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슬라이딩 모드를 이용한 로봇의 강건 추적제어

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A Robust Tracking Control for Robotic Manipulators Using Sliding Modes

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抄 錄

시스템의 모델링 과정에서 발생될 수 있는 불확실성(uncertainty) 혹은 미지의 가반중량을 비롯한 외란에 의해 발생되는 불확실성 등을 갖고있는 로봇의 강건 추적제어기 설계를 위해 가변구조시스템(variable structure system) 이론을 적용하였다. 시스템 방정식과 연계하여 슬라이딩 모드가 존재하기 위한 조건을 구했으며, 입력에 대한 불확실성은 매칭조건(matching condition)을 가정하여 다루었다. 기존의 방법에 비해 제어기 설계과정이 간단 명료하며 요구되는 궤적에 대한 추적제어 효과 또한 매우 우수함을 컴퓨터 시뮬레이션을 통해 입증하였다.

Key Words : Robust Tracking(강건추적), Robot Control(로봇 제어), Uncertainty(불확실성), Sliding Mode(슬라이딩 모드), Matching Condition(매칭 조건)

1. Introduction

Due to the rapid expansion in applicatoin of robotic manipulators during the past decade, many studies have been done in manipulator control. Path tracking is one of the important control problems and has been considered by many investigators. The path tracking of manipulators is frequently specified as a function of time. In this paper we consider the design of feedback controllers capable of tracking such a path function. Specifically, we treat the notion of tracking in an asymptotic manner for arbitrary initial conditions. The path function to be tracked is assumed to be pre-specified for each joint, hence making desired end-point trajectory for welding, assembly and so forth. Our controller for such a path tracking is designed based on the theory of variable structure systems(VSS).

VSS theory is based on the concept of an 'attractive' manifold of the underlying state or error vector space on which desired dynamic behavior is assured [1, 2]. These systems are a special class of nonlinear systems characterized by a discontinuous control action which changes structure upon reaching a set of switching hyperplanes. A fundamental property of VSS is the sliding motion of the state on the 'attractive' manifold which is the

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intersection of the switching hyperplanes. During this sliding motion the system has invariance properties, yielding motion which is independent of certain system parameters and disturbances.

In recent years, increasing attention has been given to controller designs that utilize the VSS [3-7]. Young [3] and Ryan [4] used a hierarchial method, where the basic idea is to force the system states to the surfaces of discontinuity sequentially. Slotine and Sastry [5] on the other hand used sliding conditions to drive the system states to all the switching surfaces simultaneously. It is also very interesting that the continuous controller proposed by Corless an Leitmann [8] appears to be a variable structure controller in the limit as its saturation function tends to zero.

The methodology proposed in this paper to construct a variable structure controller for robotic manipulators subjected to bounded uncertainties is based on a special way of imposing the sliding conditions. We primarily enforce the conditions which assure that once the system state is on a sliding surface that it is driven towards to zero state. This can be accomplished by taking matching conditions to account for input possessing uncertainties and subsequently choosing appropriate positive control parameters to account for external disturbances. The design procedure proposed is straightforward and relatively simple compared with [3, 5]. Moreover, the methodology is quite general and hence may be applied to various dynamical systems having bounded uncertainties.

The format of this paper is as follows. Section 2 contains a brief description of the well known conditions for sliding motions. Section 3 presents the problem formulation and controller design for path tracking. Section 4 describes the application of the proposed methodology to the control of a three degree-of-freedom manipulator, followed by simulation results in Section 5 and conclusions in Section 6, respectively.

2. Conditions for Sliding Motions

consider the system described by

$$\dot{x}(t) = f(x, t, u) \tag{1}$$

where $f : R^n \times R \times R^m$, $x \in R^n$ and $u \in R^m$. VSS are characterized by discontinuous control laws which change structure on reaching a set of switching surfaces. This control has the form

$$u_i = \begin{cases} u_i^+(x, t), & s_i(x) > 0 \\ u_i^-(x, t), & s_i(x) < 0 \end{cases} \tag{2}$$

where u_i is the i-th control input and $s_i(x)$ is the i-th switching surface, and these are certain continuous functions. The system (1) with the discontinuous control (2) is termed as VSS since the effect of the switching surface is to alter the feedback structure of the system. The conditions for so-called siliding motion to occur on the i-th switching surface of the system may be expressed as

$$\lim_{s_i \to 0^+} \dot{s}_i < 0 \quad \text{and} \quad \lim_{s_i \to 0^-} \dot{s}_i > 0 \tag{3}$$

or equivalently

$$s_i \dot{s}_i < 0 \tag{4}$$

in the neighborhood of $s_i(x) = 0$ [1, 2]. In the sliding mode, the system satisfies the equations

$$s_i(x) = 0 \quad \text{and} \quad \dot{s}_i(x) = 0 \tag{5}$$

hence, yielding the sliding motion which is independent of certain parameter variations and disturbances. Thus VSS are effectively employed in systems with uncertainties and time-varying parameters.

3. Controller Formulation

Robot manipulators made of rigid links are generally described by dynamic equations of the form

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) + D(\theta,\dot{\theta},t) = F \quad (6)$$

where $\theta(t) \in R^n$ are the joint coordinates, $M(\theta)$ is a (nxn) inertia matrix, $C(\theta,\dot{\theta})$ is an n-vector of Coriolis and centrifugal forces, $G(\theta)$ is an n-vector of gravitational forces, $D(\theta,\dot{\theta},t)$ is an n-vector of disturbances and F is an n-vector of generalized forces applied by the actuators at the joints of the manipulator. M, C, G and D typically will consist of parameters that are not completely known. The manipulator dynamics (6) are of a nature that lends itself well to controller design using the VSS approach.

First of all let us assume that M, C, G and D are completely known, then the dynamics (6) can be embedded in the state space representation given by

$$\dot{x}(t) = f(x,t) + B(x,t)u(t) \quad (7)$$

where $x(t) \in R^{2n}$ is the state, $u(t) \in R^n$ is the control and $f(x,t)$ is defined as

$$f(x,t) \equiv [f_1(x,t), \cdots, f_n(x,t)]^T$$

where each $f_i(x,t) \in R^2$ and is of the form $f_i(x,t) = [x_{2i}, g_i(x,t)]^T$. The input matrix $B(x,t)$ in the equation (7) has n diagonal block vectors with each block vector $b_i(x,t)$ having only the last entries and the off-diagonal block vectors having zero entries. We also define are the

$$\begin{aligned} x(t) \equiv [&x_{11}(t) \quad x_{21}(t); \\ &x_{21}(t) \quad x_{22}(t); \cdots; \\ &x_{1n}(t) \quad x_{2n}(t)]^T \\ \equiv [&x_1, x_2, \cdots, x_n]^T \in R^{2n} \end{aligned} \quad (8)$$

$$u(t) \equiv [u_1(t), \cdots, u_n(t)]^T \in R^n \quad (9)$$

and

$$x_d(t) \equiv [x_{1d}(t), \cdots, x_{nd}(t)]^T \in R^{2n} \quad (10)$$

desired trajectories with each $x_{id}(t)$ in the class of continuously differentiable C^1 functions on $[t_0, \infty)$. It is noted that above formulation applies to any system that is representable as a coupled set of second order ordinary differential equations. Robotic manipulators clearly fall into this class.

As previously mentioned our control objective is to drive the tracking error associated with each generalized co-ordinate to zero asymptotically for any arbitrary initial conditions and uncertainties in prescribed sets. Now, we define the tracking error $e(t) \in R^{2n}$ as

$$e(t) \equiv [e_1(t), \cdots, e_n(t)]^T = x(t) - x_d(t) \quad (11)$$

where $e_i(t) = [x_i(t) - x_{id}(t)]$, $i = 1, 2, \cdots n$. We choose the control $u(t)$ so that each component $u_i(t)$ undergoes a discontinuity on a surface si. We select the discontinuity surfaces S in the error space as

$$S(t) = [s_1(t), \cdots, s_2(t)]^T \equiv Ge(t) \quad (12)$$

where $s_i = C_i e_i$, and G is an (nx2n) constant matrix. The (1x2) row vector C_i has constant entries of the form $[c_{1i}, 1]$.

We see that the tracking error $e(t) \to 0$ for any given initial conditions, provided the equation (12) is asymptotically stable. This can be achieved by constructing controllers $u(t)$ so as to satisfy the sliding condition (4). It can be easily shown that the sliding condition (4) is always satisfied by letting the time derivative of s_i be

$$\dot{s}_i = -k_i \cdot \mathrm{sgn}(s_i) \quad (13)$$

where k_i is a suitably selected positive constant and

$$\mathrm{sgn}(s_i) \equiv \begin{cases} 1, & \text{if } s_i > 0 \\ 0, & \text{if } s_i = 0 \\ -1, & \text{if } s_i < 0 \end{cases} \quad (14)$$

Based on this concept we construct discontinuous control laws $u(t)$ that satisfy the sliding condition (4) as

$$u(t) = [GB(x,t)]^{-1}[-K\mathrm{sgn}(S) - Gf(x,t) + G\dot{x}_d(t)] \quad (15)$$

where $K \equiv [k_1, \cdots, k_n]^T$ as a mechanism for forcing all the switching surfaces $s_i = 0$ to be sliding surfaces. We obviously require that [GB (x, t)] be a nonsingular matrix. This condition allows one to verify whether a given set of sliding surfaces is admissible. The design procedure is straightforward and requires little computational effort in the absence of uncertainties. However, the situation becomes quite different if uncertainties are added to the dynamics (7), i.e.,

$$\dot{x}(t) = f(x,t) + \triangle f(x,t) + [B(x,t) + \triangle B(x,t)]u(t) + h(t)d(t) \quad (16)$$

where $d(t) \in R^{\ell}$ is the disturbance, and $\triangle f(x,t)$ and $\triangle B(x,t)$ account for the uncertainties in the model. $\triangle B(x,t)$ and $h(t)$ have the same form as $B(x,t)$. Now we define

$\triangle f(x,t) \equiv [\delta f_1(x,t), \cdots, \delta f_n(x,t)]^T$

and each $\delta f_i(x,t)$ is of the form $\delta f_i(x,t) = [0, \sum_{j=1}^{2n} \delta a_j v_j(x,t)]^T \in R^2$, where δa_j's are unknown but bounded and is given $\hat{a}_j \le \delta a_j \le \tilde{a}_j < \infty$. We also define

$f_0(x,t) \equiv [f_{01}(x,t), \cdots, f_{0n}(x,t)]^T$

$\bar{f}(x,t) \equiv [\bar{f}_1(x,t), \cdots, \bar{f}_n(x,t)]^T$

where $f_{0i}(x,t) = [0, \sum_{j=1}^{2n} a_{0j} v_j(x,t)]^T \in R^2$

and $\bar{f}_i(x,t) = [0, \sum_{j=1}^{2n} \bar{a}_j v_j(x,t)]^T \in R^2$ with

$a_{0j} = (\hat{a}_j + \tilde{a}_i) / 2$, and $\bar{a}_j = \tilde{a}_j - a_{0j}$.

In order to deal with input possessing uncertainties, we make the following assumption.

Assumption (Matching Condition) : There exist Caratheodory functions $\triangle P(x,t) \in R^{n \times n}$, and a continuous function $\phi(x,t) \in R$, such that for all $(x,t) \in R^{2n} \times R$,

$$\triangle B(x,t) = B(x,t) \triangle P(x,t), \quad (17)$$

with $|\delta p_i(x,t)| \le \phi(x,t) < 1$, $i = 1, 2, \cdots, n$. The $\delta p_i(x,t)$ is the i-th diagonal entry of the diagonal matrix $\triangle P(x,t)$ and $|\cdot|$ denotes absolute value. This assumption is equivalent to say that the input uncertainty can be lumped with the input matrix $B(x,t)$.

In view of the above assumption, we have $1 + \delta p_i t > 0$ and

$$[(1 - \delta p_i(x,t)) \ (1 - \phi(x,t))] \ge 1. \quad (18)$$

Under the formalism given above we now propose the control structure

$$u(t) = [G\ B(x,t)]^{-1}[-(K + |G\ f(x,t) - G\dot{x}_d(t))| + |G\bar{f}(x,t)|)\,\mathrm{sgn}\ (S) - G\ f_0(x,t)] / [1 - \phi(x,t)] \quad (19)$$

In order to verify the sliding condition (4) we compute

$$\begin{aligned} s_i\dot{s}_i &= s_i[C_i\dot{x}_i(t) - C_i\dot{x}_{id}(t)] \\ &= s_i[C_i f_i(x,t) + C_i\delta f_i(x,t) \\ &\quad + (1 + \delta p_i(x,t))C_i b_i(x,t)u_i(t) \\ &\quad + C_i h_i(t)d_i(t) - C_i x_{id}(t)] \end{aligned} \quad (20)$$

On substituting for u_i from (19) in (20) we get

$$\begin{aligned} s_i\dot{s}_i &= [(C_i f_i(x,t) - C_i\dot{x}_{id}(t))s_i - \\ &\quad \omega|C_i f_i(x,t) - C_i x_{id}(t)||s_i|] \\ &\quad + [(C_i\delta f_i(x,t) - \omega C_i f_{0i}(x,t))s_i \\ &\quad - \omega\ |C_i\bar{f}_i(x,t)||s_i|] \\ &\quad + [C_i h_i(t)d_i(t)s_i - \omega k_i|s_i|] \end{aligned} \quad (21)$$

where $\omega = (1 + \delta p_i(x,t)) / (1 - \phi(x,t))$.

we know that $\omega \ge 1$ from (18). Thus it is clear that the first two bracketed terms on the right hand side of equation (21) is always negative. If $[C_i h_i(t) d_i(t) s_i - \omega k_i |s_i|]$ is non positive then it follows that $s_i\dot{s}_i < 0$, for all $i = 1, 2, \cdots, n$. The design problem then is to select k_i appropriately so that

$$C_i h_i(t)\ d_i(t)\ s_i \le \omega\ k_i|s_i|$$

$$\text{i.e.} \quad k_i \ge \left| \frac{C_i h_i(t)\ d_i(t)}{\omega} \right| \quad (22)$$

Since $d_i(t)$ is an unmeasurable disturbance in general it is not feasible to determine a specific k_i. However, if k_i is chosen sufficiently large then the sliding condition can be satisfied. If the disturbance is bounded then k_i can be determined by knowing the bound on $d_i(t)$.

Remark : The above controller structure is arrived at by imposing the condition $\dot{s}_i=-k_i \,\mathrm{sgn}(s_i)$ in the absence of a disturbance, or $\dot{s}_i=-k_i \,\mathrm{sgn}(s_i)+\gamma(t)d_i(t)$ in the presence of a disturbance.

4. Illustrative Example

Consider the three-degree-of freedom manipulator of Figure 1. The manipulator has

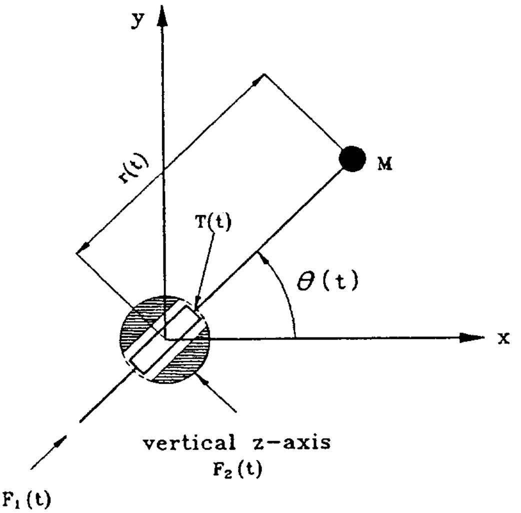


Fig.1 Three-degree-of-freedom manipulator

one rotational and translational joint in the (x, y) plane, and the arm can be lifted along the vertical z-axis which constitutes the third degree of freedom. The dynamic equations of this configuration follow directly from an application of Lagrange's equation [9]. By assuming normalized unit mass and unit length of the arm and upright column, and neglecting the gravity force, we obtain the following dynamic equations.

$$\ddot{r}(t) = r(t)\dot{\theta}^2(t) - \dot{\theta}^2(t)/(2+2M) + F_1(t)/(1+M) \qquad (23\text{-a})$$

$$\ddot{\theta}(t) = (-2(1+M)r(t) + 1)\dot{r}(t)\dot{\theta}(t)/((5/6) - r(t) + (1+M)r^2(t)) + T(t)/((5/6) - r(t) + (1+M)r^2(t)) + d(t) \qquad (23\text{-b})$$

$$\ddot{z}(t) = F_2(t) / (1+M) \qquad (23\text{-c})$$

where an unknown but bounded external torque disturbance $d(t)$ and a variable payload M bounded as $0 \le M_{min} \le M \le M_{max}$ are imposed to demonstrate the robustness of our control scheme.

We introduce state variables

$$x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [r(t), \dot{r}(t), \theta(t), \dot{\theta}(t), z(t), \dot{z}(t)]^T \qquad (24\text{-a})$$

and inputs

$$u(t) = [u_1(t), u_2(t), u_3(t)]^T = [F_1(t), T(t), F_2(t)]^T \qquad (24\text{-b})$$

to obtain the following problem statement which is of the form (16). Thus

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1 x_4^2 + \Delta f_2(x,t) + (1+\Delta b_1)u_1(t) \\
\dot{x}_3 &= x_4 \qquad (25) \\
\dot{x}_4 &= x_2 x_4 + \Delta f_4(x,t) + (1+\Delta b_2)u_2(t) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= (1+\Delta b_3)u_3(t)
\end{aligned}$$

where

$$\begin{aligned}
\Delta f_2(x,t) &= -x_4^2 / (2+2M) \\
\Delta f_4(x,t) &= (-(1+M)x_1^2 - x_1(1+2M) + (1/6))x_2 x_4 / ((5/6) - x_1 + (1+M)x_1^2) \qquad (26\text{-a})
\end{aligned}$$

$$\begin{aligned}
\Delta b_1 &= -M / (1+M) \\
\Delta b_2 &= (1+6x_1-6x_1^2(1+M)) / (5-6x_1+6x_1^2(1+M)) \\
\Delta b_3 &= -M / (1+M) \qquad (26\text{-b})
\end{aligned}$$

The control objective is to force $r(t)$, $\theta(t)$ and $z(t)$ to track asymptotically the desired trajectories $r_d(t)$, $\theta_d(t)$ and $z_d(t)$ respectively. We choose desired trajectories as

$$r_d(t) = \begin{cases} 0.75(1-\sin(\pi t/10)) + 0.25[\mathrm{m}] & , t \le 5 \text{ sec.} \\ 0.25 \ [\mathrm{m}] & , t > 5 \text{ sec.} \end{cases}$$

$$\theta_d(t) = \begin{cases} 2\pi \sin(\pi t/10) \ [\mathrm{rad}] & , t \le 5 \text{ sec.} \\ 2\pi \ [\mathrm{rad}] & , t > 5 \text{ sec.} \end{cases} \quad (27)$$

$$z_d(t) = \text{constant} \ (0.6 \ [\mathrm{m}])$$

so that the desired output motion (end-point trajectory) in the (x, y) plane becomes

$$\text{x-coordinate} : r_d(t) \ \cos(\theta_d(t))$$
$$\text{y-coordinate} : r_d(t) \ \sin(\theta_d(t)). \quad (28)$$

It can be figured out that the desired trajectories given by the equation (27) are clearly in the class of C^1. And we know that the output motion in the (x, y) plane is snail-type required frequently in the community of various welding. We define

$$x_d(t) \equiv [x_{1d}, x_{2d}, x_{3d}, x_{4d}, x_{5d}, x_{6d}]^T$$
$$\equiv [r_d(t), \dot{r}_d(t), \theta_d(t), \dot{\theta}_d(t), z_d(t), \dot{z}_d(t)]^T. \quad (29)$$

From (12) we select sliding surfaces as

$$s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 5(x_1 - x_{1d}) + (x_2 - x_{2d}) \\ 7(x_3 - x_{3d}) + (x_4 - x_{4d}) \\ 5(x_5 - x_{5d}) + (x_6 - x_{6d}) \end{bmatrix}. \quad (30)$$

This gives the matrix

$$G = \begin{bmatrix} 5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 1 \end{bmatrix} \quad (31)$$

Matching condition (17) can be shown to hold by considering $\triangle P(x,t) = \mathrm{diag}\ (\triangle b_i(x,t))$ for which it is readily shown that $|\delta p_i(x,t)| = |\triangle b_i(x,t)|$. Thus, from (26) we see that if $(M_{max} - M_{min})$ is sufficiently small, then there exists a $\phi(x,t)$ such that $\phi(x,t) < 1$. Now we choose the payload $M = M(t) = 0.3 + 0.3 \ \sin(3t)$ Kg producing $M_{min} = 0$ Kg, $M_{max} = 0.6$Kg, and $\phi(x,t) = \phi = 0.8$. According to the design procedure, we obtain the following results after some algebraic manipulations.

$$\mathbf{GB} = \mathbf{I}_{3\times 3}$$
$$Gf(x,t) - G\dot{x}_d(t) \equiv [\eta_1(x,t) \quad \eta_2(x,t) \quad \eta_3(x,t)]^T$$
$$Gf_0(x,t) \equiv [f_{01}(x,t) \ f_{02}(x,t) \ f_{03}(x,t)]^T$$
$$G\bar{f}(x,t) \equiv [\bar{f}_1(x,t) \quad \bar{f}_2(x,t) \quad \bar{f}_3(x,t)]^T \quad (32)$$

where

$$\eta_1(x,t) = 5x_2 + x_1 x_4^2 - 5x_{2d} - \dot{x}_{2d}$$
$$\eta_2(x,t) = 7x_4 + x_2 x_4 - 7x_{4d} - x_{4d}$$
$$\eta_3(x,t) = 5x_6 - 5x_{6d} - \dot{x}_{6d}$$
$$f_{01}(x,t) = -(1/2.6)x_4^2$$
$$f_{02}(x,t) = (-1.3x_1^2 - 1.6x_1 + (1/6))x_2 x_4 \ / \ (1.3x_1^2 - x_1 + (5/6))$$
$$f_{03}(x,t) = 0$$
$$\bar{f}_1(x,t) = f_{01}(x,t)$$
$$\bar{f}_2(x,t) = f_{02}(x,t)$$
$$\bar{f}_3(x,t) = f_{03}(x,t).$$

Now from (19) we can construct the sliding mode feedback controllers as

$$u_1(t) = (-(k_1 + |\eta_1(x,t)| + |\bar{f}_1(x,t)|) \ \mathrm{sgn}(s_1) - f_{01}(x,t)) \ / \ (1-\phi)$$
$$u_2(t) = (-(k_2 + |\eta_2(x,t)| + |\bar{f}_2(x,t)|) \ \mathrm{sgn}(s_2) - f_{02}(x,t)) \ / \ (1-\phi)$$
$$u_3(t) = (-(k_3 + |\eta_3(x,t)|)\mathrm{sgn}(s_3)) \ / \ (1-\phi). \quad (33)$$

On the other hand, the dynamics (25) can be simply reduced to the form of (7) in the absence of the uncertainties M(known pay load only)and d(t), hence the tracking controllers can be constructed from (15) as

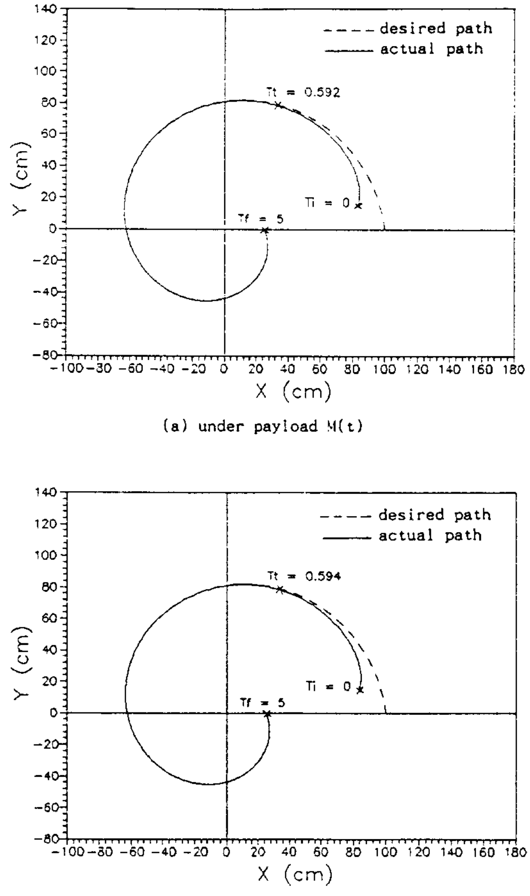
$$
\begin{aligned}
u_1(t) &= -k_1\ \mathrm{sgn}(s_1) + 5x_{2d} + \dot{x}_{2d} \\
&\quad - 5x_2 - x_1x_4^2 + (1/2)x_4^2 \\
u_2(t) &= (5/6 - x_1 + x_1^2)(-k_2\ \mathrm{sgn}(s_2) \\
&\quad + 7x_{4d} + \dot{x}_{4d} - 7x_4) \\
&\quad - (-2x_1 + 1)x_2x_4 \qquad (34) \\
u_3(t) &= -k_3\ \mathrm{sgn}(s_3) + 5x_{6d} + \dot{x}_{6d} - 5x_6
\end{aligned}
$$

5. Simulation Results and Discussions

As mentioned before, the value of k_i in the control laws should be chosen according to the magnitude of the external disturbances and process ancertainties. We assume the torque disturbance varies with respect to time as $d(t) = 0.3\ \sin(70t)$ N · m so that the disturbance is bounded such as $d(t) \in [-0.3, 0.3]$. For simulations k_1, k_2 and k_3 are adopted to be 1, 2 and 1 respectively. And we chose the initial conditions to be

$$
x(0) = [85\mathrm{cm},\ 0\mathrm{cm/sec},\ 10\mathrm{deg},\ 0\mathrm{deg/sec},\ 60\mathrm{cm},\ 0\mathrm{cm/sec}]^T
$$

Figure 2 shows the results of asymptotic tracking for the desired end-poing trajectory described by the equation (28) under variable payload M(t) and full payload (M_{max}) when we use the discontinuous control laws (33). The payload and disturbance variations are presented in Figure 3. In Figure 2, T_i, T_t and T_f represent initial, tracking and final time (second) respectively. The determination of tracking time was based on that the tracking error between desired and actual trajectories enters below one percent. The tracking time may be decreased by increasing the value of



(a) under payload M(t)

(b) under full payload

Fig. 2 End-point trajectory with discontinuous control laws (equation (33))

k_i and ϕ. This, however makes that the discontinuities of the control efforts incerase almost proportionately causing the generation of higher chattering which is undesirable in practice. Figure 4 shows the discontinuous control histories under variable payload M(t). Figure 5 presents the end-point trajectory with the discontinuous control laws given by the equation (34). The asymptotic tracking was achieved under no payload whereas it was not produced under full payload as expected.

Tracking time is considerably sensitive to the gradient vectors of sliding surfaces given C_i in equation (12), since the matrix GB(x, t) affects

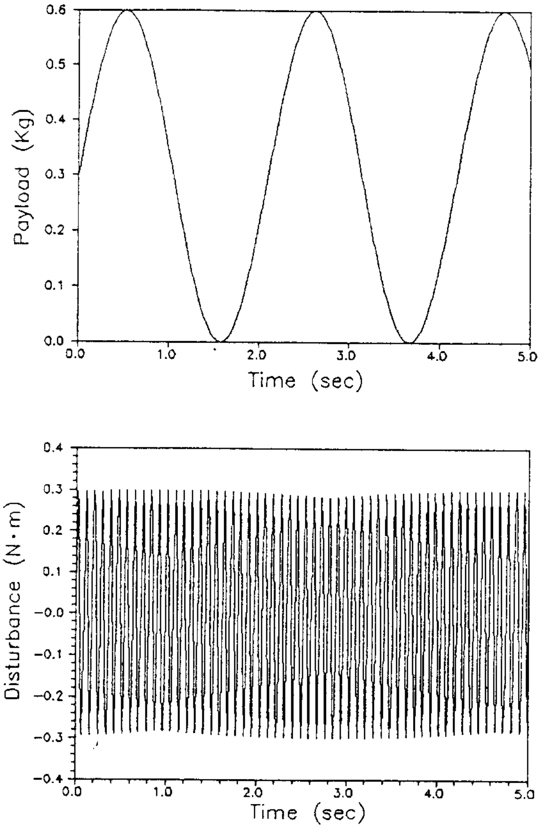


Fig. 3 Payload and disturbance variations

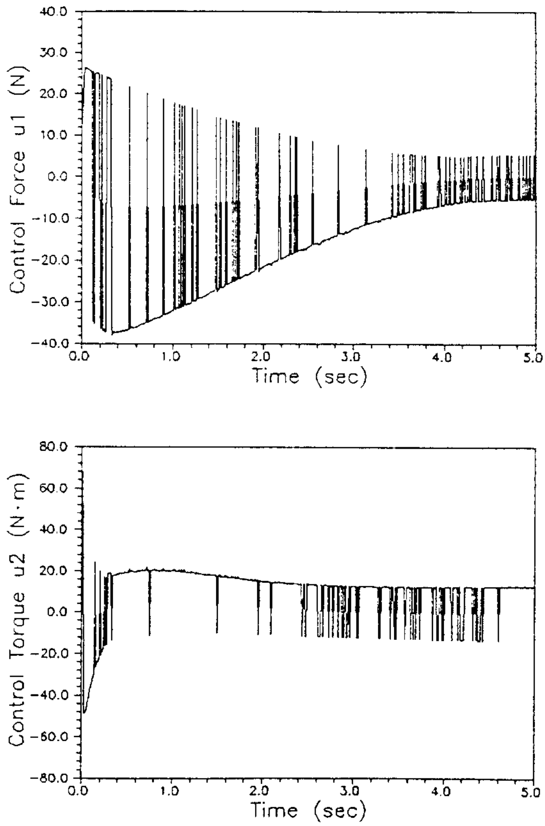
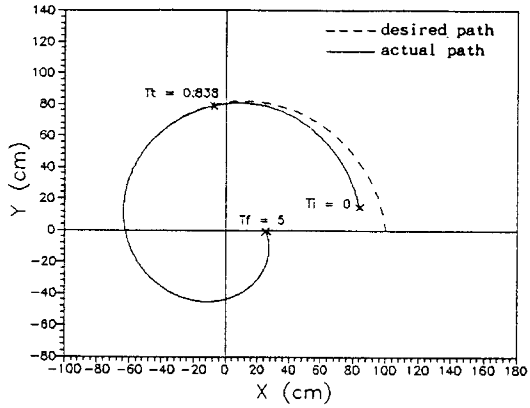


Fig. 4 Discontinuous control efforts under variable payload M(t)

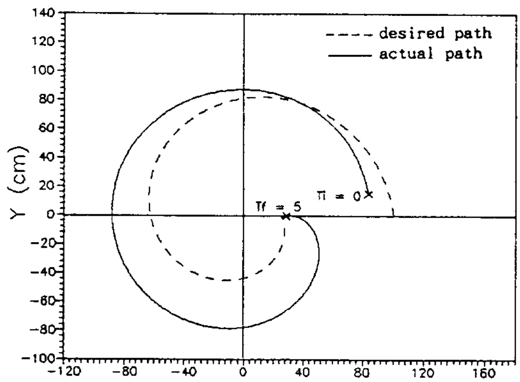
the rate of convergence to the sliding surfaces. For the linear systems the magnitude of C_i is chosen according to the intrinsic characteristics of the system such as unmodelled high frequency of the actual plant and the desired eigenvalues to be located. However, in general it is hard to select the optimal Ci in nonlinear time-varying systems, especially in uncertain dynamical systems. Figure 6 presents the effect of C_i to the tracking time. As expected, the tracking time decreases as c_{i1} increases. It is clearly observed from this figure that the tracking time is more sensitive to c_{i1} than c_{i2}. This fact may arise from the physical characteristics of the plant and the stiffness of the desired trajectoris. It is noted that the asymptotic tracking was not achieved for the values of $c_{11}=c_{21}=0.1$ even though these values satisfy the sliding condition (4) for the system (25). This implies that for the given desired trajectories the value of $c_{i1}=0.1$ is not enough to cope with the speed of convergence to the sliding surfaces.

It is known that the application of discontinuous control laws to practical implementation is not desirable due to chattering which causes the destruction of hardware. So we approximate these discontinuous feedback control laws by continuous ones inside the boundary layer. To do this, we replace $\mathrm{sgn}(s_i)$ in equation (33) by $\mathrm{sat}(\sigma_i)$ defined as

$$\mathrm{sat}(\sigma_i) \equiv \begin{cases} \sigma_i & , \ |\sigma_i| \le 1 \\ \mathrm{sgn}(\sigma_i) & , \ |\sigma_i| > 1 \end{cases} \tag{35}$$

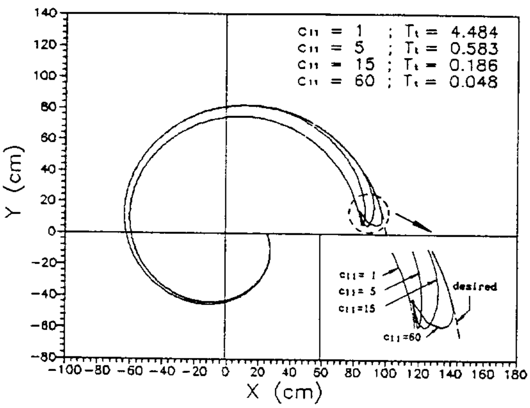


(a) under no payload

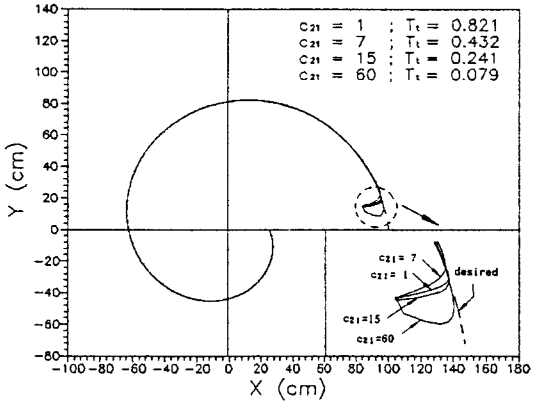


(b) under full payload

Fig. 5 End-point trajectory with discontinuous control laws (equation (34))

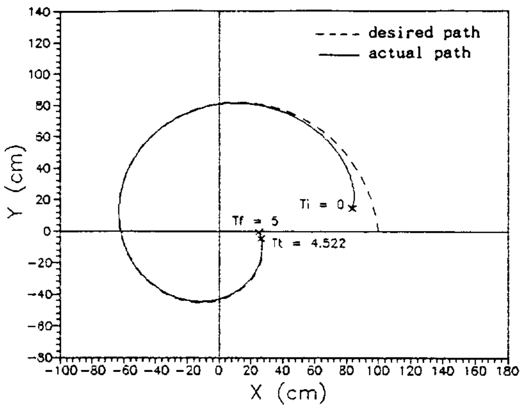


(a) c_{21} = 150, c_{31} = 50

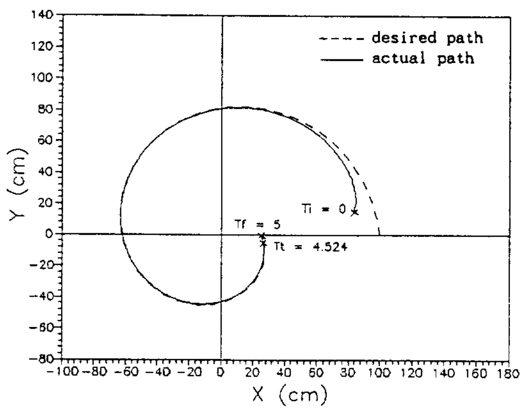


(b) c_{11} = 50, c_{31} = 50

Fig. 6 Gradient effect of sliding surfaces under variable payload M(t)



(a) under payload M(t)



(b) under full payload

Fig. 7 End-point trajectory with continuous control laws

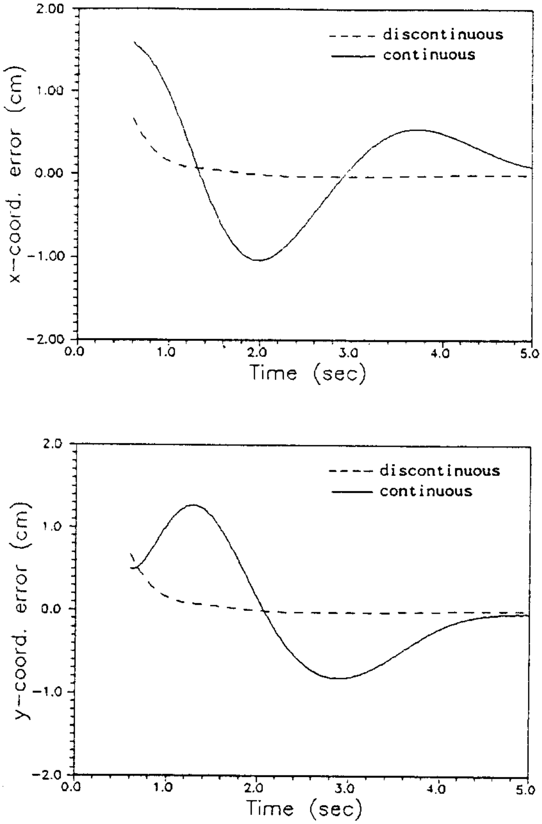


Fig. 8 Comparison of tracking error between discontinuous and continuous control laws under variable payload M(t)

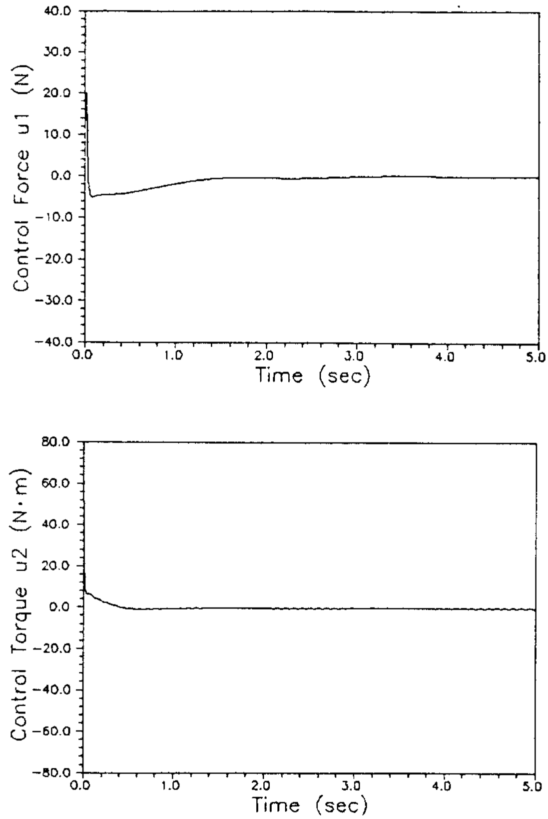


Fig. 9 Continuous control efforts under variable payload M(t)

where

$\sigma_i = s_i/(c_{ii}\varepsilon_i)$

The selection of ε_i depends on the strength of the discontinuities of control efforts. We chose $\varepsilon_1 = 0.03$m, $\varepsilon_2 = 0.05$ rad and $\varepsilon_3 = 0.05$m for the simulation. Figure 7 shows the end-point trajectory with these continuous control laws under variable payload M(t) and full payload. It is obvious from Figure 8 that the tracking error has been increased as expected. This tracking error may be reduced by decreasing the value of ε_i with trade-off the discontinuities of control efforts. Figure 9 reveals that the control efforts of approximated controllers do not indicate the chattering observed in Figure 4. Simulation results also indicate that larger control efforts are generally needed to drive the system states to the sliding surfaces, while once on these surfaces smaller control efforts are required to maintain tracking.

6. Concluding Remarks

A procedure of designing controllers for robust path tracking has been presented utilizing the theory of variable structure systems. Compared to the sliding mode controllers proposed in the prior literatures the ones advanced in here appear to be relatively simple in form and also very efficient in performance. The input possessing uncertainty was treated by imposing matching conditions via the positivity of the gain margin. The simulation results indicate excellent tracking features of the proposed control laws. The selection of gradient vector C_i, gain k_i and boundary layer width ε_i, which are vital for the success of the proposed methodology was executed in an ad-hoc way. Formally selecting these parameters in an optimal manner remains as an open research issue.

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