A Constitutive Model for Normally Consolidated Clays

정규압밀점토의 응력-변형률 구성방정식

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要　旨

정규압밀점토의 거동을 예측하기 위한 새로운 構成方程式이 제안되었으며, 그 골격은 Roscoe와 Poorooshasb의 증분응력-변형률이론의 기본개념에 근거를 두고있다.

비배수 조건에서의 有效應力經路式은 새로운 간극수압계수(C)를 이용하여 유도되었고, 여기에서 간극수압계수는 標準化된 간극수압과 應力比의 관계에서 直線의 기울기로 표시된다. 비배수 응력경로와 一定應力比經路(constant stress ratio path)를 따라 발생하는 응력의 증가량을 알게되면, 이때의 체적변형률은 간극비-대수평균 연직 응력($e-\ell np$) 관계의 線形 特性으로부터 계산될 수 있다.

또한 Roscoe와 Burland의 修正 Cam-clay 理論에서 유도된 流動法則(flow rule)을 적용하여 임의 응력점에서의 전단변형률을 예측할 수 있다.

Abstract

A new constitutive model is proposed for normally consolidated clays. A main skeleton of the proposed model is based on the concepts of the incremental stress-strain theory by Roscoe and Poorooshasb. The equation of the undrained stress path is formulated by introducing the new pore pressure parameter(C), which is the slope of the linear line in the plot of the normalized pore pressure against the stress ratio. Once the stress increment along the constant stress ratio path (followed by undrained stress path) is known, the volumetric strains are calculated from the linear characteristics between void ratio and logarithm of the mean normal stress for any stress ratio. Then the incremental shear strains are successfully predicted by applying the flow rule derived in the modified theory by Roscoe and Burland.

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1. Introduction

Several stress-strain theories such as the Cam-clay theory[6] and the modified theory[7] have been developed at the Cambridge University to describe the stress-strain behaviour of normally consolidated clays. In these theories, a fully saturated normally consolidated clay is assumed to behave as an elasto-plastic material. Based on this assumption, concepts such as yield surface and normality rule were used to obtain an expression for the strain increment ratio in terms of the stresses.

On the other hand, the incremental stress-strain theory was proposed by Roscoe & Poorooshasb[8] based on the assumption of a unique state boundary surface(SBS) for normally consolidated clays. Existence of a unique state boundary surface was studied by Henkel[4] and Roscoe et al[6].

Efforts to examine a unique SBS were also made by Balasubramaniam[1], and he found that the SBS is unique and is independent of the applied stress paths, provided that the loading steps are infinitely small.

The incremental stress-strain theory of Roscoe & Poorooshasb is mathematically similar to the Cam-Clay theory or the modified theory which is based on some of the concepts of plasticity theory and an energy equation. It therefore appears that close examination of Roscoe & Poorooshasb's theory is more fundamental to understand the basic behaviour of normally consolidated soils under the stresses.

However, though the incremental stress-strain theory is a conceptually good model, it needs quite a large number of experiments which are rather complicate.

This paper is therefore concerned with a direct application of the incremental stress-strain theory in a numerical form by using some of basic soil parameters which can be obtained in a simple manner from the results of basic soil tests.

2. Definition of Stress and Strain Parameters

The stress parameters p and q are defined by :

$$p=(\sigma_1'+2\sigma_3')/3$$

$$q=(\sigma_1'-\sigma_3')$$

where σ_1' and σ_3' are the principal effective compressive stresses, and $\sigma_2'=\sigma_3'$ under the triaxial stress system, Similarly the incremental strain parameters $d\nu$ and $d\varepsilon$ are given by :

$$d\nu=d\varepsilon_1+2d\varepsilon_3$$

$$d\varepsilon=2(d\varepsilon_1-d\varepsilon_3)/3$$

where $d\varepsilon_1$ and $d\varepsilon_3$ are the principal incremental compressive strains and $d\varepsilon_2$ is equal to $d\varepsilon_3$ under the triaxial stress system. The stress ratio, q/p, is denoted by η.

3. Basic Features of Model by Roscoe & Poorooshasb

An incremental stress-strain theory for normally consolidated clay was proposed by

ROSCOE & POOROOSHASB3), and it can be expressed by Eq. (1).

$$d\varepsilon_1 = (d\varepsilon_1 / d\eta)_\nu \, d\eta + (d\varepsilon_1 / d\nu)_\eta \, d\nu \qquad (1)$$

where $(d\varepsilon_1/d\eta)_\nu$ corresponds to the variation of $d\varepsilon_1$ with $d\eta$ in an undrained test and $(d\varepsilon_1/d\nu)_\eta$ represents the variation of $d\varepsilon_1$ with $d\nu$ in a constant-η stress path. $d\nu$ and $d\varepsilon_1$ are the incremental volumetric and axial strains respectively.

Equation (1) can be presented in a slightly different form as :

$$(d\varepsilon)_{\text{drained}} = (d\varepsilon)_{\text{undrained}} + (d\varepsilon / d\nu)_\eta \, d\nu \qquad (2)$$

This equation was derived based on the assumptions that :

(a) the undrained stress path can be normalized with respect to the preshear stress(p_o). Therefore, the shear strain in an undrained test is only a function of the stress ratio.

(b) the slope $(d\varepsilon/d\nu)_\eta$ in the (ν, ε) plane during anisotropic consolidation (constant-η) tests is only a function of the stress ratio.

(c) the volumetric strain(ν) is a function of η and p throughout the state boundary surface.

4. Stress-Strain Behaviour under Undrained Conditions

4.1 Undrained Stress Path

Figure. 1 shows the typical undrained stress paths of soft Bangkok Clay followed by the isotropically consolidated specimens in the (q, p) plot.

The shape of undrained stress paths are found to be virtually similar, and the normalized stress paths with respect to the preshear consolidation stresses(p_o) are approximately identical to each other as shown in Fig. 2.3)

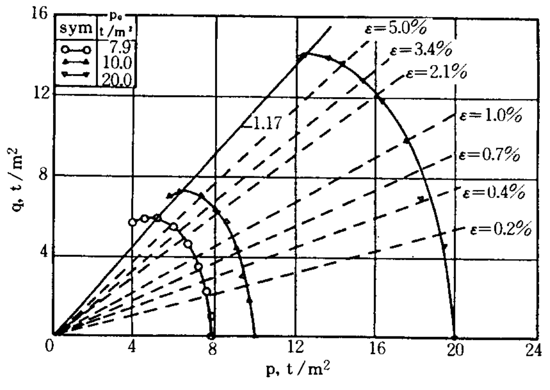

Fig 1. Undrained Stress Paths for Normally Consolidated Clays(after Handali, 1986)

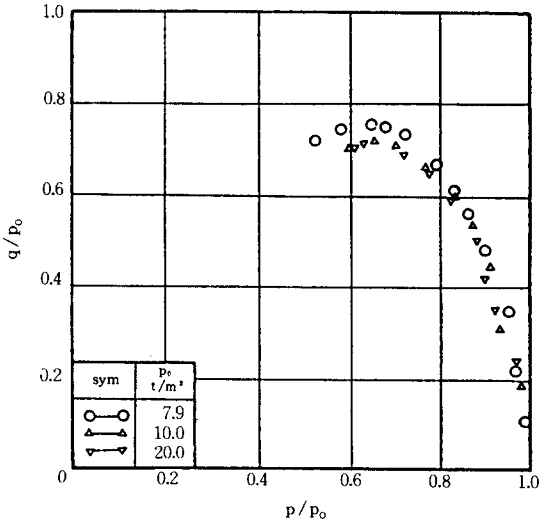

Fig 2. Normalized Stress Paths for Normally Consolidated Clays(after Handali, 1986)

4.2 Shear Strain Contours

The dashed lines superimposed in Fig. 1 are constant shear strain contours, and contours are linear and pass through the origin. A similar finding was reported by Balasubramaniam and Chaudhry[2]. This implies that, for all specimens of normally consolidated clay sheared under undrained conditions, the(η, ε) relationship is independent of the preshear consolidation stress and thus the shear strain during undrained test is only dependent on the magnitude of the stress ratio, η.

4.3 (u/ p_o, η) Relationship

In Fig. 3 the pore pressure of the normally consolidated specimens is plotted against the stress ratio, η. It is to be noted that all three specimens display linear variations and these lines pass through the origin of the (u, η) plot. The normalized pore pressure with respect to preshear consolidation pressure(p_o) also varies linearly with the stress ratio in Fig. 4. This finding implies that the normalized pore pressure(u/p_o) is a unique function of the stress ratio, η.

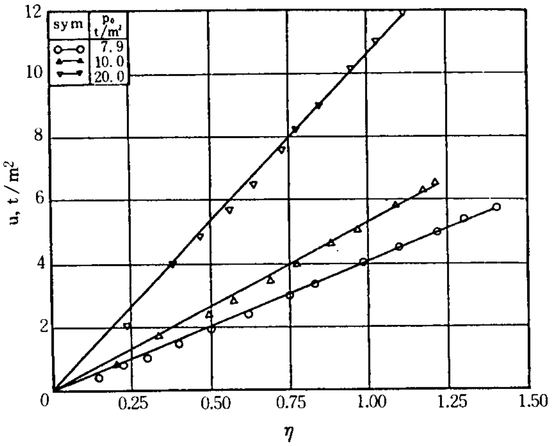

Fig 3. (u, η) Plot for Normally Consolidated Clays(after Handali, 1986)

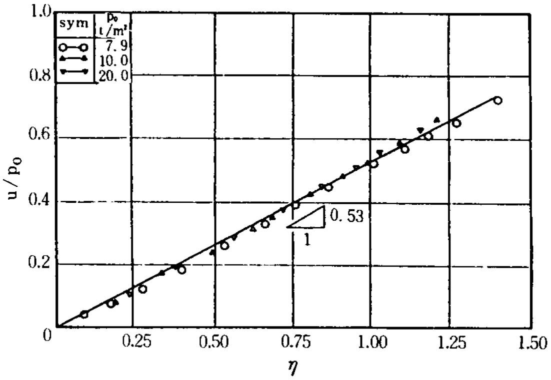

Fig 4. (u/p_o, η) Plot for Normally Consolidated Clays(after Handali, 1986)

4.4 Equation of the Undrained Stress Path

The linear relationship between u/p_o and η in Fig. 4 can be applied to formulate the equation of undrained stress path from the characteristics of the stress path as follows.

$$u = p_o + q/3 - P \tag{3}$$

$$u = C \cdot p_o \cdot \eta \tag{4}$$

where, u = pore water pressure

C=slope of linear line in the$(u/p_0, \eta)$ plot

p_0=preshear consolidation stress

Then the equation of the undrained stress path can be obtained by equating Eqs. (3) and (4) as follows :

$$\frac{p}{p_0} = 3(1-C\eta)/(3-\eta) \tag{5}$$

The stress increment or decrement along the undrained stress path at a given stress ratio can be obtained by differentiating Eq. (5)

$$dp=3p_0(1-3C)d\eta/(3-\eta)^2 \tag{6}$$

5. Constitutive Model for Normally Consolidated Clays

The basic concept of the incremental stress-strain theory represented by Eq.(2) is maintained to derive the constitutive law for the normally consolidated clay: The incremental strain components along the drained path, 'a' to 'c' in Fig. 5 are equivalent to those of the undrained path, 'a' to 'b' followed by the constant stress ratio path, 'b' to 'c', within the domain of the state boundary surface (Roscoe surface). In order to implement the theory, it is therefore necessary to calculate the shear strain along the path 'a' to 'b' (volumetric strain is zero), the shear and volumetric strains along the path, 'b' to 'c'.

5.1 Incremental Shear Strain during Undrained Test

The shear strain in undrained condition is the unique function of the stress ratio, η, as previously mentioned, and it can be represented by

$$(\varepsilon)_{\text{undrained}} = \int_0^\eta f_1(\eta)d\eta \tag{7}$$

The function $f_1(\eta)$ can be found from the transformed hyperbolic plot between (ε/η) and ε ; that is linear relationship in the $(\varepsilon/\eta, \varepsilon)$ plot with the intercept on the ε/η−axis, as represented by Fig. 6.[5]

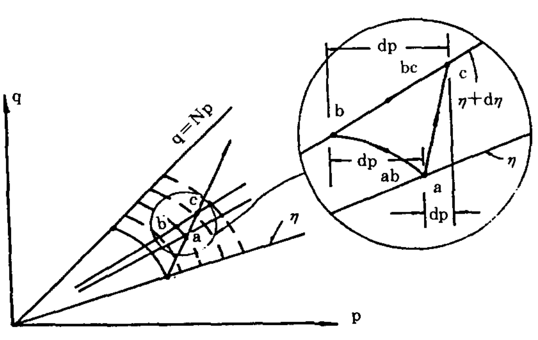

Fig 5. Components of General Stress Increments

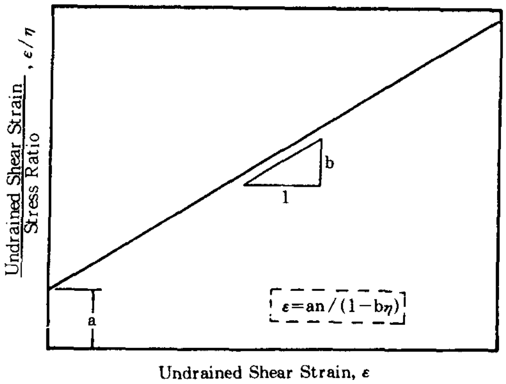

Fig 6. Typical Pattern of the Transformed Hyperbolic Plot of ε/η against ε in Undrained Test

$$\varepsilon / \eta = a + b\varepsilon \tag{8}$$

$$\varepsilon = a\eta / (1 - b\eta) \tag{9}$$

The incremental form of Eq. (9) is

$$(d\varepsilon)_{undrained} = a d\eta / (1 - b\eta)^2 \tag{10}$$

where, a = intercept on the ε / η −axis

b = slope of the linear line in the $(\varepsilon / \eta, \varepsilon)$ plot

Thus the function $f_1(\eta)$ in Eq. (7) is equivalent to the value of $a/(1-b\eta)^2$ in Eq. (10).

5.2 Incremental Volumetric Strain along the Constant-η Path

The magnitude of the volumetric strain increment followed by a constant-η stress path can be determined by:

$$(d\nu) = \frac{\lambda \, dp_{bc}}{p(1+e)} \tag{11}$$

where, 'p' and 'e' are the mean normal stress and the void ratio at point 'a' in Fig 5 respectively, and the slope of (e-ln p) plot represented by 'λ' is a constant for any stress ratio.

The stress increment along the constant-η path, 'dp_{bc}', can be estimated from the relationship in Fig. 5 as:

$$dp_{be} = dp - dp_{ab} \tag{12}$$

The magnitude of the stress decrement along the undrained stress path, dp_{ab} is equivalent to 'dp' given by Eq. (6).

By substituting Eq. (6) into Eq. (12), dp_{bc} can be expressed as:

$$dp_{bc} = dp - 3p_{oe}(1-3C)d\eta / (3-\eta)^2 \tag{13}$$

where, p_{oe} is the equivalent preshear consolidation stress at a given value of 'p' and 'η', and this can be automatically calculated from the normalization characteristics of the undrained stress path represented by Eq. (5)

The volumetric strain increment, $(d\nu)_{bc}$ can be obtained by substituting Eq. (13) into Eq. (11).

$$(d\nu)_{bc} = \frac{\lambda \, dp}{p(1+e)} - \frac{3\lambda \, p_{oe}(1-3C)d\eta}{p(1+e)(3-\eta)^2} \tag{14}$$

The plastic component of volumetric strain, $(d\nu)p_{bc}$ is then calculated by Eq. (15).

$$(d\nu)p_{bc} = \frac{(\lambda - \kappa)dp}{p(1+e)} - \frac{3\lambda \, p_{oe}(1-3C)d\eta}{p(1+e)(3-\eta)^2} \tag{15}$$

where, κ is the slope of swelling line in the (e-ln p) plot, which is also constant for any stress level.

5.3 Incremental Shear Strain along the Constant-η path

As was derived in the modified theory by Roscoe & Bunland[7], the elastic shear strain is assumed zero and the plastic strain increment ratio is defined by:

$$d\varepsilon p / d\nu p = 2\eta / (M^2 - \eta^2) \tag{16}$$

Then the shear strain increment along the constant-η path is obtained by substituting Eq. (15) into Eq. (16).

$$d\varepsilon = d\varepsilon p = \frac{2\eta}{M^2 - \eta^2} \left\{ \frac{(\lambda - \kappa) dp}{p(1+e)} - \frac{3\lambda\ p_{oe}(1-3C) d\eta}{p(1+e)(3-\eta)^2} \right\} \tag{17}$$

5.4 Summary of the model

In an undrained condition, the equation of undrained stress path is defined by Eqs. (5) and Eq. (6), the incremental shear strain can be predicted by Eq. (10).

In a drained condition, the incremental volumetric strain for any stress ratio can be calculated by Eq. (14) and the shear strain increment is obtained by summation of Eq. (10) and Eq. (17).

6. Verificaton of the model

In order to evaluate the validity of this proposed model the results of well designed triaxial compression tests for the soft Bangkok clay were used.

The average index properties of the clay are as follows: natural water content = 112 −130%, liquid limit = 118.0 ± 2%, plasticity index = 75 ± 4%. The triaxial specimens were 36㎜ in diameter and 77㎜ in height. All tests were carried out under stress controlled conditions, and a back pressure of 20.7 t/m^2 was used to saturate the specimens.

In this section, the experimentally observed strains in drained condition are compared with the strains predicted from the proposed model. A total of six fundamental soil parameters ane required in this model, of the six parameters, M, λ, κ are the same as used in the critical state model, the other three parameters such as 'a', 'b' and 'C' can be easily obtained from the undrained tests. The pore pressure parameter 'C' is evaluated in the $(u/p_o, \eta)$ plot as shown in Fig. 7, and the value of 'C' is 0.64. The undrained shear strain parameters 'a' and 'b' are determined from the linear relationship in the transformed $(\varepsilon/\eta, \varepsilon)$ plot in Fig. 8.

Isotropic consolidation and swelling tests carried out on soft Bangkok clay indicate that the value of λ is 0.50, and that of κ is 0.09. Also, the slope of the critical state line, M, is taken as 1.0.

The calculated volumetric and shear strains using the proposed model are presented in Figs. 9 and 10 respectively, and the observed strains in a conventional drained test for the specimen isotropically consolidated with 41.4 t/m^2 are compared with the predicted value. The proposed constitutive model is found to predict very closely the experimentally observed strains. It is to bo noted herein that the proposed model is limited to the triaxial stress system only. Thus, with the help of exclusive testing program, some modifications are required to extend the model for general stress system.

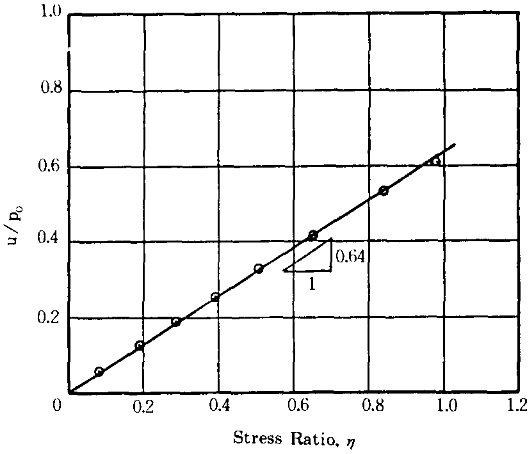

Fig 7. $(u/p_o, \eta)$ Plot for Normally Consolidated Bangkok Clay$(p_o=41.4\ t/m^2)$

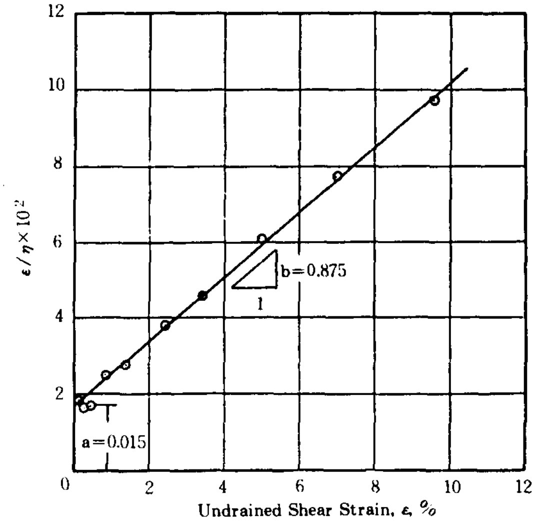

Fig 8. Transformed Hyperbolic Plot of ε/η against ε in Undrained Test$(p_o=41.4\ t/m^2)$

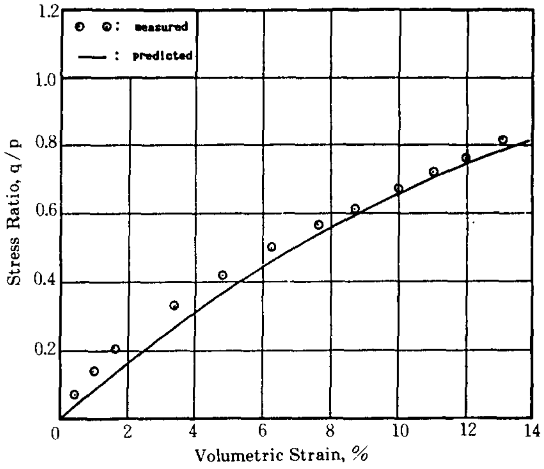

Fig 9. Measured and Predicted Volumetric Strains of Soft Bangkok Clay in Drained Condition $(p_o=41.4\ t/m^2)$

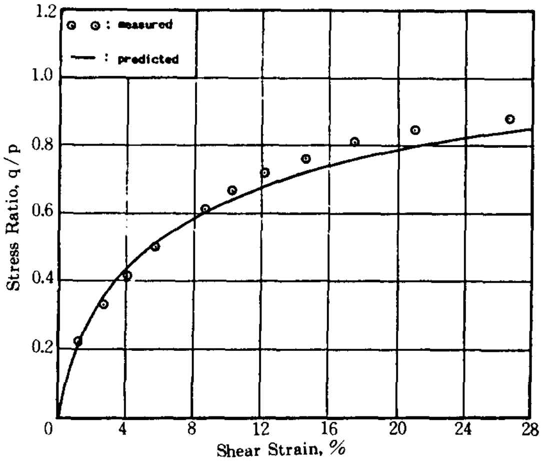

Fig 10. Measured and Predicted Shear Strains of Soft Bangkok Clay in Drained Condition $(p_o=41.4\ t/m^2)$

7. Conclusions

A new constitutive model for normally consolidated clay is proposed based on the concept of the incremental stress-strain theory by Roscoe & Poorooshasb.

The proposed model is capable of calculating the incremental strains in drained condition by using the parameters obtained from the undrained condition. The equation of undrained stress path can be formulated by introducing the new pore pressure parameter 'C', and the magnitude of stress increment followed by the undrained tests can be calculated for any value of the stress ratio. The calculation of the incremental shear strains in undrained conditions is also made by using the shear strain parameters which can be obtained from the linear relationship with intercept in the transformed hyperbolic plot of stress ratio and undrained shear strain.

The volumetric strains are calculated from the linear characteristics between void ratio and logarithm of the mean normal stress. Then the incremental shear strains are successfully predicted by applying the flow rule derived in the modified theory by Roscoe and Burland.

In general, the proposed constitutive model is found to predict very closely the experimentally observed strains.

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(접수일자 1992. 3. 3)