論文 한국정밀공학회지 제9권 제3호 1992.9
Journal of the Korean Society of Precision Engineering Vol. 9. No.3 Sep. 1992.

이 논문의 일부는 1992년도 인하대학교 학술연구비 지원에 의해 연구되었음.

Design of New Sliding Surfaces for Fast Tracking Control

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빠른 추적제어를 위한 새로운 슬라이딩 서피스 설계

최 승 복*, 박 동 원*, 이 용 복**

초 록

가변구조 제어시스템(variable structure control system)을 위해 제안된 기존의 대부분 슬라이딩 서피스(sliding surface)는 주어진 초기조건과 무관하게 설계되었으며, 서피스 계수 또는 임의로 설정되었다. 이러한 서피스를 갖는 제어시스템은 슬라이딩 운동이 일어나기 전까지 외란등에 매우 민감하며 또한 느린 추적시간(tracking time)을 초래한다. 이러한 단점을 극복하기 위해 본 연구에서는 임의로 주어진 초기조건을 항상지나며 시간에 따라 기울기 및 절편이 변하는 새로운 슬라이딩 서피스를 설계하였다. 이 서피스와 연계된 제어시스템의 슬라이딩 모드(sliding mode) 존재성을 증명하였고, 서피스의 움직임 절차를 상세히 기술하였다. 2차 선형시스템과 2자유도계 로봇의 추적제어를 통해 제안된 방법의 효율성과 우수성을 입증하였다.

1. INTRODUCTION

In recent years, great attention has been given to controller design for a path tracking problem by utilizing the thoery of variable structure systems(VSS) [1-4]. The VSS theory is based on the concept of an attractive manifold of the underlying state or error vector space on which desired dynamic behavior is assured [5]. The VSS are a special class of nonlinear systems characterized by a discontinuous control action which changes structure upon reaching a set of sliding surfaces. A salient property of the VSS is the sliding motion of the state on the sliding surface. During this sliding motion, the system has invariance properties yielding motion which is independent of certain system parameters and disturbances. Therefore the design of the sliding surface completely determines the performace of system.

Most of sliding surfaces proposed so far have been designed without consideration of given initial conditions. Using these sliding surfaces, the sliding mode occurs only after the system reaches to the surfaces. Therefore, the tracking can be hindered by the disturbance especially during the reaching phase. Furthermore, the convergence to the surfaces may only be asymptotic, so that the benefits of the VSS cannot be realized. One easy way to minimize the reaching phase, hence to get fast tracking is to employ the larger control input. Young et al. [1] used the high-gain feedback to speed up the reaching phase. However, this may cause higher chattering which is undesirable in physical system. Slotine and Sastry [2] suggested a sliding surface in the error state space in

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order to eliminate the reaching phase by imposing a constraint that the initial errors be zero. However, this situation is not gereral but strictly special. Typically the initial conditions of actual system, hence the initial errors may be located arbitrarily.

It is also known that the gradients (slopes in case of second-order system) of most of conventional sliding surfaces are usually determined in an ad-hoc manner. Ashchepkov[7] established and proved the necessary conditions of optimality of the sliding surface in the sense of speed. For given arbitrary initial conditions and properly designed controller, the optimal value of slope of the surface was determined by minimizing the quadratic performace index. However, in his specific example, he imposed initial conditions to be located on the sliding surface, hence did not treat the reaching phase. Utkin and Yang[8] also proposed various procedures for synthesizing sliding surface having optimal motion with respect to several performance indices in the sliding mode along the intersection of their surfaces. In fact, in order to get fast tracking we need to construct an optimal sliding surface minmizing a performance index evaluated at two phases: a reaching phase and a sliding phase.

In this paper, we formulate an optimal sliding surface in the sence of motion speed before introducing a new sliding surface. The optimal surface is constructed by evaluating the tracking error performance index in both reaching and sliding phases. The relationship among the performance index, slope of the surface and discontinuous control gain is presented. And a specific tracking example is given for comparison between the optimal and nonoptimal sliding surfaces. Then we introduce a new sliding surface adaptable to arbitrary initial conditions. The surface is initially designed to pass given initial errors and subsequently moves towards a predetermined surface via rotating or and shifting. We call it as a moving sliding surface (MSS) comparing with the conventional ones, for instances, employed by Slotine and Sastry[2] or Spurgeon[4]. Using the MSS, it is shown that the tracking is much faster than even the optimal one without increasing the magnitude of discontinuous control gain. To demonstrate some advantages of the proposed method, we apply the MSS to the path tracking control of a two-degree-of-freedom robotic manipulator subjected to external disturbances.

2. OPTIMAL SLIDING SURFACE

Consider typical second-order linear system described by

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = a_1x_1(t) + a_2x_2(t) + bu(t) \qquad (1)$$

$$x_1(t_0) = x_{10}, \quad x_2(t_0) = x_{20}.$$

where a_1, a_2 and $b(\neq 0)$ are known constants, and x_{10}, x_{20} are initial conditions given at initial time t_0. The control problem is to get $x(t) = (x_1(t), x_2(t))^T$ to track a desired trajectory $Xd(t) = (Xd_1(t), Xd_2(t))^T$ which belongs to the class of C^1 functions on (t_0, ∞). In other words, the controller should force the tracking error to zero asymptotically for any given initial states. Thus we define the tracking error $e(t)$ as

$$e(t) = x(t) - x_d(t) \quad \text{or}$$

$$(e_1(t), e_2(t))^T = (x_1(t)-x_{d1}(t), x_2(t)-x_{d2}(t))^T \qquad (2)$$

and also define the sliding surface $s(e(t))$ (a line in this case) in the error state space by

$$s(e(t)) = ce_1(t) + e_2(t), \quad c > 0. \qquad (3)$$

We see that the tracking error $e(t)\to 0$ for any given initial conditions provided that there exists a control $u(t)$ so as to cause the trajectory $x(t)$ to slide along the surface defined by (3). This can be achieved by satisfying the sliding condition

$$s(e(t))\dot{s}(e(t)) < 0. \tag{4}$$

We construct a discontinuous control u(t) from the concept of equivalent control as follow.

$$u(t) = [-a_1x_1(t) - (c+a_2)x_2(t) + cx_{d2}(t) + \dot{x}_{d2}(t) - k\ \mathrm{sgn}(s(e(t)))]/b \tag{5}$$

where k might be any positive number. Then the tracking problem reduces to following quation.

$$\dot{e}_1(t) = e_2(t)$$

$$\dot{e}_2(t) = -ce_2(t) - k\ \mathrm{sgn}(s(e(t))) \tag{6}$$

$$e_1(t_0) = v_1, \quad e_2(t_0) = v_2$$

From the trajectory behavior of the error states $e_1(t)$ and $e_2(t)$ in the sense of convergence speed to zero, we may assume $s(e(t_0))>0$. Then, we obtain the solution of the equation (6) in two phases: the reaching phase ($t<t_s$) and the sliding phase($t\geq t_s$), where t_s is the time at which the sliding mode begins, i.e.,

i) the reaching phase ($t<t_s$)

$$e_1(t) = k/c(-\exp(-ct)/c - t) - v_2\exp(-ct)/c + (k/c + v_2)/c + v_1$$

$$e_2(t) = k/c(\exp(-ct) - 1) + v_2\exp(-ct) \tag{7}$$

ii) the sliding phase ($t\geq t_s$)

$$e_1(t) = \{k/c^2\ (-1 + \exp((c^2v_1 + cv_2)/k)) - v_2/c\} \times \exp(-ct) \tag{8}$$

$$e_2(t) = \{k/c\ (\exp((c^2v_1 + cv_2)/k) - 1) + v_2\}\exp(-ct)$$

where $t_s = (cv_1+v_2)/k$

Our objective is to get tracking as fast as possible, hence we may choose performance index J as follow.

$$J = \int_0^\infty e_1{}^2(t)\,dt = \int_0^{t_s} e_1{}^2(t)\,dt + \int_{t_s}^\infty e_1{}^2(t)\,dt \tag{9}$$

Minimizing J, we can obtain optimal value of c (or k) with given initial errors (v_1, v_2) and k (or c).

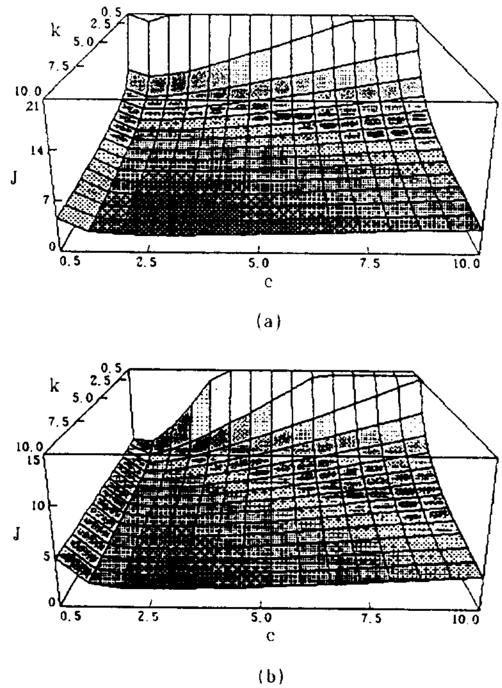


Fig. 1 Variation of the performance index : (a) $(v_1, v_2) = (2, 1)$; (b) $(v_1, v_2) = (2, -1)$.

Figure 1 presents the variation of the performace index J with respect to the values of c and k under specified initial errors. We clearly observe that the optimal value of c which minimizes the J heavily depends upon the discontinuous control gain k and also initial errors. To manifest this feature, we consider following example used by Hong and Wu [10].

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = x_1(t) + 2x_2(t) + 3u(t) \tag{10}$$

And the desired state trajectory is chosen by

$$x_{d1}(t) = 0.2t$$

$$x_{d2}(t) = 0.2$$

Then the controller(5) for satisfying the sliding condition(4) becomes

$$u(t) = [-x_1(t) - (c+2)x_2(t) + 0.2c - k\,\mathrm{sgn}(s(e(t)))]/3 \qquad (11)$$

where $k=0.5$. Figure 2 shows the state trajectory with different initial conditions. In this figure, T_t represents the tracking time determined on the basis that the tracking error between the desired and actual trajectories enters below one percent. The optimal slope c_{opt} is chosen by minimizing the performance index J in equation(9), and the value of $c=3.000$ is chosen arbitrarily. We clearly observe that the optimal sliding surface provides much faster tracking than arbitrarily chosen one which

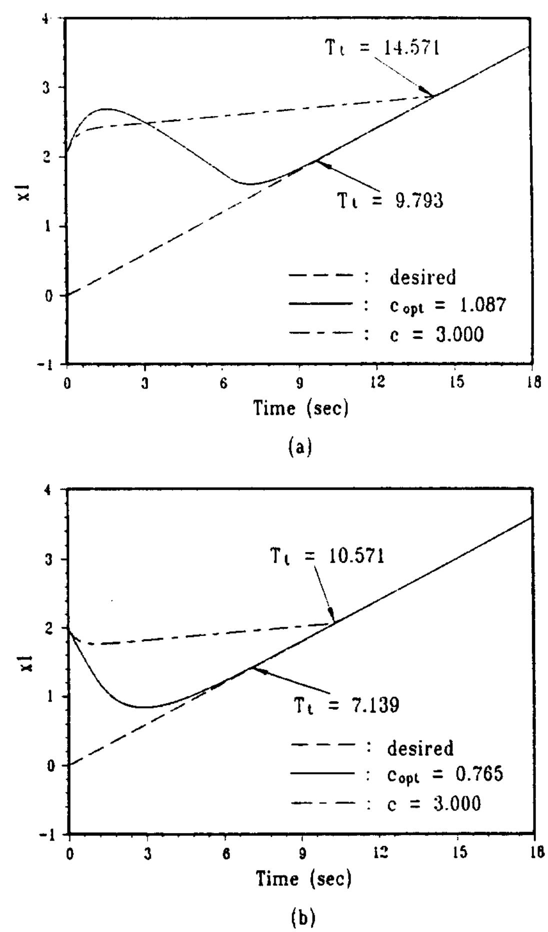


Fig. 2 State trajectories : (a) $(x_1(0), x_2(0)) = (2, 1.2)$; (b) $(x_1(0), x_2(0)) = (2, -0.8)$.

has larger magnitude of the slope. Of course, if we increase the gain k, the situation becomes quite different. This, however, may cause extreme system sensitivity to unmodelled dynamics, actuator saturation and undesirable higher chattering as well. It is noted that the sliding surface defined by the equation(3) is fixed with constatnt slope of c or c_{opt} in the error state space. In the subsequent section, we improve tracking behavior of the system without increasing the gain k by introducting a new sliding surface called the moving sliding surface.

3. MOVING SLIDING SURFACE

As mentioned in Introduction, the basic philosophy of the moving sliding surface(MSS) is that the surface is initially chosen to pass given arbitrary initial conditions, and we subsequently move the surface towards the predetermined sliding surface. The movement can be executed by rotating or/and shifting. Thus, we divide the MSS into two types and call them as the rotating sliding surface (RSS) and the shifting sliding surface(SSS), respectively. The movement for RSS is associated with time-varying slope of the surface which belongs to a *step function* to be defined below. On the other hand, the movement for SSS is accomplished by employing time-varying intercept of the surface which also belongs to a *step function*.

Definition 1. A function $\psi : R \rightarrow R$ defined on $[a, b]$ is called a *step function* if there is a partition given by $a=v_1<v_2<\cdots<v_n=b$, such that ψ is constant on each open subinterval (v_{k-1}, v_k), $1<k\le n$.

3.1 Rotating Sliding Surface

Let us define the rotating sliding surface as

$$s_r(e(t), t) = c_r(t)\, e_1(t) + e_2(t)$$
$$s_r(e(t_0), t_0) = s_{r0} = c_{r0}\, e_1(t_0) + e_2(t_0) \qquad (11)$$

We obviously see that the surface initially goes through given initial errors $e(t_0)$ with the corresponding slope c_{r0}. In other words, the representative point(RP) initially lies on the surface s_{r0} as shown in Figure 3(a). In this figure, s_p represents the predetermined sliding surface defined by $s_p = c_p e_1(t) + e_2(t)$. Before describing a moving algorithm, we summarize the argument for existence of sliding mode in a following theorem.

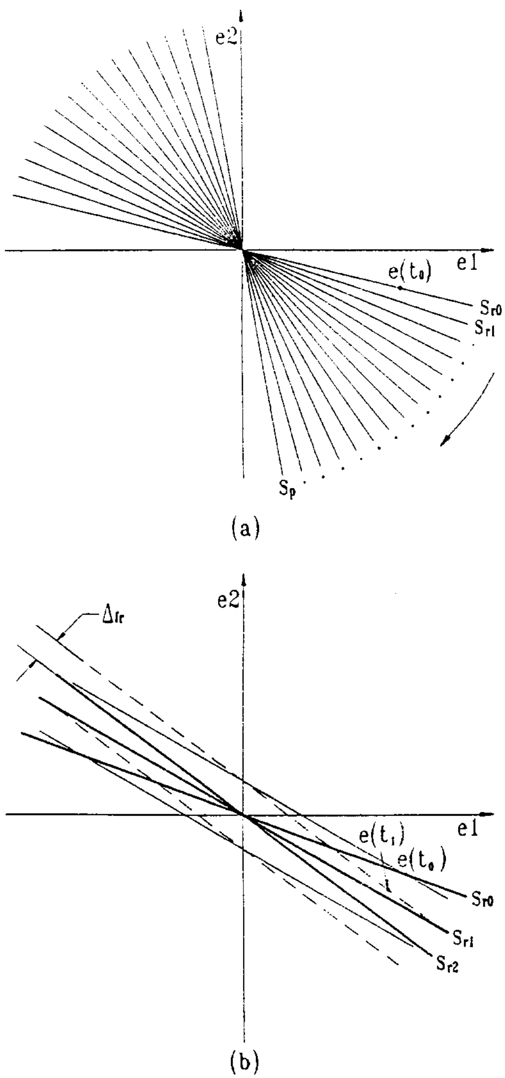


Fig. 3 Rotating sliding surface : (a) configuration; (b) mechanism.

Theorem 1. If $c_r(t)$ in equation(11) is chosen to be a *step function* for $t \in [t_0, t_r]$ with terminal values of $c_r(t_0) = -e_2(t_0)/e_1(t_0)$ and $c_r(t_r) = c_p$, and $c_r(t)$ be a constant function when $t \in (t_r, \infty)$ with $c_r(t) = c_p$, the system (1) with controller (5) incorporating the sliding surface (11) satisfies sliding condition $\dot{s}_r(e,t)\dot{s}_r(e,t) < 0$ almost everywhere.

proof : From Definition 1, there exists a partition $P = \{v_1, v_2, \cdots, v_n\}$, i.e., $t_0 = v_1 < v_2 < \cdots v_n = t_r$ such that $c_r(t)$ is constant on each open subinterval (v_{k-1}, v_k), $1 < k < n$. Trivially, P is a finite set. So we can prove that P is measurable and $m(P) = m_e(P) = 0$ [11], where m denotes Lebesgue measure and m_e stands for Lebesgue exterior measure. Therefore, since we chose $c_r(t)$ to be a *step function* on $[t_0, t_r]$, $\dot{c}_r(t) = 0$ for $t \in [t_0, t_r] - P$ and $\dot{c}_r(t) = 0$ for (t_r, ∞). Hence the control system (1) and (5) with the sliding surface (11) obviously satisfies sliding condition: $s_r(e,t)\dot{s}_r(e,t) < 0$ for $e \in R^2 - s_r$, $t \in [t_0, t_r] - P$ and $s_r(e,t)\dot{s}_r(e,t) < 0$ for $e \in R^2 - s_r$, $t \in (t_r, \infty)$. This concludes the proof.

Now we can move the sliding surface s_{r0} to the s_p by employing time-varying slope $c_r(t)$ without violating the sliding condition almost everywhere. The moving algorithm proposed in this study may be outlined as follows.

step 1. We determine an appropriate constant Δ_r required to rotate the surface and define (refer to Figure 3(b)) $\Delta_{fr} = \Delta_f + \Delta_r$, where Δ_f denotes the vicinity magnitude of the surface due to nonidealities such as delay, hysteresis and etc. The value of Δ_r plays a crucial role for improving the tracking behavior. The smaller value of Δ_r, the faster tracking time. If Δ_r approaches to zero, the RP may cross the sliding surface resulting in sluggish motion.

step 2. We calculate the initial slope c_{r0} satisfying the equation $s_{r0} = 0$ according to given initial errors $e(t_0)$; $c_{r0} = -e_2(t_0)/e_1(t_0)$.

step 3. The rotating direction is determined from the values c_{s0} and c_p, i.e., if $c < c_p$: clockwise (CW), and if $c_{s0} > c_p$: counter-clockwise (CCW).

step 4. We instantaneously rotate the s_{s0} to s_{s1} which has the slope c_{s1} obtained by solving the equation $|c_{s1}e_1(t_0)+e_2(t_0)|=\Delta_{rs}$. The larger value of two solutions c_{s1} is chosen as the slope for clockwise, and the other for counter-clockwise. The surface s_{s1} stays for a finite time (we call it as the dwelling time $(\Delta\tau)$ of the surface) before moving to the next surface s_{s2} whose slope C_{s2} is obtained by solving the equation $|c_{s2}e_1(t_1)+e_2(t_1)|=\Delta_{rs}$, where $t_1=t_0+\Delta\tau$. We know that the dwelling time $\Delta\tau$ also plays a crucial role as the Δ for the system performance. The shorter dwelling time $\Delta\tau$, the faster tracking time. If the $\Delta\tau$ is chosen to be long, the RP may cross the sliding surface. Then the control input signal which has opposite sign is activated to drive the RP to the opposite direction resulting in sluggish motion. The rotating is continuously performed in a same manner until following step is checked.

step 5. We stop the rotating under following condition, i.e., if $c_{sn}>c_p$, then fix $c_s(t)=c_p$:CW, and if $c_{sn}<c_p$, then $c_s=c_p$: CCW. The slope c_{sn} is obtained by solving the equation $|c_{sn}e_1(t_{n-1})+e_2(t_{n-1})|=\Delta_{rs}$, where $t_{n-1}=t_0+(n-1)\Delta\tau$.

From the configuration of Figure 3 one can naturally ask how about the initial errors are located in the unstable zone, i.e., the first and third quadrants. If we define the sliding surface to go through the initial conditions and the origin as well, the surface itself is unstable. Therefore, it is no doubt that the RP on the sliding surface goes away from the origin until it arrives to stable zone. From the mathematical point of view, though it is possible to drive the RP to the origin in a finite time by employing the RSS, this may cause much longer tracking time than the conventional one given by the equation (3). Furthermore, in practice this may give rise to the destruction of the physical system. To avoid this problem we propose the shifting sliding surface.

3.2 Shifting Sliding Surface

We define the shifting sliding surface as

$$s_s(e(t),t) = c_p e_1(t) + e_2(t) + \alpha(t) \tag{12}$$

$$s_s(e(t_0),t_0) = s_{s0} = c_p e_1(t_0) + e_2(t_0) + \alpha_0$$

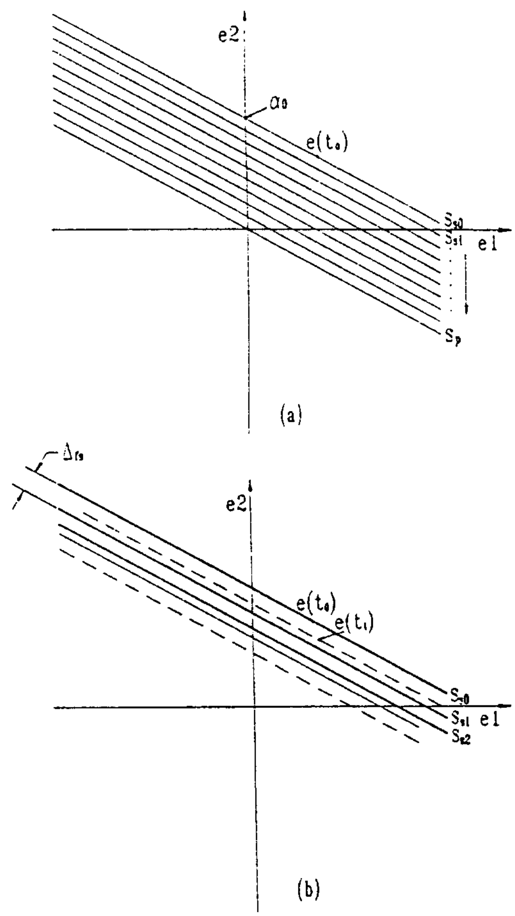


Fig. 4 Shifting sliding surface : (a) configuration; (b) mechanism.

where c_s is the slope of the predetermined sliding surface s_p, and $\alpha(t)$ is the time-varying intercept of the e_2 axis. The surface initially goes through given initial errors with appropriate initial intercept α_0 as shown in Figure 4(a). Similar to the RSS, we obtain following theorem regarding to the existence of sliding mode with the sliding surface (12).

Theorem 2. If $\alpha(t)$ in equation (12) is chosen to be a *step function* for $t \in [t_0, t\ ]$ with terminal values of $\alpha(t_0) = -e_2(t_0) - e_1(t_0)$ and $\alpha(t\ ) = 0$, and $\alpha(t)$ be a constat function when $t \in (t\ , \infty)$ with $\alpha(t) = 0$, the system (1) with controller (5) incorporating the sliding surface (12) satisfies sliding condition $s(e,t)\dot{s}(e,t) < 0$ almost everywhere.

The proof can be easily completed similar to the proof of Theorem 1. The movement is performed by calculating updated intercept $\alpha(t)$ until the surface s_0 becomes to the s_p. The algorithm to move the s_0 to the s_p is outlined as follows.

step 1. We determine an appropriate constant $\triangle$ required to shift the surface and define (refer to Figure 4(b)) $\triangle\ = \triangle\ + \triangle\ $.

step 2. We calculate the initial intercept α_0 satisfying the equation $s_{s0} = 0$ according to given initial errors $e(t_0)$; $\alpha_0 = -c_p e_1(t_0) - e_2(t_0)$

step 3. The shifting directionis is determined from the values of α_0, i.e., if $\alpha_0 > 0$; upward, and if $\alpha_0 < o$; downward.

step 4. The surface s_{s0} is immediately shifted to s_{s1} which has the intercept of α_1 obtained by solving the equation $|c_p e_1(t_0) + e_2(t_0) + \alpha_1| = \triangle_{fs}$. The larger value of two solutions α_1 is chosen as the intercept for upward, and the other for downward. The surface s_{s1} stays for a finite time $(\triangle\tau)$ before shifting to the next surface s_2 whose intercept α_2 is obtained by solving the equation $|c_p e_1(t_1) + e_2(t_1) + \alpha_2| = \triangle_{fs}$, where $t_1 = t_0 + \triangle\tau$. The shifting is continuously undertaken in a same manner until following step is checked.

step 5. We stop the shifting under following condition, i.e., if $\alpha_n > 0$, then fix $\alpha(t) = 0$; upward, and if $\alpha_n < 0$, then fix $\alpha(t) = 0$; downward. The intercept α_n is obtained by solving the equation $|c_p e_1(t_{n-1}) + e_2(t_{n-1}) + \alpha_n| = \triangle\ $, where $t_{n-1} = t_0 + (n-1)\triangle\tau$.

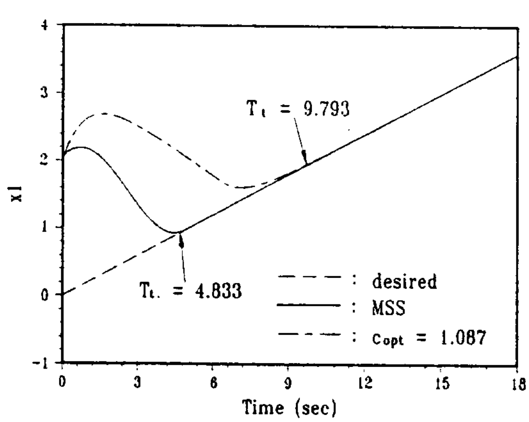
From the intuition, we may combine the RSS and the SSS to get better result in the sense of fast tracking time. For instance, if the initial errors are located in the unstable zone, the SSS is used until the RP enters to the stable zone, and subsequently RSS is employed throughout. Consequently, we may define the moving sliding surface(MSS) as

$$s_m(e(t), t) = c(t)e_1(t) + e_2(t) + \alpha(t)$$

$$s_m(e(t_0), t_0) = c(t_0)e_1(t_0) + e_2(t_0) + \alpha(t_0) \qquad (13)$$

The' intercept $\alpha(t) = 0$ for the RSS and the slope $c(t) = c_s$ for the SSS.

Figure 5 presents control responses of the system (10) obtained using the MSS (13). The dwelling time $\triangle\tau$ for both RSS and SSS is chosen to be 0.001 seconds, and the slope of the predetermined surface is taken by 5.0. The initial condition $(x_1(0),\ x_2(0)) = (2, 1.2)$ is imposed. We clearly observe that the tracking time T of the MSS is remarkably shortened



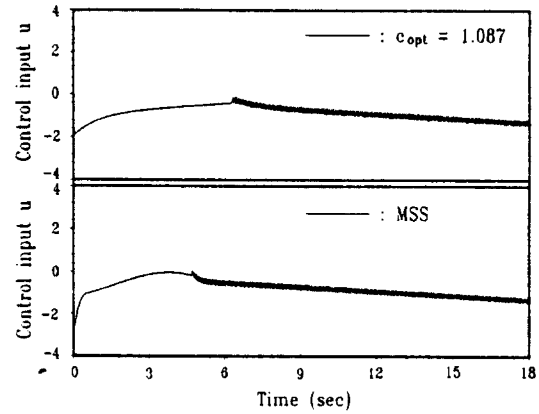


Fig. 5 Control responses with the MSS.

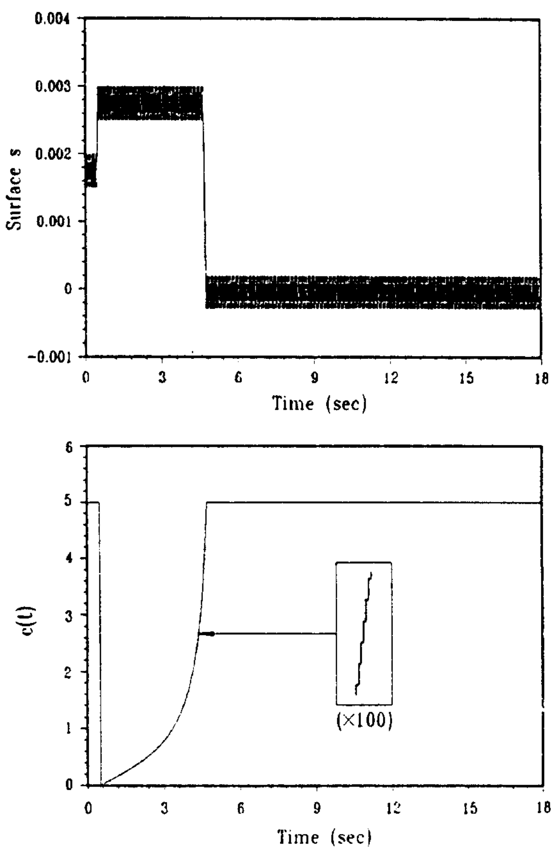


Fig. 6 Surface characteristics of the MSS.

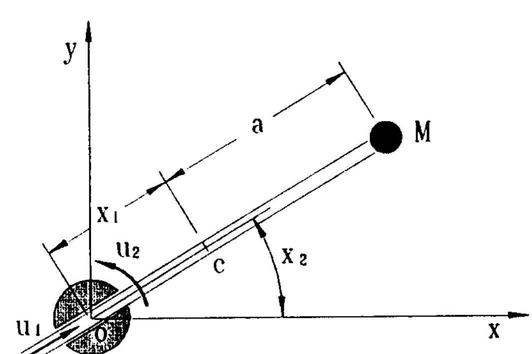
by comparing with the typical one but having optimal slope. From the input histories, we also see that the chattering magnitude of both cases are same. This improvement of the tracking behavior without increasing the gain k, hence undesirable cattering will furnish lots more benefits in practice. From the surface trajectory shown in Figure 6, we easily know that the SSS with $\Delta = 0.001$ is executed first followed by the RSS with $\Delta = 0.002$ ($\Delta = 0.001$ for the both RSS and SSS). In general, we start the movement(shifting or rotating) according to the location of initial errors. It is also observed that the RP never crosses the surface during the reaching phase. The chattering of the surface during the reaching phase is due to the imposed dwelling time $\Delta\tau$. The longer $\Delta\tau$, the larger magnitude. From the variation of the slope c(t) we observe that it indeed falls into the *step function*. In the subsequent section, we apply the MSS to the control of a two-degree-of-freedom manipulator.

4. APPLICATION TO A ROBOTIC MANIPULTOR

To illustrate the efficiency of the proposed method, a two-degree-of-freedom manipulator studied by Fu and Liao [12] is taken (see Figure 7). The equations of motion of the system are given by

$$\ddot{x}_1 = [\{\mu x_1(t) + M(x_1(t)+a)\}\dot{x}_2^2(t) + u_1(t) + d_1(t)] / [\mu + M]$$

$$\ddot{x}_2 = [-2\{\mu x_1(t) + M(x_1(t) + a)\}\dot{x}_1(t)\dot{x}_2(t) + u_2(t) + d_2(t)] / [J_1 + J_2 + \mu x_1^2(t) + M(x_1(t) + a)^2] \quad (14)$$



Fgi. 7 A two-degree-of-freedom manipulator.

where μ is the mass of the motional link, M is the payload, J_1 and J_2 are the moments of inertia of the motional link with respect to the vertical axis through c and o respectively, and $d_i(t)$ is the unknown but bounded external disturbance such that $|d_i(t)|<\delta_i$.

The control objective is to force $x_1(t)$ and $x_2(t)$ to track asymptotically the desired trajectories $x_{d1}(t)$ and $x_{d2}(t)$, respectively. Thus, the control inputs $u_1(t)$ and $u_2(t)$ should be determined to undergo that the trajectory error is to be zero asymptotically for any given initial conditons. Accordingly, in view of (13) we define the moving sliding surface as

$$s_i(t) = c_i(t)(x_i(t) - x_{di}(t)) + (\dot{x}_i(t) - \dot{x}_{di}(t)) + \alpha_i(t),\ i = 1,\ 2 \quad (15)$$

$$s_i(t_0) = c_i(t_0)(x_i(t_0) - x_{di}(t_0)) + (\dot{x}_i(t_0) - \dot{x}_{di}(t_0)) + \alpha_i(t_0)$$

Though the control problem is a multi-input case, it is treated as m silgle-input problems; the i-th sliding surface $s_i(t)$ depends only upon $\dot{e}_i(t)$ and $e_i(t)$. Hence, from the concept of equivalent control the discontinuous control laws to satisfy the sliding condition

$$s_i(t)\dot{s}_i(t) < 0 \quad (16)$$

can be obtained as follows.

$$u_1(t) = -\{\mu x_1(t) + M(x_1(t) + a)\}\dot{x}_2^2(t) + (\mu + M)\{-c_1(t)(\dot{x}_1(t) - \dot{x}_{d1}(t)) + \ddot{x}_{d1}(t) - k_1 sgn(s_1(t))\}$$

$$u_2(t) = 2\{\mu x_1(t) + M(x_1(t) + a)\}\dot{x}_1(t)\dot{x}_2(t) + \{J_1 + J_2 + \mu x_1^2(t) + M(x_1(t) + a)^2\} * \{-c_2(t)(\dot{x}_2(t) - \dot{x}_{d2}(t)) + \ddot{x}_{d2}(t) - k_2 sgn(s_2(t))\} \quad (17)$$

And the desired trajectory $x_{di}(t)$ is chosen by

$$\begin{aligned} x_{d1}(t) &= -0.75\sin(\pi t/20)\ [m] &&: t \le 10\ [sec] \\ x_{d1}(t) &= -0.75\ [m] &&: t > 10\ [sec] \\ x_{d2}(t) &= 2\pi\sin(\pi t/20)\ [rad] &&: t \le 10\ [sec] \\ x_{d2}(t) &= 2\pi\ [rad] &&: t > 10\ [sec] \end{aligned} \quad (18)$$

so that the desired output motion (end-point trajectory) in the (x, y) plane becomes

$$\begin{aligned} x\ \text{coordinate} &: (x_{d1}(t) + a)\cos(x_{d2}(t)) \\ y\ \text{coordinate} &: (x_{d1}(t) + a)\sin(x_{d2}(t)) \end{aligned} \quad (19)$$

We know that the output motion in the (x, y) plane is snail-type required frequently in the community of various control environments. For the simulation, following numerical values are employed: $\mu = M = 1kg$, $J_1 = J_2 = 1kg \cdot m^2$, $a = 1m$, $k_1 = 0.2$, $k_2 = 0.5$, $d_2(t) = 0.2\cos(5\pi t)$, $d_2(t) = 0.5\cos(5\pi t)$, and $(x_1(0),\ x_2(0),\ \dot{x}_1(0),\ \dot{x}_2(0)) = (-0.15,\ 0.349,\ -0.2,\ 0.987)$.

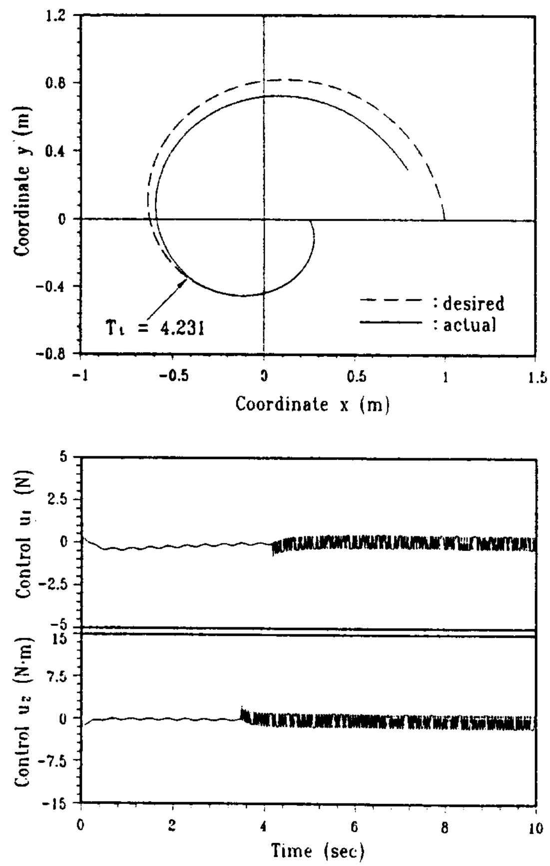


Fig. 8 Control responses using the conventional sliding surface (nonoptimal; $c_1 = c_2 = 5.0$).

Figure 8 presents the control responses obtained by employing the conventional sliding surfaces defined by

$$s_i(t) = c_i(x_i(t) - x_{di}(t)) + (\dot{x}_i(t) - \dot{x}_{di}(t)),\ i=1,2 \qquad (20)$$

where the value of constant slope c_i is chosen as 5 for i=1, 2. It is noted that the surfaces (20) are exactly same as ones employed by Fu and Liao[12]. The tracking time T_t in this figure is determined on the basis that the tracking error between the desired and actual trajectories enters below one percent. From the control imput history, we observe the disturbance effect during the reaching phase. On the other hand, Figure 9 shows the control responses obtained by employing the optimal sliding surfaces. By evaluating the performance index defined by the equation (9), the optimal value of the slope is calculated by c_{opt1} =1.512 for the $s_1(t)$ and c_{opt2} =1.351 for the $s_2(t)$, respectively. We clearly observe that the optimal sliding surface provides much faster tracking time than arbitrarily chosen one which has larger magnitude of the slope.

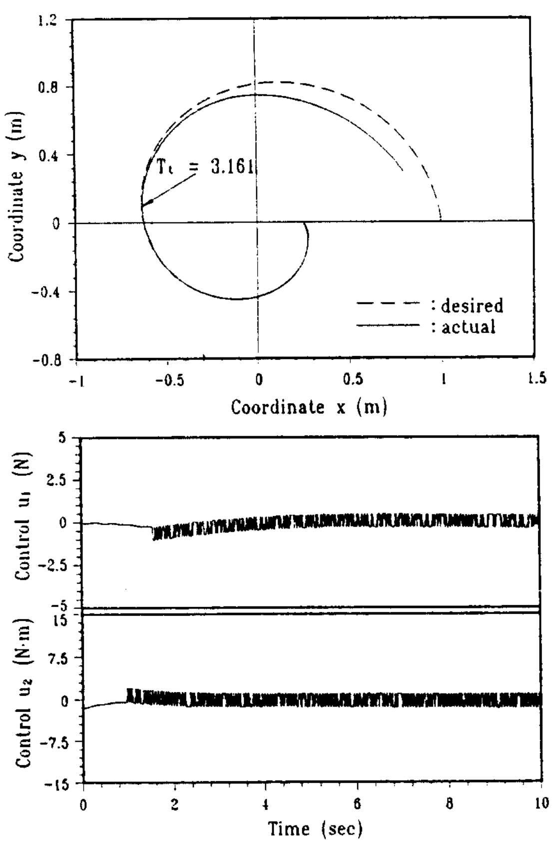


Fig. 9 Control responses using the conventional sliding surface (optimal: c_{opt1}=1.512, c_{opt2}=1.351).

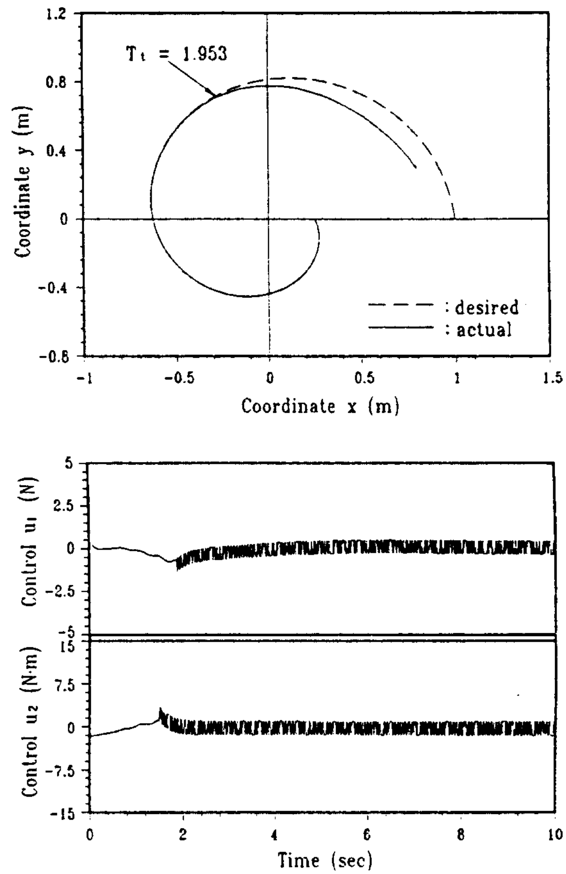


Fig. 10 Control responses using the proposed MSS.

Figure 10 presents the control responses obtained by employing the proposed MSS. We see that the tracking behavior is impressively imroved without increment of the discontinuities in the control input. For this simulation, following numerical values are used; c_i =5, Δf =0.005, Δ =0.015, Δ =0.01 and $\Delta\tau$=0.001 seconds. Simulation results in this work are quite self-explanatory justifying that the proposed method is very effective for improving the tracking behavior of the system subjected to disturbances. Without loss of generality the proposed method can be also applied to the system subjected to parameter variations.

5. CONCLUSIONS

The new type of the sliding surface called as the moving sliding surface (MSS) has been proposed to improve the tracking behavior of the second-order linear and nonlinear system. The MSS was designed first to pass given initial errors and subsequently move towards the predetermined sliding surface via rotating or/and shifting. Employing the MSS, it was possible to remarkably lessen the tracking time without increasing undesirable chattering of the control input signals. It has been shown that the proposed method could be applied to both single-input and multi-input systems. In multi-input systems, each sliding surface moves independently according to given initial errors.

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