General Response for Lateral-Torsional Buckling of Short I-Beams under Repeated Loadings

反復荷重을 받는 짧은 I형 보의 橫－비틀림 座屈의 一般的 應答에 관한 考察

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Abstract

The objective of this study is to perform extensive parametric studies of the lateral-torsional buckling of short I-beams under repeated loadings, and to gain a further insight into the lateal-torsional beam buckling problem. A one-dimensional geometrically (fully) nonlinear beam model is used, which includes superposed infinitesimal transverse warping deformation in addition to finite torsional warping deformation. A multiaxial cyclic plasticity model is also implemented to better represent cyclic metal plasticity in conjunction with a consistent return mapping algorithm.

The general response for the lateral-torsional buckling of short I-beams under repeated loadings is examined through several parametric studies around the standard case : the material yield strength, the yield plateau, the strain hardening, the kinematic hardening, the residual stresses, the load eccentricity with respect to the shear center, the height of the load with respect to the cross-section of the beam, the location of the load along the length of the beam, the dimesions of the cross-section of the beam and the fixity of the supported end remote from the load.

要 約

反復荷重을 받는 짧은 I보(Beam)의 橫－비틀림 座屈(Lateral-Torsional Buckling)에 대한 廣範圍한 Parametric Study를 遂行하여 보의 座屈現象을 좀 더 깊이 考察하고자 한다. 有限한 비틀림變形의 뒤틀림(Warping) 이외에 微少한 剪斷變形의 뒤틀림도 고려한 幾何學적 (完全) 非線形의 一次元 보를 解析的 모델로 사용하고, 또한 金屬의 周期的塑性(Cyclic Plasticity)擧動을 보다 잘 나타내기 위해 多軸 週期的塑性 모델을 Consistent Return Mapping Algorithm과 결합시켜 適用한다.

基準値 근방에서 아래와 같은 여러가지 Parameter Study를 遂行함으로써 反復荷重을 받는 짧은 I 보의 橫－비틀림 座屈의 一般的 應答을 考察한다 : 材料의 降伏強度, 降伏플래토(Yield Plateau), 變形率硬化, 移動硬化(Kinematic Hardening), 殘留應力, 作用荷重의 剪端中心에 대한 偏心率, 作用荷重의 보 斷面에 대한 높이, 作用荷重의 보 길이방향의 位置, 보 斷面의 置數, 作用荷重으로 부터 멀리 떨어진 支持端의 固定度.

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1. Introduction

Owing to their open thin-walled geometry, I-beams have a relatively low resistance to lateral-torsional buckling. General design philosophies adopt the point of view of the strength of I-beams. Such a philosophy is justified in most applications and has proven to be a valuable tool in the design of steel structures, but as a consequence, little attention has been paid to the problem of cyclic post-buckling behavior of I-beams. There are applications, such as earthquake resistant structures, for which one must consider the consequences of, and indeed design for, anticipated overloading and the possibility that these extreme loads will be repeated. In these instances the post-buckling strength of the structure is of primary concern.

Galambos[3] was the first to include the effects of residual stresses on the elastoplastic capacity of beams, and established the importance of their consideration. Woolcock and Trahair[9] considered the post-buckling of elastic beams and found that they can sustain loads in excess of the linearized bifurcation load. They correctly indicated that the additional capacity would be seldom realized due to the onset of yielding. Since most experimental investigations have been oriented toward verifying the predictions of the analytical models, and since most analytical models predicts only the buckling load, reporting of experimental data in the post-buckling range is scarce. Some data have been reported on the monotonically loaded beams (Augusti[1], Kitipornchai and Trahair[6, 7], Fukumoto, *et. al.*[2]). Hjelmstad and Lee[4] first performed the cyclic load tests. Exclusively little analysis has been done on the post-buckling problems of inelastic system.

Experimental results are often difficult to interpret because important properties such as initial imperfection, end restraints, and material properties are difficult to measure and implement. The experimental program is clearly limited in scope. Consequently, it is difficult to put the results into proper perspective. To ameliorate this condition it is needed to provide a thorough analysis of the experiments using a validated analytical model. The objective of this study is to perform extensive parametric studies of the cyclic lateral-torsional buckling of short I-beams, and to gain a further insight into the lateral-torsional buckling of short I-beams, and to gain a further insight into the lateral-torsional beam buckling problem.

Hjelmstad and Lee[5] developed a geometrically nonlinear beam model incorporated with a new cyclic plasticity model, which was verified by the same authors[4] to analyze lateral buckling of short I-beam under cyclic loading successfully. A beam model is formulated in terms of stress components and includes superposed infinitesimal transverse warping and torsional warping deformations to treat problems involving high shear and torsion. The kinematic constraint imposed in this model is appropriate for a thin-walled I-section geometry. A multiaxial cyclic plasticity model, incorporating many of the most compelling features of existing phenomenological models, is also implemented to better represent cyclic metal plasticity in conjunction with a consistent return mapping algorithm developed by Simo and Taylor[8], which is suitable for large-scale computation.

The general response for the cyclic lateral-torsional buckling of short I-beams is examined through several parametric studies around the standard case which is a close approximation of the test specimens in the experiments: the material yield strength, the yield plateau, the strain hardening, the kinematic hardening, the residual stresses, the load eccentricity with respect to the shear center, the height of the load with respect to the cross-section of the beam, the location of the load along the length of the beam, the dimensions of the cross-section of the beam and the fixity of the supported end remote from the load.

2. Analytical Model

A brief summary of important aspects of the analyticai model is presented, and test specimen (propped cantilever beam) is idealized for the analytical analysis. The properties of the standard case are also determined for parametric studies.

2.1 Description of the analytical model

The analytical model is based upon an extended kinematic assumption in which degrees of freedom are introduced for torsional warping and warping due to transverse shear. The motion of the beam is parameterized by the translation of the centroid, the finited rotation of the average cross-sectional plane, and warping out of this finitely deformed average plane. The kinematic hypothesis is used in conjunction with the finite deformation equilibrium equations of a constinuum with one-dimensional finite element interpolation of the generalized kinematic variables (three translation, three rotation, three infinitesimal transverse warping parameters, and one finite torsional warping parameter). The necessary integrations over the volume of the beam are done numerically.

The constitutive equations are treated locally using the three-dimensional equations of a continuum with a plane stress constraint. The model incorporates a proposed multiaxial cyclic plasticity in conjunction with consistent return mapping algorithm for a better representation of the cyclic loading behavior. Analytical beam model is formulated in terms of stress components and numerically treated by the finite element method.

The analytical model is described in detail by Hjelmstad and Lee[5].

2.2 Analystical model of the test specimen

The idealization of the test beam is shown in Fig. 2.1, neglecting the actual load cell in the experiments[4]. Translation, rotation, and warping are restrained at one end of the test beam. Vertical and lateral translation and torsional warping are restrained at the other end while axial extension and flexural rotations are unrestrained. Because of the presence of the load cell, the fixed end actually had a finite flexibility.

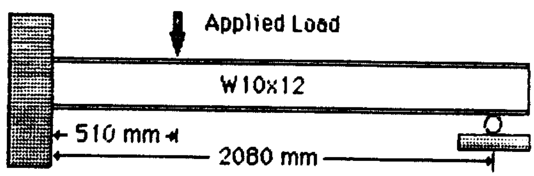

Fig. 2.1 Idealization of test beam

The load transfer mechanism is idealized using a rigid link, as shown in Fig. 2.2. The point of load application is a distance $\bar{a}$ above and $\bar{e}$ to the right of the shear center. The rigid link is modeled with a finite deformation box-section beam element, and is very stiff and remains in the elastic state throughout the

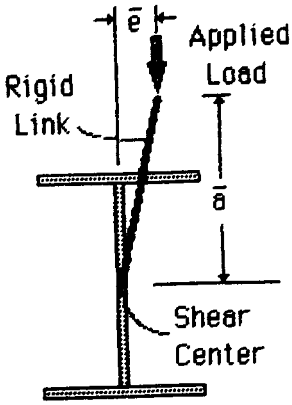

Fig. 2.2 Idealization of test beam

loading histories.

The parametric studies are carried out around a standard case which was determined to be a close approximation of the response exhibited by the test specimen 2[4]. The parametric values of the standard case are given in Table 2.1. In each of the subsequent parametric studies the standard case is perturbed to show the effect of the paramenters on the response of the beam. The close qualitative correspondence between measured and analytical response gives considerable confidence in the analytical model[4].

Table 2.1 Properties of the standard case

beam length	2080mm
location of load from fixed end	510mm
height of load above shear center	240mm
eccentricity of load position	0.25mm
depth of cross-section	250mm
width of cross-section	100mm
yield strength of material	330MPa
ultimate strength of material	475MPa
fixed end flexibility	rigid
right end condition	simple
residual stresses	none
equivalent plastic strain at onset of strain hardening	0.0235

3. Parametric Study

The general cyclic lateral buckling response of the test specimens without end flexibility and lateral bracing is examined in this chapter. The response of the systems with perturbed parameters are compared with the response of the standard model described in the previous chapter.

3.1 Effect of consititutive parameters

Several constitutive parameters are expected to have an important effect on the buckling resistance of beams because yielding tends to reduce the beam's resistance to buckling. Among these are the yield strength, the length of the yield plateau, and the strain hardening. The material properties are as follow : yield strength (σ_0)=330 MPa, ultimate strength (σ_u)=475 MPa, and equivalent plastic strain at onset of strain hardening $\bar{\varepsilon}_{sh}=$ 0.0235. Ultimate strengths of 475, 545, and 615 MPa correspond to yield strengths 330, 400, and 470 MPa, respectively, which means the shape of the strain hardenign curve is the same regardless of the value of the yield strengths. Fig. 3.1 describes the constitutive parameters mentioned above. Kinematic hard-

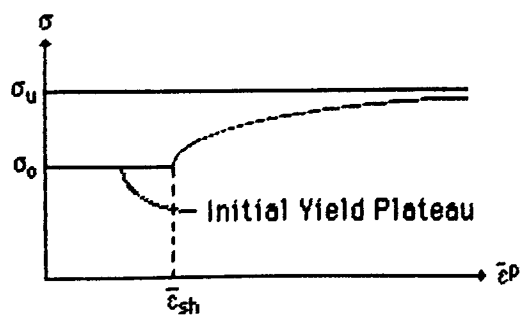

Fig. 3.1 Description of the constitutive parameters

ening employed in the proposed cyclic plasticity model is automatically included in al cases, except where this parameter is explicitly studied.

As expected, the yield strength influence the initial buckling load and post-buckling capacity of the beams, as shown in Fig. 3.2(a). While the response of initial-buckling are the same for the strain hardening case as for the perfectly plastic case at the same yield strength, the post-buckling capacity of the strain hardening case is larger than that of the perfectly plastic case for the same yield strength. The differences between them are almost the same regardless of the yield strength. The response curves of the perfectly plastic case and the strain hardening case in the post-buckling regime do not coalesce at large deformation because strain hardening has its greatest influence there. Judging from these observations, yield strength has a preponderant influence on the initial buckling load and the post-buckling behavior. Initial buckling generally occurs before strain hardening starts and hence strain hardening affects the post-buckling response.

Figs. 3.2(b,c) show the influence of each yield strength on cyclic response. As yield strength, pull yield load and asymptotic post-buckling capacity also notably increase. Differences in the cyclic responses between the strain hardening and the perfectly plastic cases are illustrated in Fig. 3.2(d-f). The latter case has a slightly smaller pull load and asymptotic post-buckling capacity than the former one in the first cycle. Observe that yield strength also has an influence on the cyclic response even the perfectly plastic case and the effect of strain hardening on the response to cyclic hardening.

The influence of the length of the initial yield plateau on the monotonic response is examined in Fig. 3.2(g). The response without an initial yield plateau has a slightly larger initial buckling load than any case with an initial yield plateau. The post-buckling responses are bounded below by the case with no plateau and above by the perfectly plastic case (infinite length plateau) for entire range of monotonic behavior. The response curves for various plateau lengths do not converge on each other at large deformation, probably due to strain hardening. Initial buckling occurs while most of the yielded material is on the yield plateau at plastic strain less than 0.01175, as evidenced by the fact that the responses for cases having a yield plateau greater than this value are identical at buckling. The effects of length of the initial yield plateau on the cyclic response of the test beams are examined in Fig. 3.2.(h). As noted previously the initial buckling load with no plateau is slightly larger than any one with plateau. Because strain hardening manifests earlier in this case the pull yield load and subsequent buckling loads also tend to be greater than the case that has a yield plateau. The observations are reinforced by comparing the other bounding case (perfectly plastic) with the standard case (see Fig. 3.2(d)). In general, one might conclude that the effects of the length of the yield plateau are minor on the initial buckling load and post-buckling behavior.

Figure 3.2(i) shows the influence on the cyclic response of the kinematic hardening model used to simulate cyclic plasticity. Due to the change in the way Bauschinger's effect is modeled in the absence of kinematic hardening, notable differences in the response during the pull recovery from buckling can be seen.

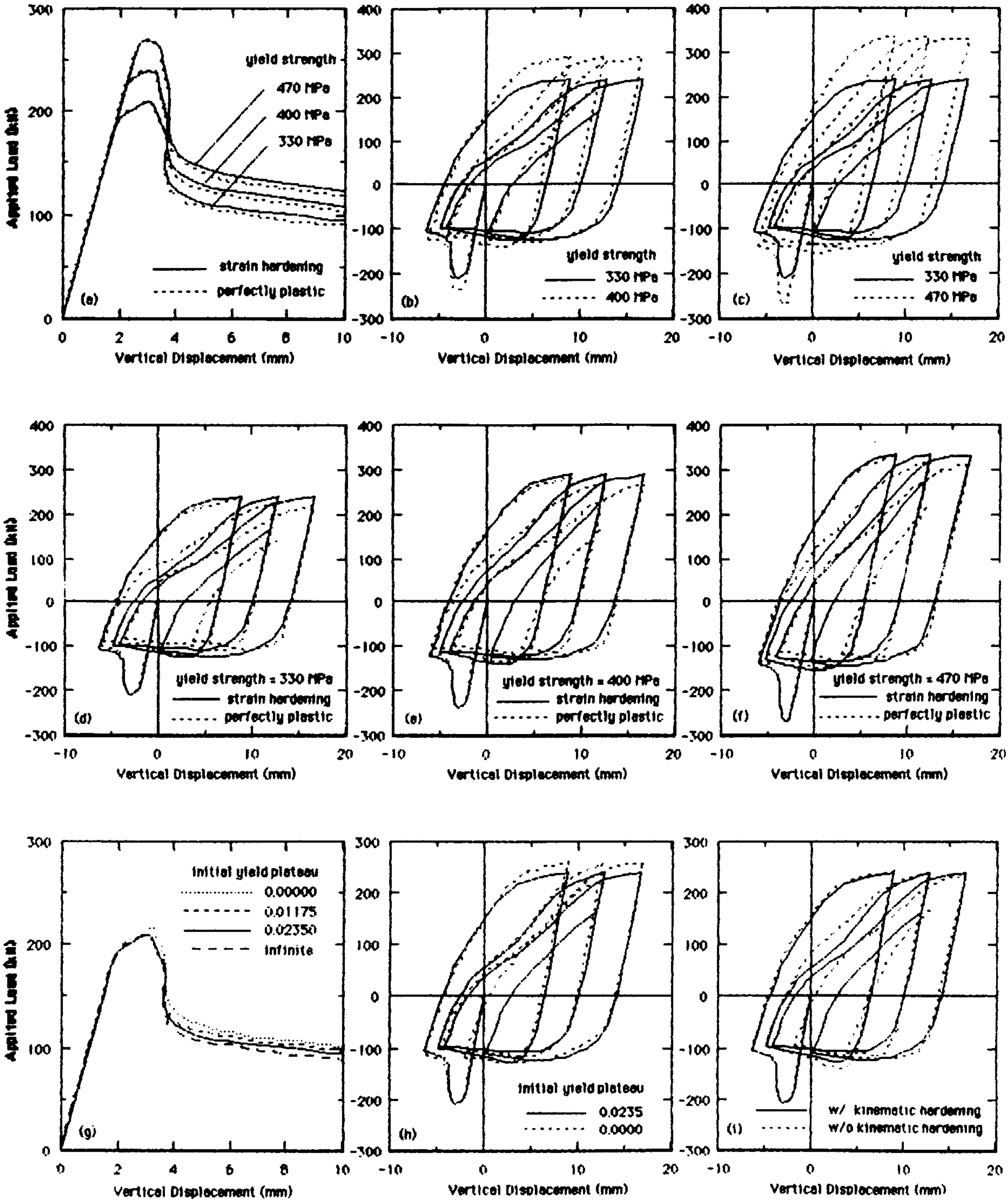

Fig. 3.2 Effect of constitutive paramenters

Qualitatively comparing these results with the cyclic load response of the test specimen 2 in the experiments (see Fig. 9[4]), one can recognize the importance of kinematic hardening to the model.

3.2 Effect of eccentrically placed load

Systems which exhibit limit load with unstable point limit behavior are generally sensitive to geometric imperfections. One of the important geometric imperfections in the propped cantilever test system is the eccentricity of the line of action of the load with respect to the shear center of the cross-section, as shown in Fig. 2.2. An eccentrically placed load will promote rotation of the cross-section prior to the buckling and will therefore reduce the magnitude of the limit capacity.

In general, it is impossible to achieve perfect placement of load in a physical test, although every effort was made to do so in the tests. In nature, even a perfect system will buckle if it passes through a bifurcation point. A perfect numerical model will not necessarily do so. In the studies performed here values of the eccentricity smaller than 0.25mm gave identical response of the system with respect to lateral buckling. Therefore the 0.25mm eccentricity is adopted as the perfect system for the analytical model.

The results of the initial monotonic buckling with values of eccentricity of 0.25, 1.25, 2.5 5.0 and 12.5mm are shown in Fig. 3.3(a). One can observe a considerable reduction in limit capacity for the modest eccentricities examined. The sharp limit response with sudden loss of capacity prevalent at small eccentricities begin to disappear at large eccentricities. On could surmise that the limit-type behavior would disappear entirely for a large enough eccentricity. For all values of eccentricity the post-buckling capacity is the same, even though for large eccentricities considerable deformations are required to achieve it. The tendency toward the same post-buckling capacity highlights the fundamental importance of this resistance parameter to the general response of these systems.

The cyclic response of the beams with initial load eccentricities is illustrated in Figs. 3.3 (b, c), covering eccentricities of 0.25, 2.5, and

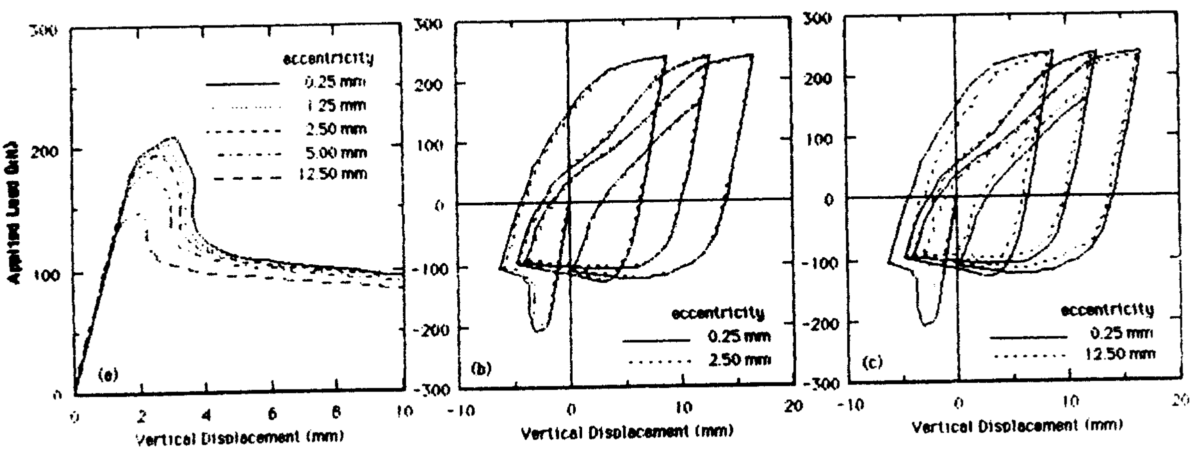

Fig. 3.3 Effect of eccentricity of load

12.5mm. One can observe that these eccentricities play a minor role in the cyclic response, the extent of influence being directly related to the magnitude of the eccentricity.

3.3 Effect of the height of the load point

It is well known that the height of the load with respect to the shear center of the corss-section has a significant effect on the linear elastic lateral buckling load. 240mm is taken as the standard value of the height of the load for the present parametric study, and a rigid link is used to apply the load remote from the shear center, as shown in Fig. 2.2.

The momotonic initial buckling and post-buckling response curves for load heights of 130, 200, 240, 280, and 350mm are shown in Fig. 3.4(a). As expected, the initial buckling load and post-buckling capacity increase with a decrease in the height of load and the rate of the loss of the post-buckling capacity is lessened as the the height of the load decreases. The response curves of the post-buckling do not coalesce at large deformation. Buckling is quite delayed for a load height of 130mm. One would thus expect that the beam would be more reluctant to buckle as the load is applied nearer to the shear center. Response to loads applied in the pull direction are expected to be stable.

The effects of load heights of 130 and 350mm on cyclic response are shown in Fig. 3.4(b,c), respectively. There is virtually no difference in the pull yield, but the height of the load has a significant influence on the limit load, the asymptotic post-buckling resonse, the post-buckling response at large deformation, and the subsequent cyclic loading response.

3.4 Effect of the load location along the beam length

The propensity of a beam to buckle laterally is directly related to the distance of the potentially destabilizing force from a pont where torsional motion is restraint. In design this distance is often called the laterally unsupported length. Qualitatively, the torsional stiffness accrues linearly with length from St. Venant resistance and cubically with length from

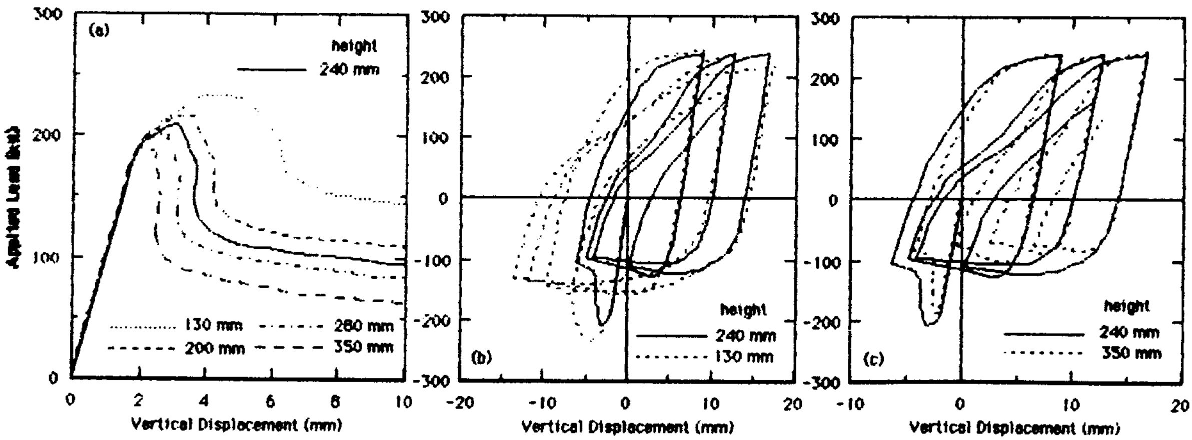

Fig. 3.4 Effect of height of load applicatoon

warping torsion resistance for an elastic beam. For short beams the warping torsional stiffness becomes so great as to practically preclude buckling.

The location of load along the length of the beam has a great influence on the initial buckling load and the post-buckling response, as shown in Fig. 3.5(a). The limit point can be seen to be sharper as the location of load approaches the middle of the beam, and initial buckling is delayed as the location of the load approaches the middle of the beam, and initial buckling is delayed as the location of the load approaches the fixed end. The response curves of post-buckling do not coalesce at large deformation.

The beam of the location of load of 380mm exhibits a strong reluctance to buckle. However, as shown in Fig. 3.5(b), even this short beam buckles at a vertical displacement of over 25mm. The circle symbols (o) on the monotonic response curves in Fig. 3.5(b) represent the points where the loading direction changes from push to pull in the cyclic loading histories. For the cyclic loading history, as shown in Fig. 3.5(c), this beam survives the first cycle without buckling but buckles in the second cycle. Since a great degree of strain

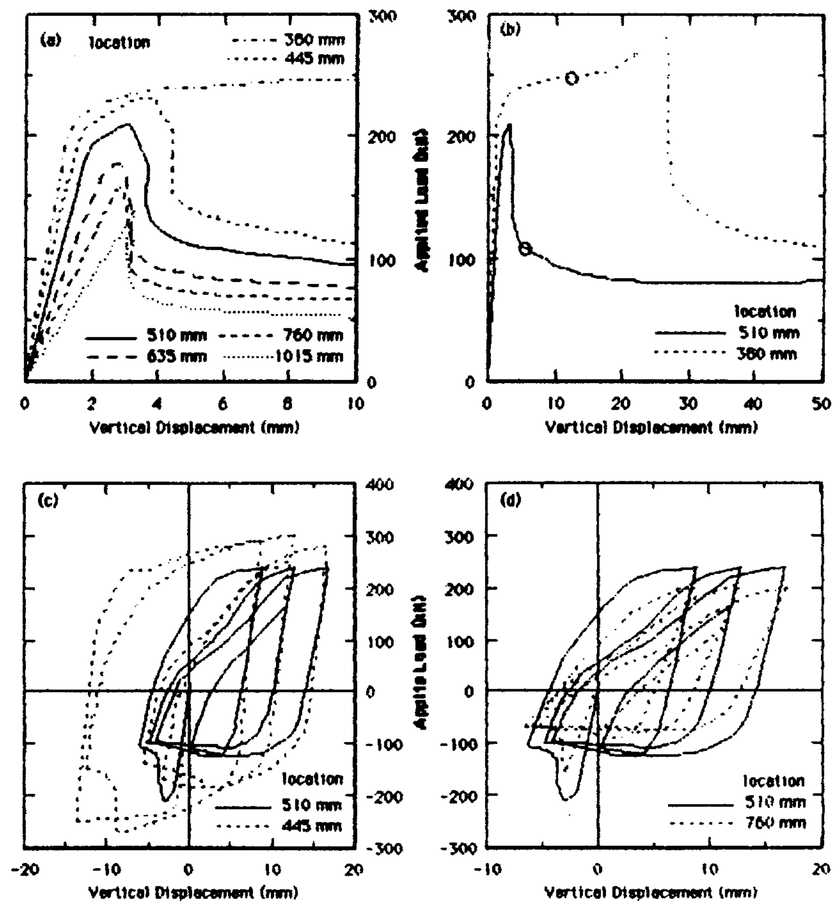

Fig. 3.5 Effect of location of the load along the length of the beam

hardening occured prior to buckling, the subsequent push and pull capacities are greater the comparison beam (ℓ =510mm). However, buckling did have a typically debilitating effect.

The cyclic response of a relatively long beam of ℓ =760mm clearly exhibits inferior behavior, as shown in Fig. 3.5(d), but the qualitative aspects of response are similar for the two cases.

3.5 Effect of cross-sectional dimensions

Resistance to lateral buckling clearly depends upon the geometric properties of the beams. In particular, the cross-sectional dimensions are expected to strongly influence the behavior. For short beams a great deal of the resistance to buckling comes from warping resistance which is dominated by the major moment of inertia of the flanges and the distance between them. Consequently, beam depth and flange width can be considered the most important geometric properties of the beam.

Since a variation of the cross-sectional dimensions with no change of length would give exaggerated results, the total lengths of the beam are chosen to give the same elastic deflection at the point of the loading as that of the standard case. All the dimensions of the beam studied are described in Table 3.1.

Table 3.1 Description of cross-section dimensions

h(mm)	b(mm)	L(mm)	ℓ(mm)	$\bar{a}$(mm)
250	100	2080	510	240(standard case)
250	125	2170	530	240
250	150	2240	545	240
375	100	2910	710	300

The monotonic buckling responses for a beam of depth 250mm (standard case) and width 100, 125 and 150mm are given in Figs. 3.6(a,d), and the cyclic responses are given in Figs. 3.6(b,c). One can observe that initial buckling is delayed by increasing the ratio of the width to height of the cross-section without a change in depth. There is a dramatic delay in initial buckling at the width of 125mm, and at the width of 150mm the beam does not buckle until well into the strain hardening regime. The asymptotic post-limit capacity also increases with the width of the beam.

Contrary to the case of variation in the width, changing the depth results in a relatively small change in the limit load, as shown in Figs. 3.6(e,f). However, the deeper beam exhibits a much sharper limit point than the standard case. The post-limit response curves converge right after initial buckling for monotonic loading, and the asymptotic post-buckling is almost the same for both. A peculiar feature can be noted in the first pull yielding region wherein a limit load occurs in the pull direction. Otherwise, the deeper section behaves like the shallower beam in the cyclic regime. One might conclude that increasing the flange width is an effective way to control buckling whereas increasing depth is not.

3.6 Effect of residual stresses

Residual stresses have long been recognized as having an important influence on the inelastic buckling of beams and columns. The beam stiffness is reduced by early yielding due to the presence of residual stresses, increasing the propensity to buckle. The pattern of residual stresses is well established for virgin

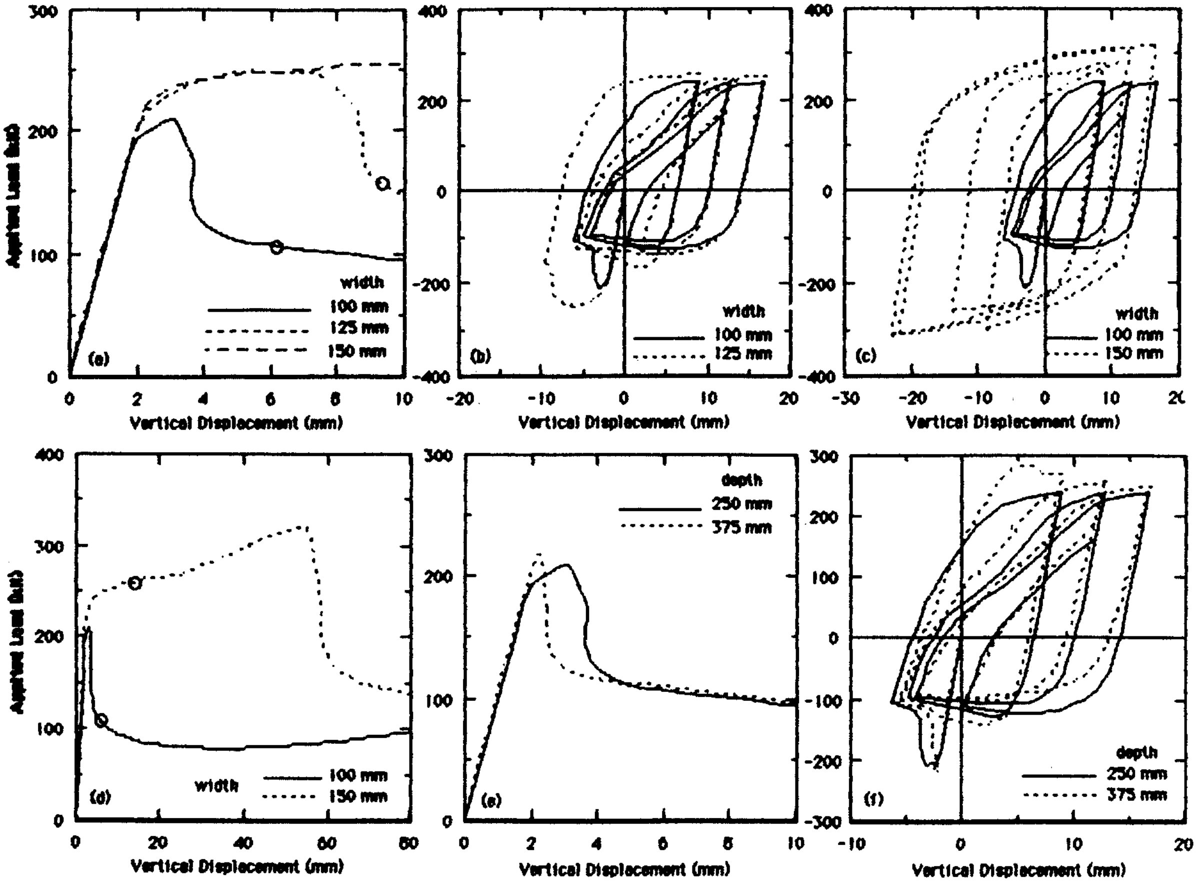

Fig. 3.6 Effect of cross-sectional dimension

sections, but this pattern may be canged by cyclic inelastic straining. A basic polynominal residual stress pattern is used for the analytical approximation, as shown in Fig. 3.7, and the maximum values range from 0 to σ_0 in steps of 0.25 σ_0 without changing the pattern.

Since yielding with residual stresses occurs well before initial buckling, the limit capacity decreases and the limit point blunts with an increase in the maximum value of residual stresses, as shown in Fig. 3.8(a). The response curves in the post-buckling range coalesce at large deformation. This feature indicates that the residual stresses have no effect on large

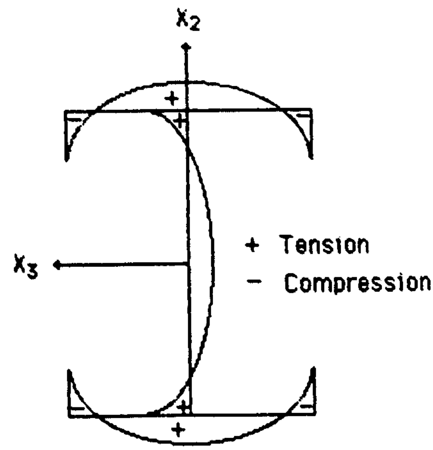

Fig. 3.7 Residual Stress distribution for typical rolled section

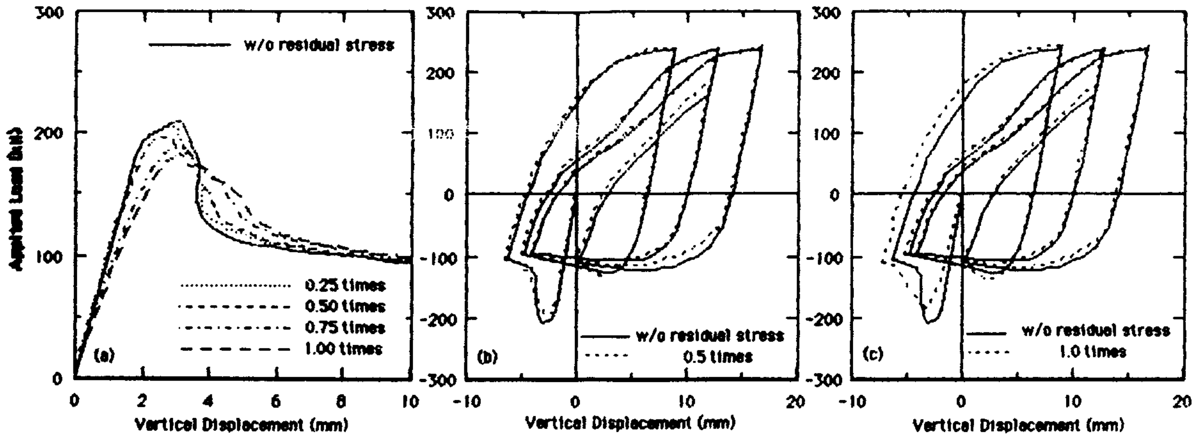

Fig. 3.8 Effect of residual stresses

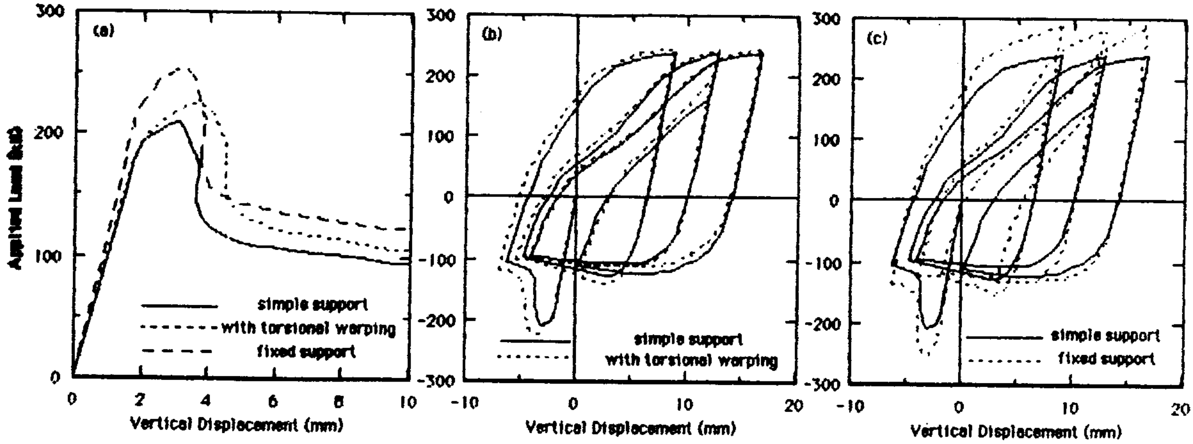

Fig. 3.9 Effect of boundary conditions on the right end

deformation behavior. They also have little effect on response to cyclic loading as shown in Figs. 3.8(b,c). The response seems to be almost recovered through the straightening of the residual twist left by inelastic buckling. Judging from this observation, the effect of the residual stresses is weaker than the influence of the residual twist of the beam left by the buckling.

3.7 Effect of right end boundary conditon

The degree of fixity at the boundary remote from the load is important to the buckling behavior. Three idealized right end boundary conditions are considered here : simple, torsional warping, and fixed. The boundary condition of the standard model is the simple support.

As expected, initial buckling and subsequent post-buckling capacities increase as right end fixity is increased, as shown in Figs. 3.9(a-c). Torsional warping restraint delays initial buckling and increases limit capacity. For the fixed support, the limit capacity increases much more over the simple support than does the addition of only torsional warping restraint but initial buckling occurs at almost the same vertical displacement as the simple case. This difference in buckling behavior could be attributed to the difference in initial stiffness. The pull yield load and asymptotic post-buckling capacity of the fixed support are much greater than those of the simple support condition. However, qualitative aspects of response for the three cases are similar for cyclic loading. From these observations, it can be recognized that restraint of torsional warping helps resist the initial buckling only, while full fixed has an effect on the response throughout the cyclic load history.

4. Summary and Conclusion

The general response for the test beams has been examined through several parametric sutudies around a standard configuration. A number of distinct features were found in this study that should be of value in the design against lateral buckling of short beams. The main observations can be summarized as follows :

(1) Effect of constitutive parameters.— The yielding strength of the material has a great influence on the initial lateral buckling capacity, the behavior at large deformation, and the response to cyclic loading. The characteristics of the yield plateau and strain hardening of mild steel strongly influence the post-buckling response but not the initial buckling. The details of modeling kinematic hardening were found to affect the response significantly. The current cyclic plasticity model did an adequate job of modeling the Bauschinger's effect in cyclic response.

(2) Effect of eccentricity placed load.— The initial horizontal eccentricity of the load with respect to the shear center has a strong influence on the limit capacity of the beam but has little effect on the post-buckling response and the response to cyclic loading, except when the initial eccentricity is quite large. The limit load is very sensitive to small load eccentricities.

(3) Effect of the height of load.— The height of the load with respect to the cross-section of the beam has a noticeable effect on both the limit capacity and asymptotic post-buckling capacity. Both capacities in crease as the load is placed close to the shear center. Furthermore, for loads placed closer to the shear center, buckling is delayed. Pull loads (Loads on the other side of the shear center) help stabilize the beam.

(4) Effect of the load location along the beam length.— The location of the load along the length of beam also has a significant effect on the limit capacity, the post-buckling capacity, pull yield load, and the deformation at which buckling commences in a cyclic loading program.

(5) Effect of cross-sectional dimensions.— A wide-section beam is better at resisting lateral buckling than is a deep-section beam. While a deeper beam can slightly improve the limit capacity, a wide-section delays or even prevents the lateral buckling of beam, because of the importance of warping resistance. Therefore, a wide I-beam would be more useful than a deep one in an application where lateral buckling resistance is important.

(6) Effect of residual stresses.— The residual stresses have an influence on the limit capacity of the beam, but have no effect at large deformation nor in the response to cyclic

loading. Residual stresses are less important in cyclic response because the residual twist in the beam left after inelastic buckling tends to overwhelm the influence of the residual stresse.

(7) Effect of the boundary condition at the right end. – The fixity of the end remote from the load has a great influence on the lateral buckling of the beam. Even addition of torsional warping restraint to the simple support condition increases the buckling load dramatically. The full fixed support has the highest limit load, but because of the increase stiffness, the beam tends to buckle at smaller deformation.

One-dimensional, geometrically nomlinear beam model used for the parametric studies appears to be an eminently suitable framework for modeling the lateral buckling of short I-beams.

The short beams exhibit sharp limit behavior with rapid post-limit loss of capacity under virgin loading. A non-zero asymptotic post-buckling capacity exists which is much smaller than the original buckling load, but not negligible.

Among the above parameters, the height of the load with respect to the cross-section of the beam, the location of the load along the beam, and a wide-section beam have noticeable effects on both the limit capacity and asymptotic post-buckling capacity.

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