# Application of Linear Goal Programming to Large Scale Nonlinear Structural Optimization

## 대규모 비선형 구조최적화에 관한 선형 Goal Programming의 응용

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요 지

본 논문은 수식화의 특이성 때문에 구조 최적화 문제에 거의 사용되지 않고 있는 선형 goal programming을 대규모 비선형 구조 최적화에 응용하는 방법을 제시한다. 이 방법은 다기준 최적화의 도구로 사용되는데 그 까닭은 goal programming이 목적함수와 제한조건등을 정의하는데 있어서 발생하는 난점들을 제거해 주기 때문이다.

이 방법은 비선형 goal 최적화 문제들의 해를 얻기 위해서 유한요소해석, 선형 goal programming 기법, 그리고 계속적인 선형화 기법을 이용한다. 즉, 대규모 비선형 구조 최적화 문제를 비선형 goal programming 형태로 전환시키는 일반적인 수식화 방법을 제시하고, 얻어진 비선형 goal 최적화 문제를 풀기 위한 계속적인 선형화 방법에 대해서도 논의한다. 설계도구로서 이 방법의 유효성을 논증하기 위하여 10, 25 및 200트러스의 사례를 가지고 응력제한조건들의 최소무게 구조 최적화 문제에 대한 해를 모색하며 이를 다른 연구결과와 비교검토한다.

Abstract

This paper presents a method to apply the linear goal programming, which has rarely been used to the structural opimization problem due to its unique formulation, to large scale nonlinear structural optimization. The method can be used as a multicriteria optimization tool since goal programming removes the difficulty in defining an objective function and constraints.

The method uses the finite element analysis, linear goal programming techniques and successive linearization to obtain the solution for the nonlinear goal optimization problems. The general formulation

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이 논문에 대한 토론을 1992년 6월 30일까지 본 학회에 보내주시면 1992년 12월호에 그 결과를 게재하겠습니다.

of the structural optimization problem into a nonlinear goal programming form is presented. The successive linearization method for the nonlinear goal optimization problem is discussed. To demonstrate the validity of the method, as a design tool, the minimum weight structural optimization problems with stress constraints are solved for the cases of 10, 25 and 200 trusses and compared with the results of the other works.

## 1. INTRODUCTION

When applying a traditional structural optimization algorithm, the user selects a single criterion which is to be maximized or minimized, and defines a set of design constraints which delimit a feasible design space. A set of design variables must also be defined. This general formulation does not allow much flexibility and has drawbacks for application in many design situations.

It is often difficult to define the problem exactly in the format required for a traditional structural optimization algorithm. For example, as the objective function and the costraints are sometimes mixed up in real design problems, the designers are often confused which one to select from the others. Design goals may be ranked in some specific order of preference but to select one as the objective of the optimization and to treat the rest as constraints are often quite difficult. In addition, design targets are generally set at optimistic levels which in practice may not be achievable. There are, however, optimization techniques which address multi-objective optimization problems and have the capacity to handle rank ordered design objectives or goals. This technique is known as "goal programming[1−6]."

In a goal programming (GP) formulation, the design goals are defined and a priority is assigned to each one. The algorithm then attempts to satisfy as many goals as possible, starting with the highest priority goal. This removes the diffculty in defining an objective function and constraints and makes goal programming an ideal design tool.

The formulation of the nonlinear structural optimization problem into a goal form and its application to a simple 3 bar truss structural optimization problem is discussed only in [7]. In this work, so as to handle more realistic structural design problem, the generalization of the approach in [7] and its application to large scale structural optimization problems are treated intensively and their feasbility is proved by comparing with known results. The finite element analysis is employed to anlayze the response of the structure during the optimization process.

## 2. FORMULATION

Traditional optimization models require the formulation of a single objective function. Linear programming (LP) is one of the most frequently used techniques of mathematical programming for the solution of the optimization problems. In LP problems the single objective function and the constraints appear as linear functions of the design variables. The LP solution procedure is based on the simplex method [8,9]. Linear goal programming (LGP) as an extension of linear programming also uses the simplex method which is modified to solve the multi-objective and multi-conflicting-objective LP problems. The terminology, basic elements and the formulation of the LGP problems are treated extensively in references [5,6] and are omitted here. In the following we discuss the formulation of the general structural optimization to LGP form.

2.1 General Goal Optimization Problem

The general goal opimization problem is:

$$\text{Minimize} \quad Z=\sum_{i=1}^{I}\omega_{ki}P_k(d_i^-+d_i^+) \quad (k=1,2,\cdots,K)$$

$$\text{subject to} \quad g_i(x)+d_i^- - d_i^+ = b_i \quad (i=1,2,\cdots,I)$$

$$\text{and} \quad x^L \le x \le x^U, \quad d_i^-, d_i^+ \ge 0$$

$$\text{to find} \quad x \in R^N \qquad (1)$$

where vector $Z$ is the objective function of $GP$ to be minimized. The dimension of $Z$ represents the number of pre-emptive priority levels. The differential weights $\omega_{ki}$ are mathematical weights which are expressed as cardinal numbers, and are used to differentiate the $i^{th}$ deviational varibles within a single $K^{th}$ priority level. The pre-emptive priority factors $P_k$ represent a ranking system which places the importance of goals in accordance with the following relationship : $P_1$(The most important goal)$\rangle P_2 \rangle\rangle\rangle P_K$ (The least important goal). $d_i^-$ and $d_i^+$ are negative and positive deviational variables that express the possibility of deviation from a right-hand-side value $b_i$(these variables are conceptually similar to slack variables in $LP$ models). $g_i$ is the goal constraints function we desire to minimize its numerical deviation from a stated right-hand-side value $b_i$ in a selected goal constraint. I is the total number of the goal constraints. This goal constraint is expressed as the flexible constraint. $x^L$ *and* $x^U$ represent the lower and upper bound of design variable, respectively. $x$ is a set of $N$ design variables we seek to determine.

The solution procedure is that we consider the objective function with the highest priority (P1) first and determine the solution to mimimize deviational variables which are related to priority P1. Move to the objective function having the next highest priority (P2) and determine the best solutions. Repeat until all priority levels have been investigated and determine the best and compromise solutions. More detailed discussion about the LGP solution procedure is given in [5,6].

2.2 Linearization of Nonlinear Optimization Problem

To use the simplex method in solving nonlinear goal programming problem, the goal constraints must be linearized. One of the most well known linearization methods is the Griffth and Stewart method [10]. This algorithm employs the Taylor series expansion [11] of the function $g_i(x)$ in Eq.(1) about the point $x^m$. This is expressed in equation form as follows:

$$g_i(x)=g_i(x^m)+\nabla g_i(x^m)\delta_x+\theta(\delta_x^2) \quad (i=1,2,\cdots,I) \quad (2)$$

where $\nabla$ is a gradient operator, $\delta_x$ is a vector representing small variations in the design variables, and $\theta(\delta_x^2)$ represents all terms of the order two and higher in variations. Neglecting the higher order terms in each expansion results in an approximate linear opimization problem [12].

2.3 Structural Optimization Problem

The traditional model of the minimum weight structural optimization problem is

$$\text{Minimize} \quad W(x)$$

$$\text{subject to} \quad G_i(x) \le 0 \quad (i=1,2,\cdots,M)$$

$$\text{and} \quad x^L \le x \le x^U$$

$$\text{to find} \quad x \in R^N \qquad (3)$$

where $W(x)$ is the objective function, representing the total weight of the structure. $G_i(x)$ represents the constraint functions including the inequality and equality constraints and it may be a function of stress, displacement, Euler buckling and natural frequency. I is the total number of constraints. $x^L$ and $x^U$ represent the lower and upper bound of design variable, respectively. For simplicity we consider the minimum weight optimization problem of truss structures with stree constraints. The structural optimization problem with the cross-sectional area of the truss members taken as the design variables becomes:

$$\begin{aligned}
&\text{Minimize} && W(x)=\sum_{j=1}^{J}\rho_j 1_j x_j \\
&\text{subject to} && \sigma_i(x)/\sigma_a - 1 \le 0 \quad (i=1,2,\cdots,M) \\
&\text{and} && x^L \le x \le x^U \\
&\text{to find} && x \in R^N
\end{aligned} \tag{4}$$

where $\rho_j$, $1_j$, $x_j$ are the density, length the cross-sectional area of $j^{th}$ truss member, $\sigma_i$ and $\sigma_a$ represent the stress component of the $i^{th}$ member and allowable stress, respectively. J is the total number of design variables or members. M is the total number of members.

The structural optimization problem, Eq.(4) can be rewritten, in a goal structural optimiz ation model, as follows:

$$\begin{aligned}
&\text{Minimize} && Z=\sum_{i=1}^{M+1}\omega_{ki}P_k(d_i^- + d_i^+) \quad (k=1,2,\cdots,K) \\
&\text{subject to} && W(x)/W_a - 1 + d_i^- - d_1^+ = 0 \\
& && \sigma(i-1)(x)/\sigma_a - 1 + d_i^- - d_i^+ = 0 \quad (i=2,\cdots,M,M+1) \\
&\text{and} && x^L \le x \le x^U,\ d_i^-,\ d_i^+ \ge 0 \\
&\text{to find} && x \in R^N
\end{aligned} \tag{5}$$

where $W(x)$ is the total weight of the structure, $W_a$ represents target weight or initial weight. Using Eq.(2) to linearize the stress constraints and neglecting the higher order terms, the structural optimization problem becomes :

$$\begin{aligned}
&\text{Minimize} && Z=\sum_{i=1}^{M+1}\omega_{ki}P_k(d_i^- + d_i^+) \quad (k=1,2,\cdots,K) \\
&\text{subject to} && Z=\sum_{j=1}^{J}\rho_j 1_j x_j / W_a + d_1^- - d_1^+ = b_1 \\
& && \nabla\sigma_{(i-1)}(x^m)^T x/\sigma_a + d_i^- - d_i^+ = b_i \quad (i=2,\cdots,M,M+1) \\
&\text{and} && x^T \le x \le x^U,\ d_i^-,\ d_i^+ \ge 0 \\
&\text{to find} && x \in R^N
\end{aligned} \tag{6}$$

where

$$b_1 = 1$$

$$b_i = 1 + \nabla\sigma_{(i-1)}(x^m)^T x^m/\sigma_a - \sigma_{(i-1)}(x^m)/\sigma_a \quad (\text{for } i=2,\cdots,M,M+1)$$

This linear goal structural optimization problem can be solved using the modified simplex method [13,14]. By solving the LGP optimization problem described, the nonlinear goal structural optimiz ation problem can be solved via an iterative process. The linearization starts about some initial value $x^{(0)}$. After linearlization, the LGP algorithm is used to determine the optimum value for the linearized equations. The new optimum values $x^{(1)}$ are improvement on the original $x^{(0)}$ values. This process continues until $\| x^{(m)} - x^{(m-1)} \|$ is less than or equal to some predetermined termination criterion. Stress in the structure members at each value of the design variabes is obtained by using the finite element anaylsis. The gradient information is obtained by using the finite difference approximation.

## 3. TEST CASES

The following is structural optimization examples to demonstrate the ability of the method in solving large scale optimization problems. The planar 10 member truss, 25 member space truss and 200 member plane truss problems were chosen, because their solutions are available in [15,16]. Schmit et al.[15] introduced approximation concepts and used the NEWSUMT algorithms which is a sequence of unconstraints minimization technique based on the extended interior function formulation. In Ref.[16] a steepest descent algoritm was used for optimal design of structures.

### 3.1 Planar 10 Member Truss

Fig.1 shows the geometry and dimensions of the 10 member truss.

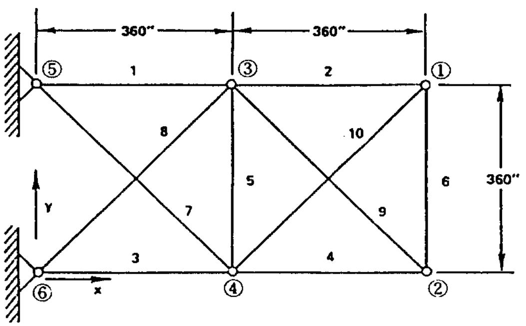


Fig.1 10 Member Planar Truss

The truss element descriptions including initial cross sectional area, minimum member size, material properties and load data are specified in Table 1.

Two cases are considered for the anaylsis. The truss is designed to withstand a single loading condition subject to weight and stress goal constraints.

Table 1. Design Data for 10 Member Planar Truss

Modulus of elasticity=$10^4$ ksi
Material density=0.10 lb/in$^3$
Lower limit on cross-sectional areas=0.10 in$^2$
Initial value of design variable=10 in$^2$
Stress limit=±25 ksi
Number of loading conditions=1

Load Data

| Load Case No. | Loading Condition | Node | Load component (kips) in Direction x | y | z |
|---|---|---|---|---|---|
| I | 1 | 2 | 0.0 | −100.0 | 0.0 |
| | | 4 | 0.0 | −100.0 | 0.0 |
| II | 1 | 1 | 0.0 | 50.0 | 0.0 |
| | | 2 | 0.0 | −150.0 | 0.0 |
| | | 3 | 0.0 | 50.0 | 0.0 |
| | | 4 | 0.0 | −150.0 | 0.0 |

Results and Discussion

The results for the optimum cross-sectional area of each member, the opimum weight, the number of iterations, the number of active constraints, the number of function evaluations, and the maximum positive deviation are given in Table 2 for case I and case II. The results obtained from the different nonlinear programming algorithms [15, 16] are also shown in Table 2.

For case I, target weight is 1585 lb and the optimum weight of 1593.18 lb is obtained from the LGP. There are 10 active constraints for the optimum design, which are the minimum size constraints on members 2, 5, 6 and 10 and stress constraints on members 1, 3, 4, 7, 8 and 9. The maximum positive deviation of goal constraints is $2.5x10^{-6}$.

For case II, the target weight is 1655 lb and the optimum weight of 1664.53 lb is obtained from the LGP. There are 10 active constraints for the optimum design, which are the minimum size constraints on members 2, 5 and 10 and the stress constraints on members 1, 3, 4 and 6−9. The maximum positive deviation of goal

Table 2. Results for 10 Member Truss

| | Optimum Cross-Sectional Area in $in^2$ | | | | | |
|---|---|---|---|---|---|---|
| | Case Ⅰ | | | Case Ⅱ | | |
| Member Number | LGP | Ref. 15 | Ref. 16 | LGP | Ref. 15 | Ref. 16 |
| 1 | 7.9379 | 7.938 | 7.9379 | 5.9477 | 5.948 | 5.9478 |
| 2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 3 | 8.0621 | 8.062 | 8.0621 | 10.0523 | 10.05 | 10.0520 |
| 4 | 3.9379 | 3.938 | 3.9379 | 3.9477 | 3.948 | 3.9478 |
| 5 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 6 | 0.1 | 0.1 | 0.1 | 2.0523 | 2.052 | 2.0522 |
| 7 | 5.7447 | 5.745 | 5.7447 | 8.5593 | 8.559 | 8.5592 |
| 8 | 5.5690 | 5.569 | 5.5690 | 2.7545 | 2.755 | 2.7545 |
| 9 | 5.5690 | 5.569 | 5.5690 | 5.5829 | 5.583 | 5.5830 |
| 10 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| Optimum Weight (lb) | 1593.18 | 1593.23 | 1593.18 | 1664.53 | 1664.55 | 1664.53 |
| No. of Iterations | 12 | 15 | 9 | 11 | 10 | 1 |
| No. of active constraints | 10 | 10 | 10 | 10 | 10 | 10 |
| No. of Funtion Evaluations | 143 | | | 132 | | |
| Max. positive Deviation | $2.5x10^{-6}$ | | | $2.6x10^{-5}$ | | |
| Max. Constraint Violation | | | $6.2x10^{-4}$ | | | $5.9x10^{-6}$ |

constraints is $2.6x10^{-5}$.

By comparing data from Ref.[15] with LGP data, the optimum values are almost the same due to the same number of active constraints.

### 3.2 25 Member Space Truss

A 25 member space truss is shown in Fig.2.

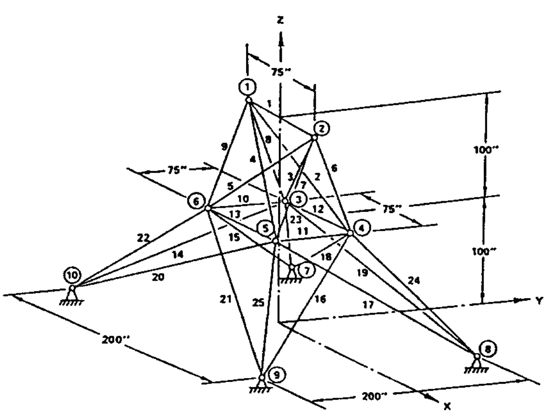


Fig.2 25 Member Space Truss

The design data for this truss are given in Table 3.

Table 3. Design Data for 25 Member Space Truss

Modulus of elasticity=$10^4$ ksi
Material density=0.10 $lb/in^3$
Lower limit on cross-sectional areas=0.10 $in^2$
Initial value of design variable=0.5 $in^2$
Stress limit=$\pm$40 ksi
Number of loading conditions=2

Load Data

| Load Condition | Node | Load component (kips) in Direction x | y | z |
|---|---|---|---|---|
| 1 | 1 | 0.5 | 0.0 | 0.0 |
| | 2 | 0.5 | 0.0 | 0.0 |
| | 3 | 1.0 | 10.0 | −5.0 |
| | 4 | 0.0 | 10.0 | −5.0 |
| 2 | 3 | 0.0 | 20.0 | −5.0 |
| | 4 | 0.0 | −20.0 | −5.0 |

Simple design variable linking is used to set the double symmetry for this truss. This results in 7 design variables. The truss is designed to withstand a double loading condition subject tc weight and stress goal constraints.

Results and Discussion

Table 4 shows the optimum values of the cross-sectional area of each member, the optimum weight, the number of iterations, the number of active constraints, the number of function evaluations, and the maximum positive deviations.

**Table 4. Results for 25 Member Truss**

| Optimum Cross-Sectional Area in in² | | |
|---|---|---|
| Member Numbers | LGP | Ref. 16 |
| 1 | 0.1 | 0.1 |
| 2, 3, 4, 5 | 0.3766 | 0.3755 |
| 6, 7, 8, 9 | 0.4705 | 0.4734 |
| 10, 11, 12, 13 | 0.1 | 0.1 |
| 14, 15, 16, 17 | 0.1 | 0.1 |
| 18, 19, 20, 21 | 0.2770 | 0.2786 |
| 22, 23, 24, 25 | 0.3823 | 0.3796 |
| Optimum Weight(lb) | 91.24 | 91.27 |
| No. of Iteration | 6 | 5 |
| No. of Active Constraints | 7 | |
| No. of Function Evaluations | 56 | |
| Max. Positive Deviation | $4.1\times10^{-4}$ | |

Also shown are the results from the different nonlinear programming method [16]. The target weight is 90 lb. The optimum weight of 91.24 lb is obtained form the LGP. There are 7 active constraints for the optimum design. Four of these constraints are related to members 3, 4, 5, 9, 18, 21, 22 and 24. The rest of the active constraints are related to the minimum size constraints on members 1, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24 and 25. The maximum positive deviation of goal constraints is $4\times10^{-4}$. In comparison with Ref.[16] the LGP yields very good results.

3.3 200 Member Plane Truss

The geometry and dimensions of a 200 member plane truss are shown in Fig.3.

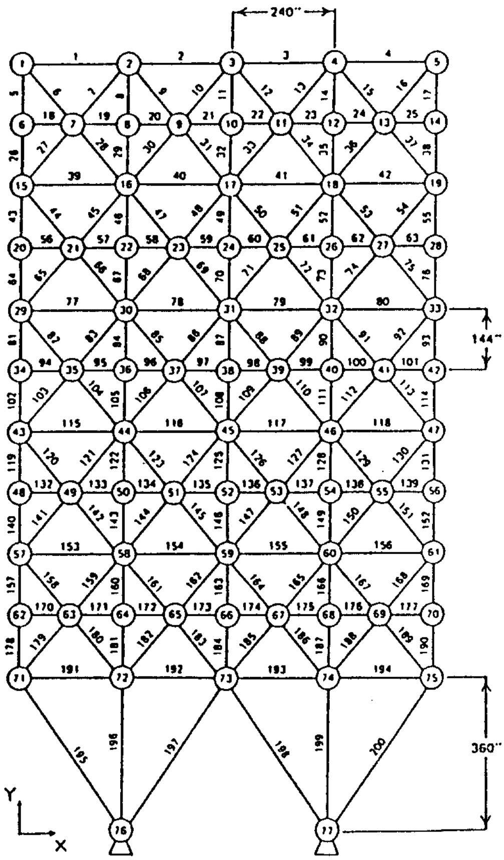


Fig.3 200 Member Plane Truss

This structure has 77 joints and 150 degrees of freedom. The structure is designed to withstand 3 loading conditions. Becuase of its symmetry the design variables are reduced to 96 variables. Table 5 gives design information for the structure. The plane truss is analyzed and designed subject to weight and stress goal constraints.

Results and Discussion

The optimum results for the 200 member plane truss are given in Table 6. Also shown are the results from the different nonlinear

**Table 5. Design Data for 200 Member Plane Truss**

| |
|---|
| Modulus of elasticity=$3\times10^4$ ksi |
| Material density=0.283 ib/$in^3$ |
| Lower limit on cross-sectional areas=0.10 $in^2$ |
| Initial value of design variable=1 $in^2$ |
| Stress limit=±30 ksi |
| Number of loading conditions=3 |

Loading Condition 1. One kip acting in position x direction at node points 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, 71.

Loading Condition 2. 10 kips acting in position y direction at node points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24,···, 71, 72, 73, 74, 75.

Loading Condition 3. Loading Conditions 1 and 2 acting together

**Table 6. Results for 200 Member Truss**

| Optimum Cross-Sectional Area in $in^2$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Member Numbers | LGP | Ref. 16 | Member Numbers | LGP | Ref. 16 | Member Numbers | LGP | Ref. 16 |
| 1,4 | 0.1 | 0.1 | 69,71 | 0.1 | 0.1 | 146 | 2.3872 | 2.4116 |
| 2,3 | 0.1 | 0.1 | 70 | 1.1678 | 1.1731 | 153,156 | 0.4268 | 0.425 |
| 5,17 | 0.2422 | 0.2469 | 77,80 | 0.2033 | 0.1892 | 154,155 | 0.1 | 0.1 |
| 6,16 | 0.1243 | 0.1185 | 78,79 | 0.1 | 0.1 | 157,169 | 1.2747 | 1.3251 |
| 7,15 | 0.1 | 0.1 | 81,93 | 0.9726 | 1.0402 | 158,168 | 0.8334 | 0.8462 |
| 8,14 | 0.3696 | 0.3675 | 82,92 | 0.4335 | 0.4108 | 159,167 | 0.1 | 0.1 |
| 9,13 | 0.1 | 0.1 | 83,91 | 0.1 | 0.1 | 160,166 | 4.6461 | 4.588 |
| 10,12 | 0.1 | 0.1 | 84,90 | 2.2790 | 2.2576 | 161,165 | 0.1 | 0.1 |
| 11 | 0.2457 | 0.2454 | 85,89 | 0.1 | 0.1 | 162,164 | 0.1295 | 0.1276 |
| 18,25,56,63, | | | 86,88 | 0.1 | 0.1 | 163 | 2.6533 | 2.6772 |
| 94,101,132,139, | 0.1 | 0.1 | 87 | 1.4448 | 1.4567 | 171,172,175,176 | 0.1 | 0.1 |
| 170,177 | | | 95,96,99,100 | 0.1 | 0.1 | 173,174 | 0.1 | 0.1 |
| 19,20,23,24 | 0.1 | 0.1 | 98,98 | 0.1 | 0.1 | 178,190 | 1.6080 | 1.6597 |
| 21,22 | 0.1 | 0.1 | 102,114 | 1.3059 | 1.3481 | 179,189 | 0.1158 | 0.1156 |
| 26,38 | 0.5755 | 0.5815 | 103,113 | 0.1028 | 0.1111 | 180,188 | 0.9145 | 0.9181 |
| 27,37 | 0.1 | 0.1 | 104,112 | 0.5003 | 0.4795 | 181,187 | 4.9795 | 4.9213 |
| 28,36 | 0.1809 | 0.1736 | 105,111 | 2.6123 | 2.5909 | 182,186 | 0.1348 | 0.1339 |
| 29,35 | 0.7030 | 0.7008 | 106,110 | 0.1029 | 0.1007 | 183,185 | 0.1 | 0.1002 |
| 30,34 | 0.1 | 0.1 | 107,109 | 0.1 | 0.1 | 184 | 2.9866 | 3.01 |
| 31,33 | 0.1 | 0.1 | 108 | 1.7781 | 1.7922 | 191,194 | 1.2073 | 1.2392 |
| 32 | 0.5790 | 0.5792 | 115,118 | 0.3054 | 0.2932 | 192,193 | 0.8204 | 0.8521 |
| 39,42 | 0.1127 | 0.1035 | 116,117 | 0.1 | 0.1 | 195,200 | 2.2635 | 2.3257 |
| 40,41 | 0.1 | 0.1 | 119,131 | 1.1883 | 1.2625 | 196,199 | 5.9840 | 5.9232 |
| 43,55 | 0.6547 | 0.676 | 120,130 | 0.6168 | 0.6050 | 197,198 | 2.5530 | 2.5718 |
| 44,54 | 0.2653 | 0.2489 | 121,129 | 0.1 | 0.1 | | | |
| 45,53 | 0.1 | 0.1 | 122,128 | 3.3982 | 3.3587 | Optimum | | |
| 46,52 | 1.2710 | 0.1 | 123,127 | 0.1 | 0.1 | Weight | 7472.7 | 7488 |
| 47,51 | 0.1 | 0.1 | 124,126 | 0.1157 | 0.106 | (lb) | | |
| 48,50 | 0.1 | 0.1 | 125 | 2.0538 | 2.0771 | No. of Iteration | 52 | 15 |
| 49 | 0.8344 | 0.8402 | 133,134,137,138 | 0.1 | 0.1 | No. of Active | 96 | 90 |
| 57,58,61,62 | 0.1 | 0.1 | 135,136 | 0.1 | 0.1002 | Constraints | | |
| 59,60 | 0.1 | 0.1 | 140,152 | 1.5216 | 1.5748 | No. of Function | 5141 | |
| 64,76 | 0.9881 | 1.008 | 141,151 | 0.1246 | 0.1325 | Evaluations | | |
| 65,75 | 0.1 | 0.1 | 142,150 | 0.6913 | 0.6817 | Max. Positive | $6.7\times10^{-4}$ | |
| 66,74 | 0.3276 | 0.3103 | 143,149 | 3.7315 | 3.692 | Deviation | | |
| 67,73 | 1.6044 | 1.5955 | 144,148 | 0.1183 | 0.1092 | Max. Constraint | | $5.2\times10^{-4}$ |
| 68,72 | 0.1 | 0.10 | 145,147 | 0.1 | 0.1 | Violation | | |

Table 7. Active Constraints at the Optimum Design for 200 Member Plane Truss

| Design Variables | |
|---|---|
| (i) Active Stress Constraints (57) | (ii) Active Minimum size Constraints (39) |
| 3,4,6,9,13,15,16,19, 20,22,23,25,28,31,33, 34,37,38,40,41,43,46, 49,50,51,52,54,55,56, 58,59,61,63,64,67,68, 69,70,71,73,74,76,77, 79,81,82,85,86,87,88, 89,91,92,93,94,95, and 96. | 1,2,5,7,8,10,11,12, 14,17,18,21,24,26,27, 29,30,32,35,36,39,41, 44,45,47,48,54,57,60, 62,65,66,72,75,78,80, 83,84, and 90. |

programming methods[16].

The target weight is 7480 lb. The opimum weight obtained from the LGP is 7472.7 lb. The active constraints are 96 for the optimum design. 57 of these constraints are related to stress constraints and the rest are related to the minimum size constraints. The detailed active constraints are shown in Table 7.

The maximum positive deviation of goal constraints is $6.7 \times 10^{-4}$. In comparison with Ref. [16], 0.2% weight reduction is achieved by the LGP.

## 4. CONCLUSION

A method is developed for solving large scale nonlinear structural optimization problems using linear goal programming. The method combines the finite element analysis and linear goal programming with successive linearization to solve the nonlinear structural opimization problem. The method has a wide range of applicability as a design tool, in which the ability to solve multi-conflicting objective problems is one of the great advantages of the goal programming.

The structural optimization examples performed demonstrates the ability of the method in solving large scale structural design problems. While the method is applied only to truss structures with stress constraints, it can be used for the design of other types of structures with different types of constraints. For some types of constraints such as displacement constraints a move limit techniques should be employed with the successive linearization to assure convergence.

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