sec. intervals.

All beams were tested in three point bending mode according to the RILEM recommendation. The beam test was performed in a 133 KN MTS Servo-Hydraulic Universal Testing Machine, with an approximately constant rate of deflection, which was chosen so that the maximum load was reached within 30-60 seconds after the start of the test. The data were acquired by IBM-PC/AT system, and the were sampled at 1 sec. intervals.

4. RESULTS and DISCUSSION

4.1 Results

Results of compressive strength, modulus of elasticity, and splitting tensile strength at 28 days are presented at Table 4. Results of fracuture energy GF and characteristic length l_{ch} at 28 days are given on Table 5.

Table 4. Results of hardened concrete specimens at 28 days.

Properties	MIX1	MIX2	MIX3	MIX4	MIX5	MIX6
Compressive Strength (MPa)						
F_c1	53.4	47.2	38.0	26.5	48.5	47.6
f_c2	51.4	48.6	36.1	26.6	48.4	45.1
f_c3	54.1	47.2	36.2	25.6	48.2	47.4
f_c(average)	53.0	47.7	36.8	26.2	48.4	46.7
Modulus of Elasticity(GPa)						
F_c1	27.58	25.51	24.96	23.31	28.06	23.37
f_c2	27.24	25.86	25.03	22.96	28.13	24.62
E_c(average)	27.41	25.69	25.00	23.14	28.10	24.00
Splitting Tensile Strength(MPa)						
F_t1	4.2	4.1	3.6	3.1	4.2	3.9
f_t2	4.8	3.9	4.0	3.4	3.8	3.9
f_t3	4.3	4.0	3.9	3.5	4.2	4.0
ft(average)	4.4	4.0	3.8	3.3	4.1	3.9

4.2 Influence of the Water to Cement Ratio

The influence of the water-to-cement ratio on the concrete properties was determined by performing regression analysis on the exper-

Table 5. Fracture energy and characteristic length results

MIXTURE# (5 beams / Mixture	Water / Cement Ratio (kg/kg)	Slump (cm)	Modulus of Elasticity (GPa)	Splitting Tensile Strength (MPa)	Fracture Energy G_F (N/m)	Characteristic Length l_{ch} (mm)
MIX 1 av.	0.41	5±1	27.41	4.4	95.12	135
MIX 2 av.	0.50	5±1	25.69	4.0	84.57	136
MIX 3 av.	0.60	5±1	25.00	3.8	79.25	137
MIX 4 av.	0.70	5±1	23.14	3.3	69.83	148
MIX 5 av.	0.50	<1	28.10	4.1	91.64	153
MIX 6 av.	0.50	20±3	24.00	3.9	82.74	131

imental data. The most important relationships are the ones required to establish mix design diagrams such as the one presented in a conceptual format in Fig. 1. Mix design diagrams can be obtained by plotting the data from Tables 3, 4 and 5. In this work, they are established for the following properties: compressive strength-f_c, modulus of elasticity-E_c, and fracture energy-G_F. Fig. 4 shows the dependence of the compressive strength on the water - to - cement ratio, the aggregate to cement ratio, and the cement content for a given consistency of the fresh concrete. Least-square analysis shows a strong correlation among the variables when the equations presented in section 2 are used. Sometimes there is a tendency to propose new mathematical equations even when only limited experimental results are available. The author believe that only when there is a large number of experimental results, at least 35, a new formulation is justified. In this experimental program only six different mix proportions were used, therefore the behavior laws previously described described are used because they were based on the thousands of different mix proportions.

Fig. 5 and 6 show the mix design diagram obtained when the modulus of elasticity and the fracture energy are selected as performance criteria for the case of a constant consistency of fresh concrete. Once the consistency is kept constant it is possible to easily observe how the water-to-cement ratio influences the concrete properties.

The comparative analysis of Fig. 4, 5 and 6 shows the great influence of the water-to-cement ratio on the three properties studied. For example, even though mix 4 (Table 3) has the highest aggregate content it is the one that has the lower modulus of elasticity and the lowest fracture energy. This ob-

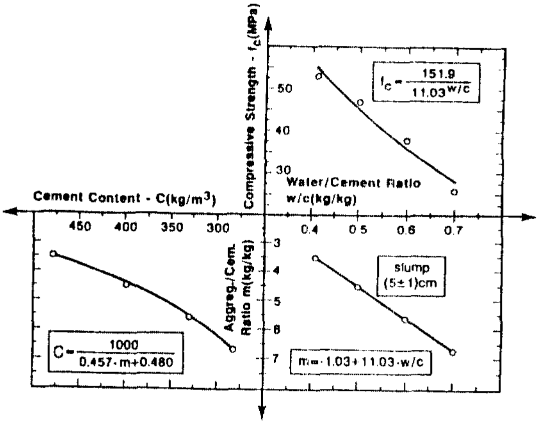

Fig. 4 Mix design diagram for a given consistency of fresh concrete. Compressive strength as the design criterion.

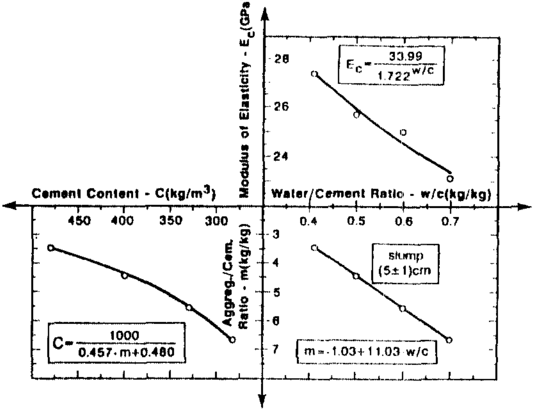

Fig. 5 Mix design diagram for a given consistency of fresh concrete. Modulus of elasticity as the design criterion.

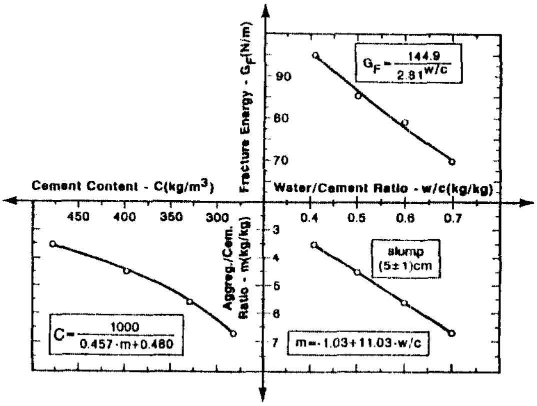

Fig. 6 Mix design diagram for a given consistency of fresh concrete. Fracture energy as the design criterion.

servation, apparently contradictory, is explained by the fact that both the modulus of elasticity and fracture energy mainly on the strength of the matrix and its bond to the aggregate.

4.3 Influence of cement content

The influence of the cement content on mechanical properties of concrete was studied under two conditions:concrete with same consistency and concrete with same water-to-cement ratio. Fig. 7-10 show the results for a constant consistency of fresh concrete equal to(5±1) cm slump. The compressive strength, modulus of elasticity, and fracture energy increases with increasing cement content, as indicated in Figs. 7 to 9. For a constant consistency, an increase in cement content reauires a decrease in the water-to-cement ratio and therefore producing a stronger and stiffer matrix.

Fig. 11 through 14 show the influence of the cement content on the mechanical properties of concrete for a water-to-cement ratio constant and equal to 0.50. It should be noted under this condition the dependency of the compressive strength, modulus of elasticity, and fracture energy to the cement content (Figs. 11-13) is significantly different from the condition of constant consistency(Figs. 7-9). Fig. 11 indicates that for a constant water-to-cement ratio the compressive strength is basically independent of the cement content, as expected from the Abram's and Power's models. An increase in cement content results in a reduction in the modulus of elasticity, as shown in Fig. 12. This result is expected because an increase in cement content leads to a reduction in the aggregate content and since the aggregate has a higher modulus of elasticity than the cement paste, reducing the aggregate content would cause a decrease in the elastic modulus of concrete. The fracture energy decreases with the cement content for a constant water-to-cement ratio, as shown in Fig. 13. The reason for such decrease is similar to the elastic modulus dependency, that is, there is less aggregate with increasing cement content. The characteristic length decreases with increasing cement content both for constant consistency and constant water-to-cement ratio, as indicated in Figs. 10 and 14. For the condition of constant consistency, this decrease is associated to lower water-to-cement ratio for higher cement consumption. Petersson(10) also verified that

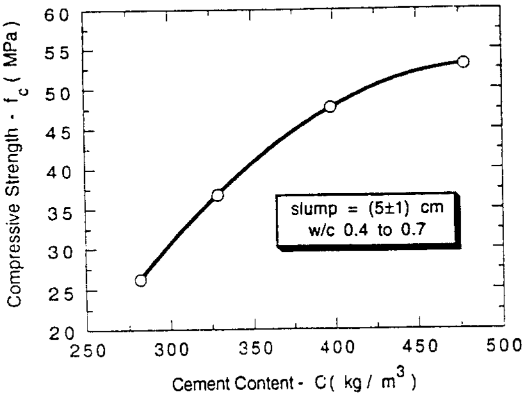

Fig. 7 Effect of cement content on the compressive strength for a constant consistency.

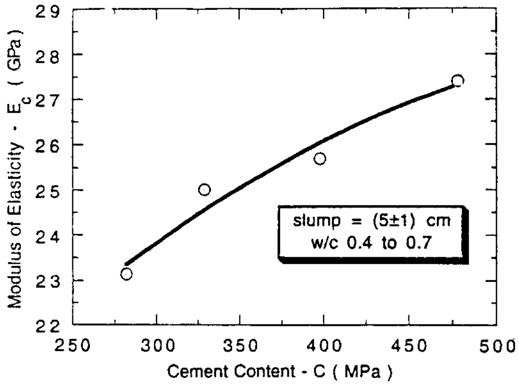

Fig. 8 Effect of cement content on the modulus of elasticity for a constant consistency.

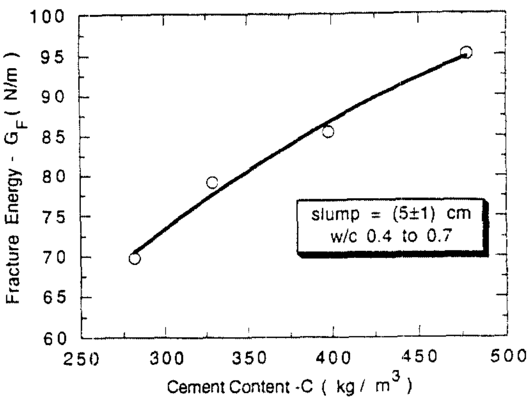

Fig. 9 Effect of cement content on the fracture energy for a constant consistency.

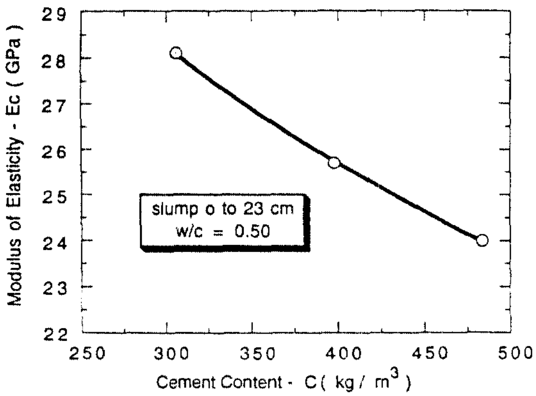

Fig.12 Effect of cement content on the modulus of elasticity for a constant water-to-cement ratio.

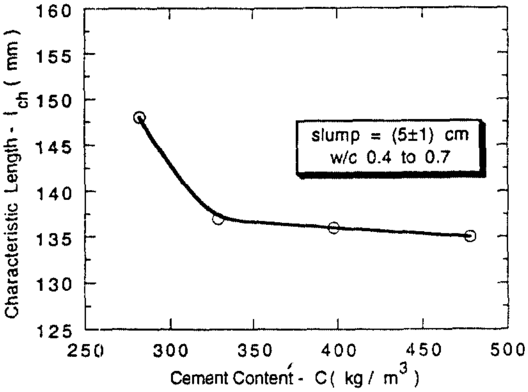

Fig. 10 Effect of cement content on the characteristic length for a constant consistency.

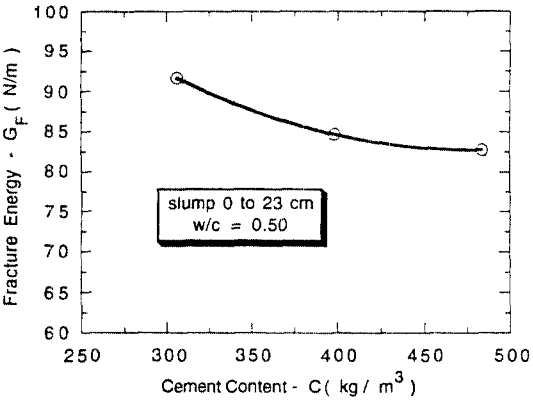

Fig.13 Effect of cement content on the fracture energy for a constant water-to-cement ratio.

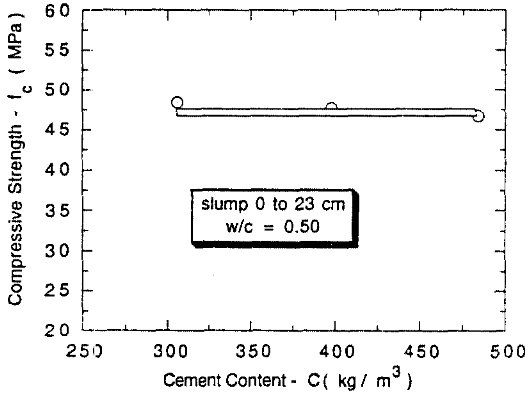

Fig. 11 Effect of ecment content on the compressive strength for a constant water-to-cement ratio.

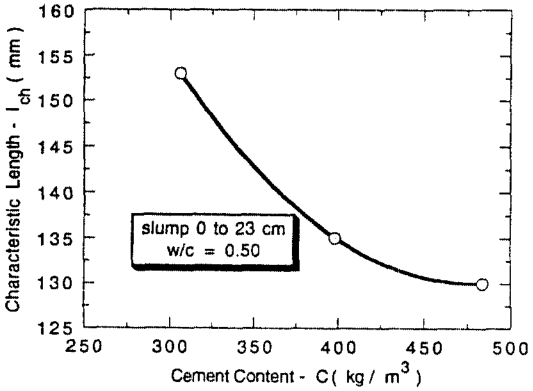

Fig.14 Effect of cement content on the characteristic length for a constant water-to-cement ratio.

the characteristic length decreases for lower water-to-cement ratios. For the condition of constant water-to-cement ratio, the decrease of the characteristic length with the cement content is due to a reduction of the amount of aggregate and therefore reducing the capacity of energy absorption.

4.4 Influence of aggregates-to-cement ratio

Fig. 15－18 indicate the influence of the aggregate content on the mechanical properties of concrete for a constant consistency. Under this condition, an increase in aggregate to cement ratio causes an increase in the water-to-cement ratio leading to a reduction in compressive strength, modulus of elasticity, and fracture energy. Normally it is expected that the modulus of elasticity and fracture energy will increase with increasing percentage of a ggregate, however as indicated in Figs. 16 and 17, this does not happen because the increase of elastic modulus of fracture energy is over-shadowed by a weaker and less stiff matrix. When the consistency is not kept constant but the water-to-cement ratio is, the matrix has the mechanical properties approximately constant, so both the modulus of elasticity and fracture energy in crease with higher contents of aggregates as shown in Figs. 20 and 21. For a constant water-to-cement ratio, the compressive strength is not affected by the aggregate to cement ratio again confirming Abrams' and Power's models. The characteristic length increases with the aggregates to cement ratio both constant consistency and constant water-to-cement ratio, as shown in Figs. 18 and 22. For the condition of constant consistency, the increase is associated to a higher water-to-cement ratio, as discussed in section 4.3; for the condition of constant water-to-cemet ratio, the increase of the characteristic length with higher aggregate concentration is due to significant influence the aggregate has on the elastic modulus and fracture energy of concrete. Note that the rate of increase of the characteristic length for the constant water-to-cement ratio is superior than for the constant consistency. One explanation is that while the tensile strength remains about the same for constant water-to-cement ratio, both the elastic modulus and fracture energy have significant increase and therefore increasing the characteristic length $(E G_F / f_t^2)$.

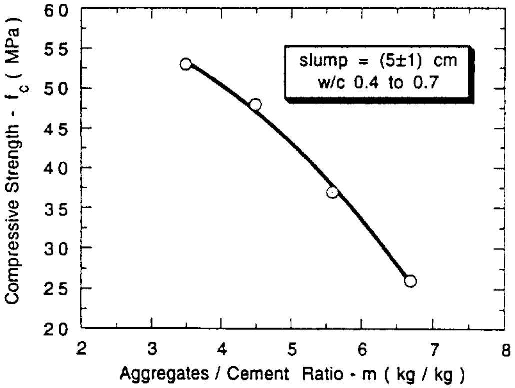

Fig. 15 Effect of aggregates to cement ratio on the compressive strength for a constant consistency.

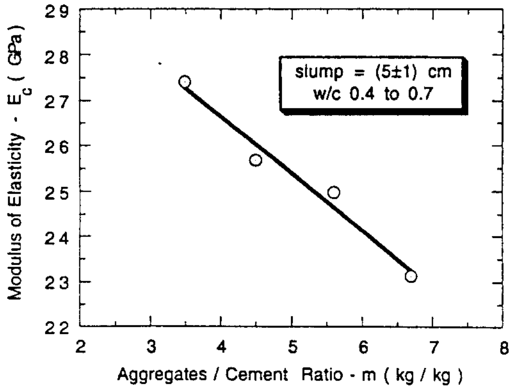

Fig. 16 Effect of a ggregates to cement ratio on the modulus of elasticity for a constant consistency.

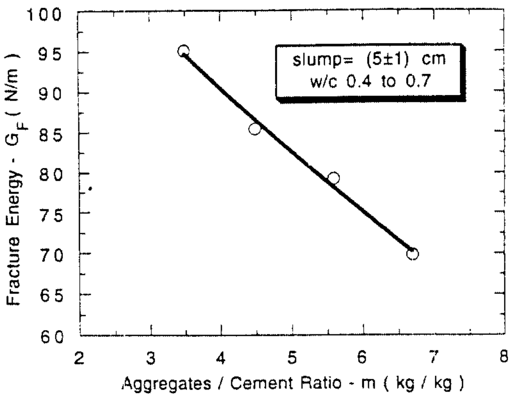

Fig. 17 Effect of aggregates to cement ratio on the fracture energy for a constant consistency.

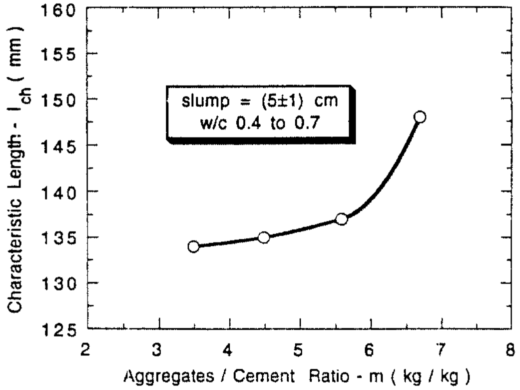

Fig. 18 Effect of aggregates to cement ratio on the characteristic length for a constant consistency.

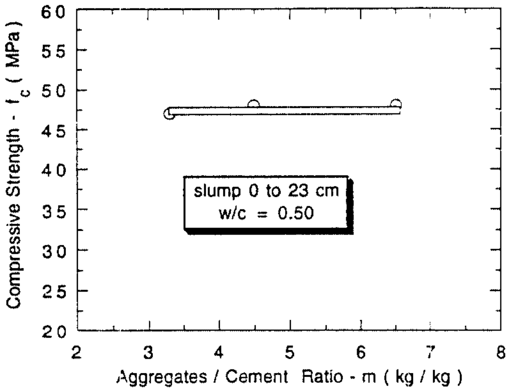

Fig. 19 Effect of aggregates to cement ratio on the compressive strength for a constant water-to-cement ratio.

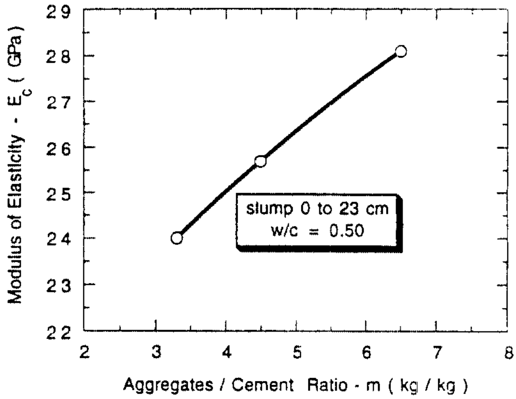

Fig. 20 Effect of aggregates to cement ratio on the modulus of elasticity for a constant water-to-cement ratio.

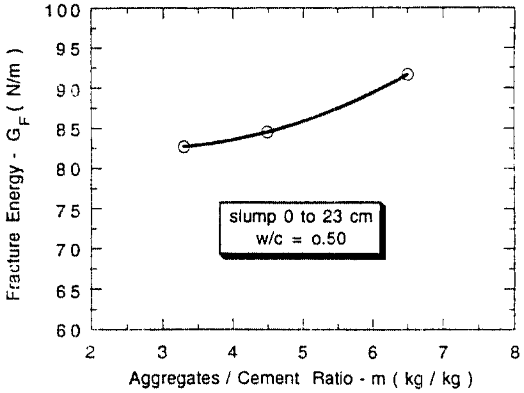

Fig. 21 Effect of aggreagtes to cement ratio on the fracture energy for a constant water-to-cement ratio.

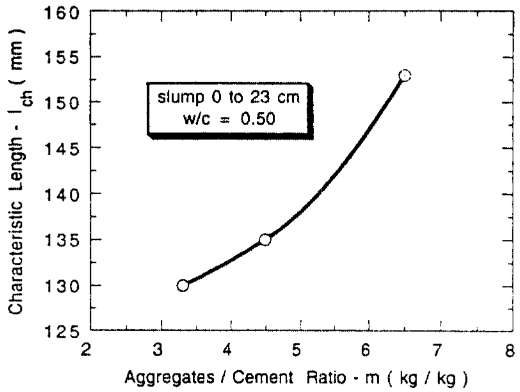

Fig. 22 Effect of aggregates to cement ratio on the characteristic length for a constant water-to-cement ratio.

5. CONCLUSIONS

A useful mix design diagram can be obtained by combining three relationships dealing with the properties of fresh and hardened concrete in one graph. Using all information in such diagram, it is possible to determine mix proprotions which comply with the predefined performance criteria. This diagram also allows the comparison of resulting properties from concrete mixtures with different mix proportions.

The effects of the mix parameters on the properties of concrete are easy to determine once the analysis is performed under two conditions:constant consistency and constant water-to-cement ratio. Mix parameters may have different effects on the concrete properties;for example a decrease in cement content or an increase in aggregate content leads to an increase in the fracture energy for a constant water-to-cement ratio and leads to a decrease in the fracture energy for a constant workability.

In this work special attention was given to design of mixtures with given fracture mechanics properties and modulus of elasticity. The resulting mix design diagrams allow the precise determination of a concrete mixture for a specified fracture energy or modulus of elasticity. This approach can also be used for many properties of hardened concrete:permeability, flexural strength, electrical resistivity, carbonation, among others.

감사의 글

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