Application of the Stochastic Finite Element Method to Structural System Reliability Analysis

확율유한요소법의 구조시스템신뢰성해석에의 적용

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SUMMARY

This paper is an attempt to account for the uncertainty of the residual strength in the reliability analysis of structural systems. For this purpose the stochastic finite element method(SFEM) is linked to the system reliability analysis procedure. The stochastic finite element is known to be able to a more explicitly consider the effect of unceraintes of material and geometric variables on those of load effects in structural analysis procedure. The method has been applied to system as well as component reliability analysis of a plane structure. Comparison of the results by the present approach is made with the method in which the residual strength of failed component is treated as deterministic variable. Several case studies have been carried to show the effect of uncertainty in residual strength of a member after failure. Is has been conformed that reidual strength very much affect the system reliability level. It can be, hence, concluded that the uncertainties in the post-ultimate behaviour may have to be taken into account in the system reliability analysis for a better a ssessment of the system reliability especially for a structure of which member behaviour is modelled as asemi-brittle model. And then the stochastic finite element method can efficiently evaluate the system reliability.

요 약

이 논문에서는 구조시스템신뢰성해석에 있어서 부재의 파괴후 잔류강도의 불확실성을 고려하였다. 이를 위하여 확율유한요소법(Stochastic Finite Element Method: SFEM)을 시스템신뢰성해석과정에 접합하였다. 확율유한요소법은 신뢰성해석 시 재료와 기하학적 변수의 불확실성을 좀더 함축적으로 고려할 수 있는 것으로 알려져있으며, 본논문에서 이 방법을 구조부재와 구조시스템의 신뢰성해석에 적용해 보았다. 이 논문

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이논문에 대한 토론을 1992년 3월 31일까지 본학회에 보내주시면 1992년 9월호에 그결과를 게재하겠습니다.

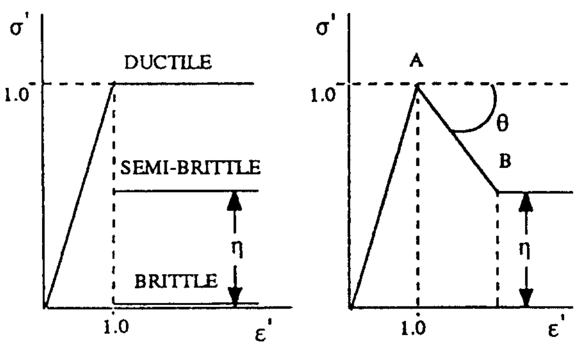
의 방법과 파괴된 부재의 잔류응력을 확정적으로 취급하는 방법과 그 결과를 비교하였으며, 부재가 파괴된 후 그 잔류강도의 불확실성이 구조시스템신뢰성에 주는 영향을 보기위해 여러 경우를 고찰해 보았다. 그결과로부터 부재의 파괴 후 잔류강도가 구조시스템신뢰성에 대단히 큰 영향을 준다는 것을 다시 확인할 수 있었다. 이논문의 여러경우에 대한 연구로 부터 좀 더 나은 구조시스템신뢰성의 평가를 위해서 부재의 파괴후 거동이 갖는 불확실성을 구조시스템신뢰성해석시, 특히 부재의 파괴후 거동이 semi-brittle인 경우에, 고려해야 한다는 결론을 내릴 수 있겠다. 이점을 받아들인다면 확율유한요소법이 구조시스템신뢰성해석에 있어서 적합한 방법일 것이다.

1. INTRODUCTION

During the last decade a general fraework for the reliability assessment of structural systems has been well established and it is matured tó apply the method to real structures. At these days' with regard to the methods for reliability analysis the advanced first-order reliability method(AFORM) is well accepted in assessing structural reliability for a single limit state equation. Many works have been carried out on the structural system reliability analysis. In system reliability analysis it has been well recognised that the post-ultimate behaviour of a member after failure much affect the residual strength of structural system and consequenntly on the system reliability(e.g, see references 1, 2, 3 and so on). The post-ultimate behaviour is usually characterised by the post-ultimate slope, θ[1] and the residual strength parameter, η as shown in Fig.1 which are commonly treated as deterministic variables at present. With reference to some sxperimental works, for example for the structural member found in offshore platforms,[4-6] there may be sufficiently large uncertainties in the post-ultimate slope and the residual structural parameter. Hence they should likely be treated as random variables.

The reliability analysis procedure commonly used at present is that structural analysis is carried out just once and the load effects, say stresses and displacements, are directly input into the reliability analysis procedure. And so the effect of variation of material and geometric variables on the variation of load effects are disregarded in the structural analysis and the level of uncertainties in load effects are assumed to be the same as those of loadings itself. In this paper for convenience the method adopting this assumption is termed as the "ordinary reliability method (ORM)" to distinct it from the stochastic finite element method. This approach gives reasonable level of structural reliability when the uncertainties in material and geometric variables are comparatively small and do not affect the variation of load effects. When the variation of material and geometric variables are considerably great, the result by this approach may be, however, well outside the true solution. To solve this problem the stochastic finite element method has been proposed and applied to reliability assessment of structures.[7-13] In this method the general procedure of the ordinary reliability method is to be followed and the structural analysis is repeated to get the sensitivities of basic variables. This approach can, hence, more explicity account for the effect of uncertainties in material and geometric variables and may give a closer solution to the true solution than the ordinary reliability method even for a single limit state equation, that is the component reliability analysis. One of major shortcomings is that the computational time is very much more expensive than the ordinary reliability method since the structural analysis should be repeated many times to get the gradient of limit state equation to random variables (at every iteration steps when iterative method is used). In spite of this the stochastic finite element method seems to be an adequate method to account for the uncertainties of the post-ultimate behaviour, say the residual strength and the post-ultimate slope. These are a kind of material variables.

In the next section detailed is the formulation procedure of a kind of stochastic finite element method. In system analysis the residual strength parameter only is considered, that is, the post-ultimate behaviour of member after failure is modelled into the two-state model as in Fig.1 (a). For the system reliability analysis, so called, the extended incremental load method developed by the present author is used to get the limit state equation of failure mode[3] and the identifying procedure proposed also in reference 3 is used. Several case studies for a simple plane frame structure have been carried out with varying the mean and COV of the residual strength parameter to investigate their effects on the system reliability. Comparison is made between the cases when the residual strength parameter is deterministic and when it is probabilistic.



(a) two-state model (b) three-state model

Fig.1 Typical Model of Post-Ultimate Behaviour

2. FORMULATION OF STOCHASTIC FINITE ELEMENT METHOD

The apparent difference of the stochastic finite element method from the ordinary reliability method is that uncertainties in material and geometric variables can be explicitly accounted for. A few algorithms have been proposed.[7-10] Reference 11 well summaries the state of-the-art in this area. In this paper the algorithm proposed by Kiureghian and Taylor[10,12] is adopted. An essential point in applying the method is to find the partial derivatives of limit state equation to random variables. Let divide the random variables into two groups: resistance variable vector $\{r\}$ and load variable vector $\{q\}$, that is, random variable vector $\{x\}$ is:

$$\{x\}=(\{r\},\ \{q\}) \tag{1}$$

and the limit state equation is expressed in terms of random variables as:

$$g(\{x\})=g(\{x\}),\ \{q\}) \tag{2}$$

Using the displacement method of structural analysis for a linear system of N degrees of freedom, the stiffness equation is given as:

$$[K]\{U\}=\{F\} \tag{3}$$

where $[K]$ is the stiffnes matrix of total structural system. $\{U\}$ and $\{F\}$ are nodal displacement and nodal force vectors, respectively. The elements of $[K]$ in general contains random variables such as material and geometric properties, and the vector $\{F\}$ contains geometric properties and the applied loads. It is clear that $[K]$ and $\{F\}$ are random and hence $\{U\}$ would be also random. For a linear structural system the load effects $\{q\}$ would be stress of elements or nodal displacement. For a component considered now the load effect is obtained from:

$$\{q^{(e)}\}=[B]\{u^{(e)}\} \quad (4)$$

in which superscript (e) is the element which contains the component considered now and matrix $[B]$ is the load effect-nodal displacement relation matrix of which each element is a function of material and geometric variables. At the current design point $\{x^*\}$ the limit state equation is $g(\{x^*\})=g(\{r^*\},\ \{q^*\})$ where $\{r^*\}$ is explicity known in terms of $\{x^*\}$. $\{q^*\}$ is given using Eqs.(3) and (4) as:

$$\{q^*\}=[B]^T([K]^{-1}\{F\})^{(e)}_{x=x^*} \quad (5)$$

in which the curled bracket of vectors $\{x\}$ and $\{x^*\}$ and omitted. Suerscript (e) is added to denote that the nodal displacement vector is related to element (e) is sorted out using the element topology data (as well known). The partial derivatives of limit state equation (2) to random variables is given by:

$$\{\frac{dg}{dx_i}\}_{x=x^*}=[\{\frac{dg}{dr_k}\}^T\{\frac{dr_k}{dx_k}\}+\{\frac{dg}{dq_k}\}^T\{\frac{dq_k}{dx_k}\}]_{\substack{x=x^*\\ r=r^*\\ q=q^*}} \quad (6)$$

All terms can be easily calculated except the term of$\{dq_k/dx_i\}$. Using Eq.(5) the derivative is given as follow:

$$\{\frac{dq}{dx_i}\}_{x=x^*}=[\{\frac{dB}{dx_i}\}^T\{u^{(e)}\}+[B]^T(\{\frac{dU}{dx_i}\})^{(e)}]_{x=x^*}=[\frac{d[B]^T}{dx_i}\{u^{(e)}\}+[B]^T(\frac{d[K]^{-1}}{dx_i}\{F\}+[K]^{-1}\frac{d\{F\}}{dx_i})^{(e)}]_{x=x^*} \quad (7)$$

It is easily shown that

$$\frac{d[K]^{-1}}{dx_i}=-[K]^{-1}\frac{d[K]}{dx_i}[K]^{-1} \quad (8)$$

Then Eq.(7) becomes

$$\{\frac{dq}{dx_i}\}_{x=x^*}=[\{\frac{dB}{dx_i}\}^T\{u^{(e)}\}+[B]^T[[K]^{-1}(-\frac{d[K]}{dx_i}\{U\}+\frac{d\{F\}}{dx_i})^{(e)}]]_{x=x^*} \quad (9)$$

where $\{U\}$ is obtained from Eq.(3) at the current design points.

After calculating partial derivative $\{dq_k/dx_i\}$ from Eq.(9) the partial derivative of Eq.(6) is completely calculated. Once after obtaining the partial derivative of limit state equation the iterative procedure[14] can work. The load effect in the limit state equation (2) is implicity a non-linear function of random variables. The above formulation of the stochastic finite element method has a merit that the available computer code for the ordinary reliability analysis can be used without much modification.

3. NUMERICAL EXAMPLES OF COMPONENT RELIABILITY ANALYSIS

Two plane frame structure models shown in Figs.2 and 3 are selected for the present study. Comparison is made between the component reliability indices by the ordinary reliability method and by the stochastic finite element method.

The simple protal frame model shown in Fig. 2 is frequently selected in the system reliability analysis and sensitivity stody. It has four beam elements and five nodes. Both nodes of an element are treated as components. Component failure is assumed to occur when bending moment at a particular element is reached the plastic bending moment, and the limit sate equation is given by:

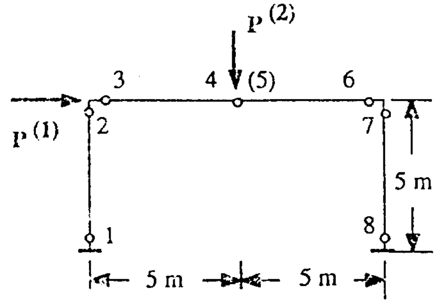
$$g(r,q)=r-q \quad (10)$$

where γ is plastic bending moment as strength and q bending moment as load effect. Data for this model are also shown in Fig.1. For simplicity all variables are assumed to be normally distributed and statistically independent. There are 8 random variables for each component (see Table 1). In the stochastic finite element analysis, the structural analysis procedure is repeated many time. A variable of which sensitivity factor is less than ε_α, after the first iteration, is treated as a deterministic one form the second iteration to reduce the number of random variables and computational time, where ε_α is a prescribed small number. Doing this is expected not to effect the result. For example Table 1 compares the results of Component 7 in Fig.2 when $\varepsilon_\alpha=0$ and 0.01, respectively and when COV of material and geometric variables, say E, A and I, is 20%. It can be seen that treating the variabless of which sensitivity factor are less than $\varepsilon_\alpha(=0.01)$ does not affect the result.

For most steel structures the COVs of elastic modulus and geometric variables are comparatively smaller than those of load effects and usually have the value ranging 4 to 10%. To see the effect of uncertainties in such variables on the reliability level, reliability indices of 7 componets in Fig.2 (Component 5 is the same as Component 4) are evaluated with varying the COV from 10% to 30%. These range may not be, of course, realistc and just for illustration. When applying the ordinary reliability metho, the limit state equation is given by:

$$g(r,q)=r-(q^{(1)}+q^{(2)}) \qquad (11)$$

in which $q^{(1)}$ and $q^{(2)}$ are bending moments due to load $P^{(1)}$ and $P,^{(2)}$ respectively. Table 2 shows the results by the ordinary reliability method and by the stochastic finite element method. The stochastic finite element method gives smaller reliability indices than the ordinary reliability method as can be expected. As it is seen, the difference of the reliability indices by the two method is small for this structure model. Even when the COVs of E, A and I are 30%, the difference lies 3 to 9% except Components 2 and 3. In the case that the COVs are less than 15%, the difference is less than about 5% for all components. This means that when COVs of material and geometric properties are less than about 15%, the ordinary reliability method gives reasonable reliability indices and within this range the stochastic finite element method could not keep its merit.



comp.	A_k	I_k	R_k
1,2	4.0	3.58	0.075
3,4	4.0	4.77	0.101
5,6	4.0	4.77	0.101
7,8	4.0	3.58	0.075

k =component number
A_k =cross sectional area(x10^{-3} m^2)
I_k =moment of inertia(x10^{-5} m^4)
mean yield stress=276 MPa
R_k=mean strength(=plastic bending moment, Mn)
$P_{(1)}$ =0.02 MN,$P_{(2)}$=0.04MN
COV of R_k=5%, COV of $P_{(1)}$ and $P_{(2)}$=30%

Fig.2 Portal Frame Model

As a reinforced concrete structure model, a 5 story-3 bay building shown in Fig.3 is considered.[12] The uncertainties of material and geometric properties of RC structures are comparatively greater than steel structures. Data for reliability analysis of this model are listed

Table 1 Sensitivity Factor of Random Variables for Component 7 of Portal Frame Model

(1) when $\varepsilon_{\alpha}=0$

variable	design point	α
R_7	0.7364E-01	0.2770
E	0.2100E+06	0.02506E-3
A^1	0.4000E-02	−0.9939E-04
I^1	0.3677E-04	−0.1038
A^2	0.4001E-02	−0.1306E-02
I^2	0.4631E-04	0.1112
$P^{(1)}$	0.2343E-01	−0.4373
$P^{(2)}$	0.5319E-01	−0.8419

$\beta=1.306$ $P_f=0.0958$

No.of iteration = 25

(note) A^1,I^1 = A and I of Components 1,2,7 & 8

A^2,I^2 = A and I of Components 3,4,5 & 6

(2) when $\varepsilon_{\alpha}=0.01$

varible	destgn point	α
R_7	0.7364E-01	0.2770
E	0.2100E+06	0.0000
A^1	0.4000E-02	0.0000
I^1	0.3677E-04	−0.1038
A^2	0.4001E-02	0.0000
I^2	0.4631E-04	0.1112
$P^{(1)}$	0.2343E-01	−0.4373
$P^{(2)}$	0.5319E-01	−0.8419

$\beta=1.306$ $P_f=0.0958$

No.of iteration = 2

Table 2 Reliability Indices to Changes in COVs of E, A and I of Portal Frame Model

comp.	by ORM*	by the stochastic finite element method			
		COV⁺=10%	15%	20%	30%
1	5.647	5.612	5.564	5.485	5.348
2	4.331	4.230	4.125	4.000	3.687
3	6.093	5.928	5.763	5.565	5.212
4 (5)	1.979	1.968	1.954	1.934	1.877
6	3.179	3.159	3.138	3.113	3.058
7	1.322	1.318	1.313	1.306	1.288
8	2.349	2.321	2.287	2.243	2.130

ORM* = ordinary reliability method

COV⁺ = COV of material and geometric variables of all components

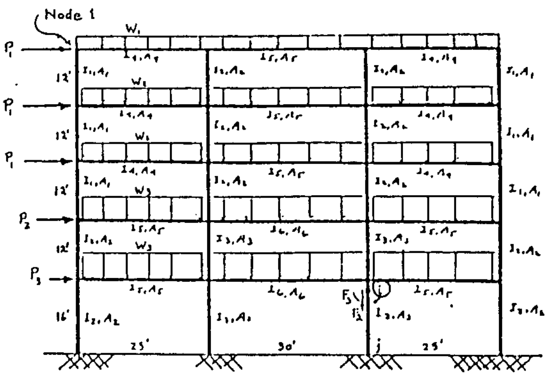


Fig.3 5 Story-3 Bay Building (after reference 12)

Table 3 Data for 5 Story-3 Bay Building(after reference 12): unit=kips, ft

variable	mean	COV	dist.type	varible	mean	COV	dist.type
W_1	6.00	0.18	log-normal	I_5	1.63	0.24	normal
W_2	7.50	0.18	log-normal	I_6	2.69	0.12	normal
W_1	8.00	0.18	log-normal	A_1	3.36	0.18	normal
P_1	22.5	0.40	extreme type-I	A_2	4.00	0.18	normal
P_2	20.0	0.40	extreme type-I	A_3	5.44	0.18	normal
P_3	16.0	0.40	extreme type-I	A_4	2.72	0.33	normal
E_1	454.0	0.09	normal	A_5	3.13	0.33	normal
E_2	497.0	0.08	normal	A_6	4.01	0.33	normal
I_1	0.94	0.12	normal	R_1	700.0	0.14	log-normal
I_2	1.33	0.12	normal	R_2	500.0	0.10	log-normal
I_3	2.47	0.12	normal	R_3	1400.0	0.11	log-normal
I_4	1.25	0.24	normal				

in Table 3. The COV of geometric variables ranges from 12% to 33%.

The limit sate equation concerns the reliability of element i-j of the model(see Fig.3). A Node i is taken here which is under the combined axial force and bending moment. For the purpose of illustration the equation given by Eq. (12) is taken as in reference 12:

$$g(\{x\})=1-\frac{F_2}{A_3R_1}-\frac{F_3}{I_3R_2[1-\frac{F_2}{R_3A_3}]} \tag{12}$$

where F_2 is the axial force and F_3 is the bending moment at node i. In Table 3 the distributed loads W_1, W_2 and W_3 have the same probabilistic characteristics except mean values and this also works for the concentrated loads P_1, P_2 and P_3. The load cases can be hence grouped into two cases:

Load case1 : distributed loads W_1, W_2 and W_3

Load case2 : concentrated loads P_1, P_2 and P_3.

For the ordinary reliability analysis the limit state equation can be expressed referring to Eq. (12) as:

$$g(\{x\})=1-\frac{(F_2^{(1)}+F_2^{(2)})}{A_3R_1}-\frac{F_3^{(1)}+F_3^{(2)}}{I_3R_2[1-\frac{(F_2^{(1)}+F_2^{(2)})}{R_3A_3}]} \tag{13}$$

in which superscript (1) and (2) are refer to the load effects due to Load cases 1 and 2, respectively. The stochastic finite element method is applied to Node i with data in Table 3 and another case that COV's of elastic modulus, sectional area and moment of inertia are uniformly given as 10% is also carried out. Results are summarised in Table 4. With data in Table 3, the stochastic finite element method gives 20% smaller reliability index than the ordinary reliability method. This difference may be due to that the large values of COV's of sectional area and moment of inertia much pull down the reliability when using the the stochastic finite element analysis. When COV's of all material and geometric variables are uniformly given as 10%, the difference is about 6%.

Table 4 Summary of Reliability Analysis for Building

	by ORM*	by stochastic finite element method	
		COV-I**	COV-II***
β	2.74	2.20	2.57
P_f	0.303E-02	0.0138	0.508E-02

ORM* =ordinary reliability method

CIV-I** =COV values of E, A and I in Table 4

COV-II***=COV of E, A and I of all member are 10%

4. APPLICATION TO SYSTEM RELIABILITY ANALYSIS

When using the stochastic finite element method, the computational cost is in general much more expensive than the ordinary reliability method even in the component reilability analysis since structural analysis is repeated tens of time to get the gradients of limit state equation to random variables. If the stochastic finite element method is linked to the system reliability analysis procedure, the computing time must be tremendous. Considering this point, the portal frame model in Fig.2 is selected again to illustrate the application of the present stochastic finite element analysis procedure to the system reliability analysis of a structure of which member behaviour is not ductile. The two-state model for post-ultimate behaviour shown in Fig.1 (a) is employed and the residual strength parameter only is considered.

4.1 Sensitivity Study

Firstly, investigated is the effect of uncertainties material and geometric variables on the reliability of failure path (called "path reliability" hereafter). For this, the path 7-4-8-2 shown in Fig.4 is selected which is the most important failure mode (or the most dominant failure mode) of the portal frame model, Fig.2 when the behaviour of all member is ductile.[15] Table 5 shows the reliability indices till system failure occurs with varying COVs of all material and geometric variables, say E, A and I, from 0 to 30%. It is shown that for the present frame model, the uncertainties in E, A and I do not affect the path reliability index especially as components failure is in progress. This can be also conformed from the sensitivity factors of such variables. To compare the sensitivity factor of random variables, the reidual strength parameter, η is included as another random variable. The path 7-4-8-2 is selected again for this purpose. As well recognised, since the important failure modes (or dominant failure modes) are depending on the post-ultimate behaviour after failure, the path 7-4-8-2 may not be the most important one when the post-ultimate behaviour of members is not ductile. Selecting the path 7-4-8-2 is just for illustration in this section.

As previously described, the two state model for the post-ultimate behaviour is employed. Four cases are considered as follows.

Case 1 : η=0.9 COV of E,A & I=20%
Case 2 : η=0.9 〃 =30%
Case 3 : η=0.5 〃 =20%
Case 4 : η=0.5 〃 =30%

COV of η, V_η is given as 10% for all cases. Table 6 shows the sensitivity factors of random variables after the first iteration. It is shown that the sensitivities of limit state equation(10) to variables E, A and I are negligiblly small even the case when their COVs are 30%. Their uncertainties, hence, do not give any influence on the path reliability index. Based on this finding the material and geometric variables (E, A and I) will be treate as deterministic ones hereafter.

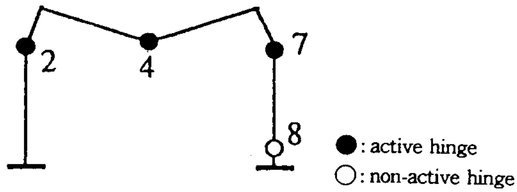


Fig.4 Failure Mode of Path 7-4-8-2

Table 5 Path Reliability Index of path 7-4-8-2 to Changes in COVs of E,A and I

COV of E, A & I	7	7-4	7-4-8	7-4-8-2
0%	1.322	1.868	2.096	2.489
10%	1.318	1.865	2.096	2.489
20%	1.306	1.858	2.095	2.489
30%	1.288	1.846	2.094	2.489

(note) COV of η=10%, COV of load: V_L=30%

Table 6 Sensitivity Factors of Random Variables in Limit State Equation after the First Iteration for Path 7-4-8-2 of Portal Frame Model

variable	η=0.5		η=0.9	
	$V_{E,A,I}$=20%	$V_{E,A,I}$=30%	$V_{E,A,I}$=20%	$V_{E,A,I}$=30%
R_2	0.5868E-01	0.5868E=01	0.6116E-01	0.6116E-01
E	−0.8895E-04	−0.8895E-04	0.6386E-04	0.6386E-04
A^1	−0.1221E-09	−0.1832E-09	−0.2389E-09	−0.3583E-09
I^1	−0.1149E-04	−0.1723E-04	−0.5417E-05	−0.8126E-05
A^2	−0.4787E-10	−0.7180E-10	0.1719E-10	0.2578E-10
I^2	−0.8981E-04	−0.1347E-03	0.5308E-04	0.7963E-04
$\eta_{7,8}$	0.1056	0.1056	0.6116E-01	0.6116E-01
η_4	0.2844	0.2844	0.1647	0.1647
P^{m}	−0.9179E-09	−0.9179E-09	−0.2461E-06	−0.2461E-06
P^{∞}	−0.9389	−0.9389	−0.9786	−0.9786
$R_{7,8}$	0.5281E-01	0.5281E-01	0.3058E-01	0.3058E-01
R_4	0.1422	0.1422	0.0823E-01	0.8237E-01

(note) A^1,I^1 = A and I of Components 1,2,7 & 8
A^2,I^2 = A and I of Components 3,4,5 & 6
$V_{E,A,I}$=COV of E, A & I COV of η: V_η=10%.

The load effect of component are mainly depending on loading variables. When any component has failed, it affects the stiffness matrix as well as the residual strength of a structure system, and consequently on the redistribution of load effect of survival components in a structure. Investigated is the relative influence of the residual strength parameter and load on the path reliability index. Several case studies have been carried with the path 7-4-8-2 as above. The range of mean and COV of η, and COV of load are listed as follows.

1) mean of η =0.5, 0.9 for all members
2) COV of η : 0, 10, 20, 30(%)
3) COV of load : V_L=10, 20, 30(%)

Altogether results of 24 cases are produced by applying the stochastic finite element method. Table 7 and Fig.5 show the path reliability indices to changes in uncertainties in η and load (Fig.5 is a graphic representation of Table 7). As far as the present numerical results are concerned, it can be drawn that:

1) the path reliability index depends more on the uncertainty of load than that of residual strength.
2) on the basis of the above finding, it may be natural that the path reliability is not sensitive to change in V_η when V_L is large, say 30% and this be due to that the large value of V_L overwhelms the effect of V_η. In similar when V_η is large, the path reliability index is not much affected by V_L.
3) as a typical case when V_L is 30%, the path reliability index, when V_η is 30%, is 12 and 18% less than the case when V_η is 0%, i.e., η is deterministic, and mean of η is 0.5 and 0.9, respectively.

From these discussions, although the effect of V_η on the path reliability inex is relatively smaller than that of V_L, it is clear that V_η gives nearly the same order of influence on the path reliability index as V_L and the path reliability index is also much sensitive to changes in V_η when V_L is comparatively small. This also conforms that the uncertainty in the residual strength of a failed component is important in the system reliability analysis.

4.2 Case Studies

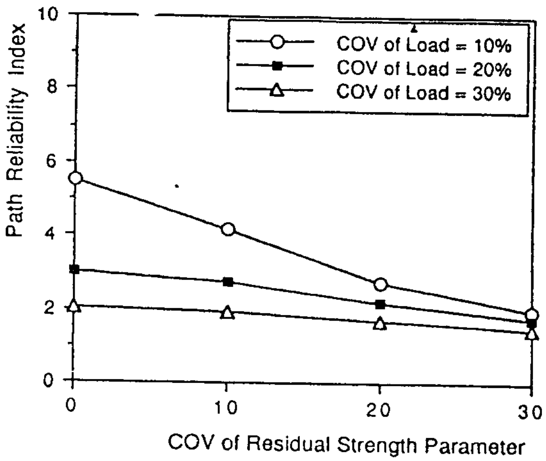
As mentioned before, the important failure mode is much dependent on post-ultimate behaviour of members. This section is concerned with the most important failure with varying V_η (COV of η) and V_L(COV of load) within the same range for Table 7 and Fig.5, and when η=0.5 as follows.

1) mean of η = 0.5 for all members
2) COV of η: V_η=0, 10, 20, 30(%)
3) COV of load : V_L=10, 20, 30(%)

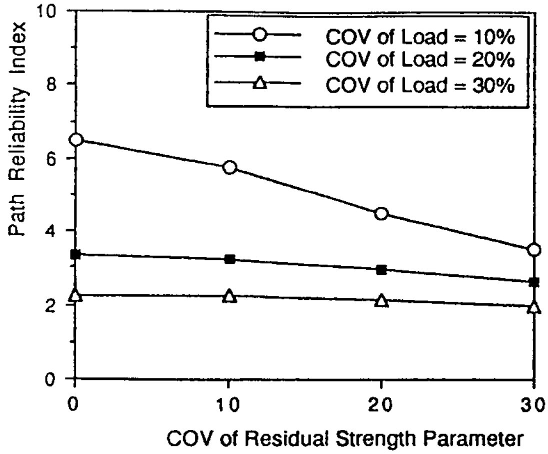
Fig.6 shows the result of case study. The most important failure mode of each case is shown with its path reliability (β_{path}) and the corresponding failure probability($P_{f,path}$). In identifying the most important failure mode, the procedure in reference 3 is adopted which considers the deterministic criteria as well as the probabilistic criteria. From Fig.6 it is shown that nearly the same tendency in effects of changes in V_η and V_L on the path reliability index as seen in Table 7 and Fig.5 in the previous section, and the uncertainty in η much decrease the path reliability index. When the post-ultimate behaviour of all members is ductile, the mot important failure mode is path 7-4-8-2 and its path reliability index is 2.489(see Table 5). When the post-ultimate behaviour of all members is non-ductile, its path reliability index much decreases as shown in Table 7 and Fig.5. However as seen in Fig.6, the path is not the most important one any more in the case of non-ductile system. This is due to the different

:-distribution of load effects from the ductile stem.

Path 4-7-8-3 at third and fourth row of right ımn in Fig.6 is actually one of the important ure mode of ductile system when V_L is 30% its path reliability index is 2.489 which ıe same as that of path 7-4-8-2 . As shown `ig.6 when V_L is 30%, η=0.5, and V_η is 20 30%, its path reliability is decreased by 65 and 70%, respectively. This typically : the effect of residual strength of failed nents on the residual strength of structural ., re-distribution of load effect and cons- y on the path reliability. Comparing path and path 7-4-8-2 when η=0.5 and V_η= : 7-4-8-2. This difference of reliability :tween the two paths is also due to the .ıt re-distribution effect between failure paths.



(a) η=0.9



(b) η=0.5

Fig.5 Reliability Index vs V_η and V_L of Path 7-4-8-2

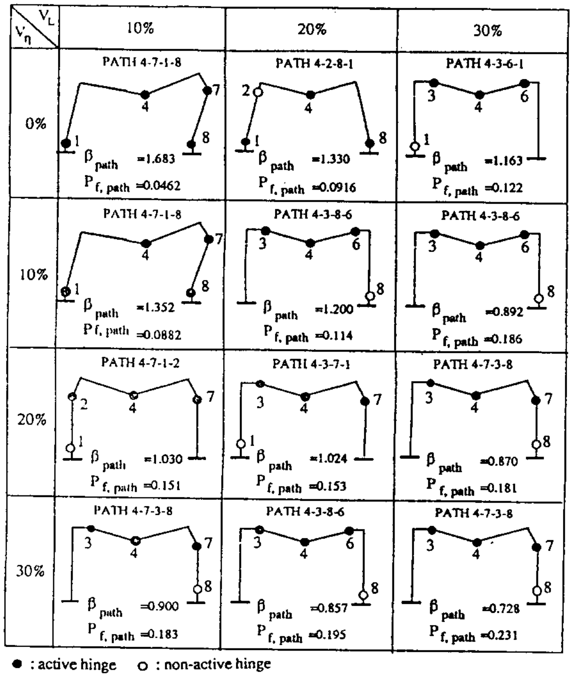


Fig. 6. Most Important Failure Modes to Changes in V_η and V_L when η=0.5

Table 7 Reliability Index of Path 7-4-8-2 to Changes in Uncertainties of Residual Strength Parameter and Load

(a) η=0.9

V_η \ V_L	0.1	0.2	0.3
0%	5.514	3.007	2.041
10%	4.180	2.723	1.945
20%	2.771	2.193	1.772
30%	1.992	1.742	1.476

(b) η=0.9

V_η \ V_L	0.1	0.2	0.3
0%	6.484	3.367	2.262
10%	5.772	3.254	2.262
20%	4.531	2.973	2.130
30%	3.534	2.633	1.994

5. DISCUSSION AND CONCLUSIONS

This study has concerned with the application of the stochastic finite element method to the reliability analysis of structural system. Formulation of the present stochastic finite element analysis in detailed

From the component reliability analysis the stochastic finite element method may have advantage over the ordinary reliability method when COVs of material and geometric variables are larger than 15 to 20%. This figure, of course, depends on the structure types. With regard to the results of system reliability analysis for a simple plane frame structure, it has been found that the path reliability index of failure mode is not affected by the variation of COVs of material and geometric variables, even when the COVs are large enough such as 30%. The residual strengths of failed components, as another material variables, give much lowering effect on the path reliability, rather than material and geometric variables, which is due to the effect on the residual strength and the re-distribution of load effects on components of structural system.

If one examine the previous experimental works for structural members found in offshore platforms, buildings and so on, it would be recognised that the post-ultimate behaviour is mainly non-ductile and there is sufficiently enough uncertainty in the post-ultimate behaviour. It can be, hence drawn that the uncertainty in the post-ultimate behaviour should likely be accounted for in the system reliability analysis, as concerned in this paper, for a better assessment of system reliability. And then the stochastic finite element method becomes adequate method in doing that.

As previously montioned, the post-ultimate behaviour is usually characterised by the residual strength parameter and the post-ultimate slope. This paper is basically an attempt to illustrate the effect of uncertainy in the residual strength on the path reliability of failure mode with the two-state model. More work would be needed in the application of the stochastic finite element method to reliability analysis of structural system before applying to real structures, and the uncertainties not only in the residual strength but also in the post-ultimate slope should be quantified for various tupes of structure members under possible loading conditions through experiments and non-linear structural analysis. This extension will be presented at the judicial conference in the near future.

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