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플렉시블한 관절을 갖고 있는 로보트의 강인한 디지탈 추적 제어에 관한 연구

김 진 걸*

A Study on the Robust Digital Tracking Control of a Robot with Flexible Joints

Jin-Geol Kim*

초 록

플렉시블한 관절을 가지고 있는 로보트에 대하여 계단(step) 혹은 경사(ramp) 함수로 기준입력이 주어졌을 때, 모델의 불확실성이나 외란 아래에서 추적오차가 없는 디지탈 제어기법을 제시하고 실제로 실험을 통하여 제어기를 MC 68000으로 구현함으로써 제안한 제어 알고리즘의 유용성을 보였다. 제어기 설계에 있어서 고려한 사항들은 비선형 마찰력, 중력에 의한 외란, 구동기와 부하사이의 뒤틀림 스프링 효과, 그리고 측정할 수 없는 상태변수들이다. 또한 미분불가능한 쿨롱마찰로 인하여 전체 시스템이 이산/연속 혼성 시스템이 되어 출력에서 나타나는 리미트 싸이클을 해석/추정하기 위하여 혼성 시스템에서의 기술함수(Describing Function)법을 제시하였다.

Key Words : Digital control, Nonlinear control system, Robots, Limit cycles, Coulomb friction, Delayed output feedback

I. INTRODUCTION

In certain applications of industrial robots, it was pointed out that it is necessary to accurately track a moving reference point, that is, to track a ramp reference input with zero steady state error. In this paper, experimental results of an actual implementation of a robust digital tracking controller of a geared robotic manipulator are presented. It is known (Brady, 1989; Spong, 1986) that gears in the robots create the problems of joint flexibility, backlash and friction, speed limitation, and dominance of rotor inertia.

One of the most widely used methods for designing a tracking controller, continuous and discrete, for a robotic manipulator is the PID controller coupled with an additional feedforward loop for velocity compensation. The method, however, has the following serious shortcomings : (a) The feedforward compensation is basically an open loop control, which is based on an exact mathematical model. Since the actual system parameters are always different from those of the model, there will always be a nonzero steady state error. (b) When high ratio gears are used in a manipulator drive system, there is a substantial amount of spring

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effects in the system. In this case, a drive must be modeled by at least a fourth order differential equation. Since the PID controller uses only two state (position and velocity) variables in the feedback loop, the resulting controller is basically a partial state variable feedback controller. As such, it is, in general, rather difficult to obtain a satisfactory response. (c) The Coulomb frictions, which exist in most of robotic manipulators, are usually ignored in controller design. However, the investigation shows that the Coulomb frictions play a critical role in the control system. The nonlinear frictions cause a nonzero steady state error. If the controller gain is increased to reduce the steady state error, then the system often goes into a limit cycle oscillation. (d) Because of the Coulomb friction, even when the system gain is small, a closed loop system often exhibits a limit cycle. To eliminate or at least reduce the magnitude of the limit cycle, it is, in general, necessary to feedback the entire state variables.

The investigation further shows that the interactions between the joints as well as the nonlinear effects due to Coriolis and centrifugal forces are often negligible compared to the spring effects and the Coulomb frictions. The same conclusions were also reported by other investigators (Good, 1985; Sweet, 1985; Vidyasagar, 1987). Thus, in this study, the joint drive systems are decoupled, and each joint is investigated separately. The nonlinear effects due to the Coriolis and centrifugal forces are also ignored. However the spring effects and Coulomb frictions are fully incorporated into a fourth order system model.

The effective inertia of each joint, of course, is dependent on the geometric configuration of the manipulator at each instance and also on the mass of the object grasped by the manipulator. The maximum value is used for the load inertia in this investigation. The variation of inertia value during an actual operation of a manipulator is accommodated by designing a controller which is robust with respect to the variation in the load inertia.

The Coulomb friction is discontinuous when the velocity is zero. Thus the manipulator system under consideration cannot be linearized. Also, when a digital controller is implemented, the nonlinear system cannot be represented as a nonlinear discrete system. It is, thus, necessary to consider the system as a continuous system driven by discrete control function.

In order to track a ramp input with zero steady state error, the system is first augmented by two additional discrete error integrators. In the manipulator under consideration, only the angular position is measured. Although a tachometer could have been used to measure the velocity, it was not implemented because of the lack of physical space on the manipulator. To compensate for the missing three state variables, three delayed values of the output (angular position) variable as well as the variable itself are used in the feedback controller. The feedback gain parameters are obtained by minimizing a quadratic cost functional for the discrete linear system which is obtained by discretizing the continuous system with the nonlinearities removed. Then the resulting digital controller is applied to the original continuous nonlinear system. This often results in the actual closed loop system going into a limit cycle. In this case, the describing function method for the hybrid system is developed and is applied to readjust the feedback parameters so that the limit cycle is eliminated.

The controller designed by the above method tracks ramp input functions with zero steady state error. It is robust in the sense that as long as the closed loop system remains stable, the response tracks the input with zero steady state error for any variations in the system parameters. This is also true for any disturbances such as the gravitational pull as long as the disturbance is either a step or ramp function.

II. MODEL

The manipulator under consideration is represented by the diagram shown in Fig. 1. In the figure, F_m and F represent the Coulomb frictions and T_d represents the disturbance torque due to the gravitational pull. The latter

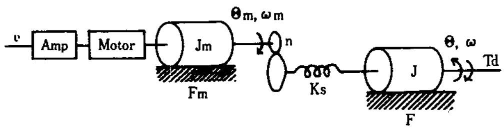


Fig. 1 Robot Drive System

is regarded as an external disturbances to the system. u is the control input to the amplifier for driving the motor. The above drive system can be modeled by the block diagram shown in Fig. 2.

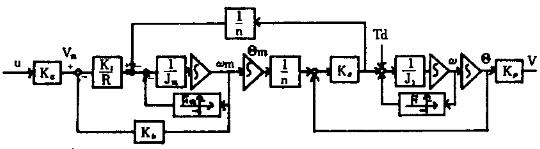


Fig. 2 Block Diagram of the Robot Drive System

In the above figure, the variables are defined as follows;

u : input voltage to the amplifier

V_a : the voltage applied to the motor

T_d : disturbance torque due to the gravitational pull

v : potentiometer output of position measurement

θ : load position angle

ω : load angular velocity

θ_m : motor shaft angle

ω_m : motor shaft angular velocity

The motor inductance and the gear backlash are experimentally found to be insignificant (Brady, 1987; Sweet and Good, 1985). Hence they are ignored in the block diagram. The system parameters are :

$K_p=4.8$ volts/rad : potentiometer constant

$R=8.19$ Ohms : motor winding resistance

$K_b=0.0388$ volts·sec/rad : motor back emf constant

$K_t=0.0388$ N·m/A : motor torque constant

$K_a=6$: power amplifier constant

$n=436.7$: gear ratio

$K_s=200$ N·m/rad : spring constant

$F_m=0.004264$ N·m : motor Coulomb friction

$J_m=0.000003$ kg·m^2 : motor inertia

$F=0.207$ N·m : load side Coulomb friction

$J=0.3$ kg·m^2 : maximum load inertia

Define the state variable vector x by

$x=(x_1, x_2, x_3, x_4)^T$

$x_1=v$: load angular position measured by a potentiometer

$x_2=\omega$: load angular velocity

$x_3=\theta_m/n$: motor position divided by the gear ratio

$x_4=\omega_m/n$: motor velocity divided by the gear ratio

Then the system equation in vector form is given by

$$\dot{x}=\begin{bmatrix} 0 & 4.8 & 0 & 0 \\ -139 & 0 & 667 & 0 \\ 0 & 0 & 0 & 1 \\ 72.8 & 0 & -350 & -61.3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 3.33F(x_2) \\ 0 \\ 736F_m(x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 21.7 \end{bmatrix} u + \begin{bmatrix} 0 \\ 3.33T_d \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

III. DIGITAL TRACKING CONTROLLER

To design a digital controller, one must first choose the sampling time. In this experiment, the sampling frequency of 20 Hz, or the sampling time of $T=0.05$ sec. is chosen.

Let r be the ramp reference input. The objective is to design a digital control as a feedback controller of the measurable variables only such that $x_1=r$ in steady state for all step or ramp reference input function r under either a constant or ramp disturbance T_d. It is further required that the settling time is less than 5 sec. And the overshoot is less than 20%.

It has been shown (Kim, 1988) that if the system is augmented by two additional digital error integrators at the output side, then the output of the augmented system can be forced to follow the given reference input r(t). Therefore, the system (1) is now augmented by introducing additional state variables x_5 and x_6 which are defined by

$$\begin{aligned} x_5((k+1)T) &= x_5(kT)+Tx_6(kT) \\ x_6((k+1)T) &= x_6(kT)+T\{x_1(kT)-r(kT)\} \end{aligned} \tag{2}$$

where T, the sampling time, is 0.05 sec. Then the augmented system is given by

$$\dot{x}=\begin{bmatrix} 0 & 4.8 & 0 & 0 \\ -139 & 0 & 667 & 0 \\ 0 & 0 & 0 & 1 \\ 72.8 & 0 & -350 & -61.3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 3.33F(x_2) \\ 0 \\ 736F_m(x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 21.7 \end{bmatrix}u + \begin{bmatrix} 0 \\ 3.33T_d \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_5((k+1)T) &= x_5(kT)+Tx_6(kT) \\ x_6((k+1)T) &= x_6(kT)+T\{x_1(kT)-r(kT)\} \\ x_5(t) &= x_5(kT), \quad kT \le t < (k+1)T \\ x_6(t) &= x_5(kT), \quad kT \le t < (k+1)T \end{aligned} \tag{3}$$

It should be noted that the augmented system (3) is a hybrid system consisted of continuous and discrete systems. Also, since the variables x_5 and x_6 are externally introduced to the system, they are measurable and hence are available for feedback in the controller design.

To find a tracking digital control, consider the original nonlinear system with $F=0$, $F_m=0$, and $T_d=0$.

$$\dot{x}=\begin{bmatrix} 0 & 4.8 & 0 & 0 \\ -139 & 0 & 667 & 0 \\ 0 & 0 & 0 & 1 \\ 72.8 & 0 & -350 & -61.3 \end{bmatrix}x+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 21.7 \end{bmatrix}u \tag{4}$$

The corresponding discrete linear system for the sampling time $T=0.05$ sec. is given by

$$x((k+1)T)=\begin{bmatrix} 0.307 & 0.180 & 3.324 & 0.314 \\ -4.743 & 0.307 & 22.766 & 0.292 \\ 0.032 & 0.003 & 0.847 & 0.139 \\ 0.535 & 0.153 & -2.569 & -0.411 \end{bmatrix}x(kT) + \begin{bmatrix} 0.010 \\ 0.142 \\ 0.012 \\ 0.301 \end{bmatrix}u(kT) \tag{5}$$

Augmenting the above system with the variables x_5 and x_6, which were difined in eq. (2) and letting $\hat{x}=(x_1, x_2, x_3, x_4, x_5, x_6)^T$, the following augmented linear system is obtained.

$$\hat{x}((k+1)T)=$$

$$\begin{bmatrix} 0.307 & 0.180 & 3.324 & 0.314 & 0 & 0 \\ -4.743 & 0.307 & 22.766 & 0.292 & 0 & 0 \\ 0.032 & -0.003 & 0.847 & 0.139 & 0 & 0 \\ 0.535 & 0.153 & -2.569 & -0.411 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.05 \\ 0.05 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\hat{x}(kT) + \begin{bmatrix} 0.010 \\ 0.142 \\ 0.012 \\ 0.301 \\ 0 \\ 0 \end{bmatrix}u(kT) - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.05 \end{bmatrix}r(kT) \tag{6}$$

Suppose a digital feedback controller u(kT) stabilizes the augmented linear system (6) when $r=0$. If the same digital controller is applied to the original augmented nonlinear system (3) with bounded nonlinearity, then it is shown (Kim, 1988) that either the augmented nonlinear system (3) with $T_d=0$ and $r=0$ is asymptotically globally stable or the system response goes into a limit cycle. If a limit cycle is present as a result of applying the feedback controller, then the feedback parameters are readjusted by applying the describing function method to eliminate the limit cycle. The resulting system then becomes asymptotically globally stable. It is further shown (Kim, 1988) that if a digital feedback control u(kT) makes the augmented nonlinear system (3) with bounded nonlinearity for $r=0$ and $T_d=0$ asymptotically globally stable, then the system response x_1 follows any given step or ramp reference input r with zero steady state error under any step or ramp disturbance T_d. In addition, the closed loop system is robust with respect to the system parameter variations in the sense that as long as the closed loop system (3) with $r=0$ and $T_d=0$ remains globally asymptotically stable, the output of the system follows the given reference input r with zero steady state error, that is, the control u(kT) is a robust tracking digital controller.

To derive a tracking digital controller, a digital controller, which stabilizes the augmented linear system (6) when $r=0$, is obtained first. In general, the controller must be given as a feedback of the entire state variables. In the present case, however, only the output x_1 and the augmented variables x_5 and x_6 are available for feedback. It is shown (Kim and Chyung, 1984; Na, 1986) that, in this case, there exists a delayed feedback control of the form

$$u(kT)=-\{K_1x_1(kT)+K_2x_1((k-h)T)+K_3x_1((k-2h)T)+K_4x_1((k-3h)T)+K_5x_5(kT)+K_6x_6(kT)\} \quad (7)$$

which stabilizes the linear system (6) when $r=0$. Here the positive integer h is the basic time delay. Fig. 3 shows the closed loop system when the above controller is applied to the original augmented nonlinear system (3). To find a controller which satisfies the given transient requirements, define the cost function J by

$$J=\sum_{k=0}^{\infty}\{\hat{x}(kT)^TQ\hat{x}(kT)+u(kT)^2\} \quad (8)$$

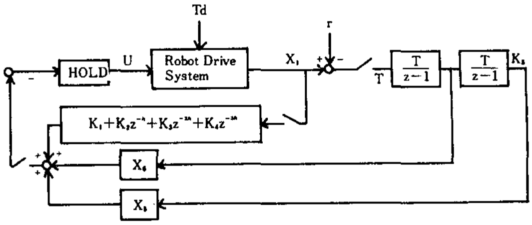


Fig. 3 Delayed Feedback Control System

where $Q=Q^T\leq 0$ is a positive semidefinite matrix and the element q_{55} is nonzero. Then, the feedback gains Ki's in eq. (7) are chosen so that the above cost J is minimized subject to the constraint given by eq. (6) with $r=0$ and the initial condition.

$$\hat{x}(kT)=(x_1,x_2,x_3,x_4,x_5,x_6)^T=(1,0,0,0,0,0,)^T \quad (9)$$

where $k=-3h, -3h+1, \ldots, 0$. The matrix Q should be chosen so that the given transient constrains are satisfied.

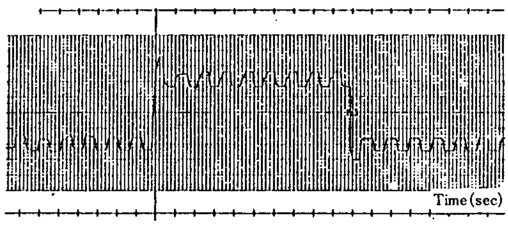
For the current implementation, the basic delay $h=2$ is chosen. After investigating several weighting matrices, the following Q matrix is chosen for the cost function J.

$$Q=\begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix} \quad (10)$$

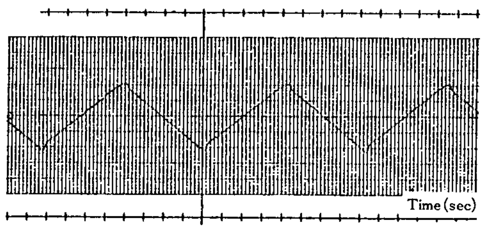
The corresponding optimal delayed feedback digital controller is obtained numerically and is found to be

$$u(kT)=-\{2.83x_1(kT)+1.21x_1((k-2)T)-0.045x_1((k-4)T)+0.27x_1((k-6)T)+8.84x_5(kT)+8.67x_6(kT)\} \quad (11)$$

When the above controller is implemented on the actual nonlinear manipulator, the system responses are as shown in Fig. 4. Fig. 4(a) is the response to a step input with a magnitude of 0.2 volt, and Fig. 4(b) is the response to a ramp input with a slope of 0.05 volt/sec. Although the ramp response follows the input with zero steady state error, the step response clearly shows the existence of a limit cycle. Therefore, it is now necessary to readjust the feedback parameters using the describing function method.



(a) Step response(10 mV/div or 0.12°/div, 1sec/div)



(b) Ramp response(50 mV/div or 0.6°/div, 1sec/div)

Fig. 4 System Responses ($K_1=2.83$, $K_5=8.84$, $K_6=8.67$)

Ⅳ. LIMIT CYCLE ELIMINATION

The controller obtained in the previous section causes a limit cycle. To apply the describing function method to eliminate the limit cycle, it is necessary to rearrange the block diagram. When the load side Coulomb friction F is reflected to the motor side through the gear train, its value is found to be 0.000474 Nm. Since the motor side Coulomb friction is 0.004264 Nm, the load side Coulomb friction, when reflected to the motor side, is only about one tenth of the motor side friction. Thus, the load side friction is ignored in the describing function analysis. By combining a single block D(z), which is the fundamental transfer function of the discrete part, and replacing the holding circuit by the transfer function $(1-e^{-ST})/s$, the block diagram given in Fig. 3 can be rearranged as shown in Fig. 5. It is suggested (Kim, 1988) that for the describing function analysis in nonlinear sampled-data system, the sampling operation can be replaced by the transfer function of 1/T since describing function is concerned only with the fundamental frequency component. In Fig. 5, the motor side Coulomb

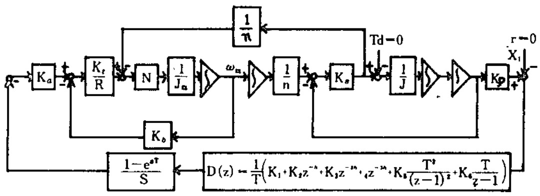


Fig. 5 Delayed Feedback System with the Nonlinearity in the Forward Path

friction is also replaced by an equivalent nonlinea element N in the forward path (Won, 1985). For the describing function analysis for step reference inputs, we can let $T_d=0$ and $r=0$ without loss of generality. From Fig. 5, overall fundamental transfer function L(s) of the linear part is given by

$$L(s) = \frac{K_t}{RJ_m ns}\left[K_b n + \frac{K_s K_p}{Js^3+K_s s}\frac{RJs^2}{K_t K_p n} + \frac{1-e^{-sT}}{s}K_a\frac{K_s K_p}{Js^3+K_s s}D(e^{sT})\right] \quad (12)$$

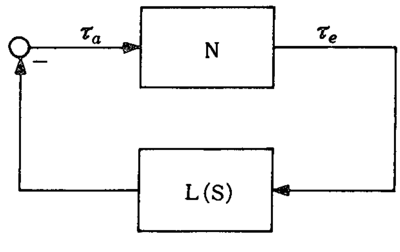


Fig. 6 Closed-loop System for Describing Function Analysis

Fig. 6 shows the simplified block combining the linear part into a single block for the describing function analysis.

Suppose the closed loop system is in a limit cycle. Let $\tau_a = M\sin(\omega t)$, $\tau_n = M_2\sin(\omega t+\phi)$ and $x_1 = M_1\sin(\omega t+\phi)$ be the fundamental frequency components of τ_a, τ_n and x_1, respectively. Let $N_{df}(M)$ be the describing function of the nonlinearity N. The describing function $N_{df}(M)$ is defined by

$$N_{df}(M) = \frac{M_2}{M}\exp(j\varphi) \qquad (13)$$

Let $\lambda = 0.004264/M$, $\alpha = \sin^{-1}(\lambda)$. Then, the describing function $N_{df}(M)$ is given (Shinners, 1978) by

$$N_{df}(M) = \sqrt{a^2+b^2}\exp(j\ \tan^{-1}(b/a)) \qquad (14)$$

where, for $\lambda > 0.536$,

$$a = 1-\lambda^2\ ,\ b = 2\lambda\sqrt{(2/\pi)^2-\lambda^2} \qquad (15)$$

and for $\lambda > 0.536$,

$$a = \frac{1}{\pi}[\pi-(\alpha-\beta)-\sin(\alpha)(\cos(\alpha)+\cos(\beta))-\cos(\beta)(\sin(\alpha)+\sin(\beta))]$$

$$b = \frac{1}{\pi}(\sin(\alpha)+\sin(\beta))^2$$

$$\cos(\beta)-\beta\sin(\alpha)+\cos(\alpha)-(\pi-\alpha)\sin(\alpha)=0 \qquad (16)$$

After a lengthy but routine manipulation and substituting the actual parameter values as well as h=2 and T=0.05 in L(s) of eq. (12), and also letting $s=j\omega$, the transfer function $L(j\omega)$ of the linear part of the system is given by

$$\begin{aligned}L(j\omega) = {} & \frac{61.3}{\omega^2(666.7-\omega^2)}[5.7\omega^2+\frac{22660}{\omega} \\ & \{K_1\sin(0.05\omega)-K_2\{\sin(2.05\omega) \\ & -\sin(2\omega)\}-K_3\{\sin(4.05\omega) \\ & -\sin(4\omega)\}-K_4\{\sin(6.05\omega) \\ & -\sin(6\omega)\} \\ & -0.00125K_5\{\sin(0.05\omega) \\ & -\sin(0.1\omega)\}/\omega \\ & +0.05K_6\sin(0.05\omega)\}] \\ & +j\frac{61.3}{\omega^2(666.7-\omega^2)}[\omega^3-666.7\omega \\ & +\frac{22660}{\omega}\{K_1\{1-\cos(0.05\omega)\} \\ & -K_2\{\cos(2.05\omega)-\cos(2\omega)\} \\ & -K_3\{\cos(4.05\omega)-\cos(4\omega)\} \\ & -K_4\{\cos(6.05\omega)-\cos(6\omega)\}-0.00125K_5 \\ & \{\cos(0.05\omega)-\cos(0.1\omega)\}/\omega \\ & +0.05K_6\cos(0.05\omega)\}]\end{aligned} \qquad (17)$$

The magnitude and frequency of a potential limit cycle can now be investigated by studying the intersection point of the loci of $-1/N_{df}(M)$ and $L(j\omega)$ on the complex plane. For the feedback controller given by eq. (7), the general shape of the loci are as shown in Fig. 7 and detailed loci are given in Fig. 8.

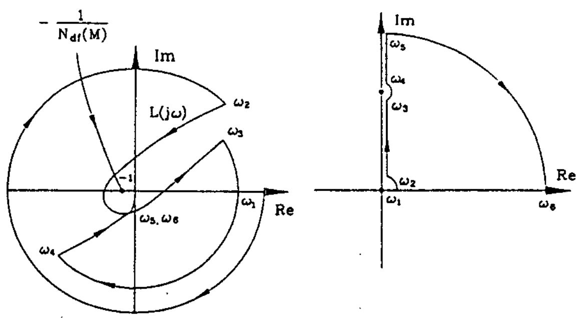


Fig. 7 Overall Loci of $-1/N_{df}(M)$ and $L(j\omega)$ (Not to Scale)

For a controller given by eq. (11), the locus of $L(j\omega)$ is labeled as $L'(j\omega)$ in Fig. 8. There is indeed an intersection. The magnitude and frequency of the limit cycle are M=0.95 and ω=1.8. The corresponding magnitude at the output x_1 is M_1=0.04. The magnitude and frequency of the actual response are, from Fig. 4(a), M_1=0.045 and ω=1.09, which are in a reasonable agreement with the predicted values.

To eliminate the limit cycle, it is necessary to avoid the intersection. However, from Fig. 7, it is obvious that the intersection cannot be removed by readjusting the feedback parameters, for eliminating the intersection will, in general, cause the system to be unstable, that is, the $L(j\omega)$ plot will encircle the (-1, 0) point. Therefore, the parameters are now readjusted so that the magnitude of the limit cycle is acceptably small. It can be seen from Fig. 8 that the magnitude is reduced if the locus

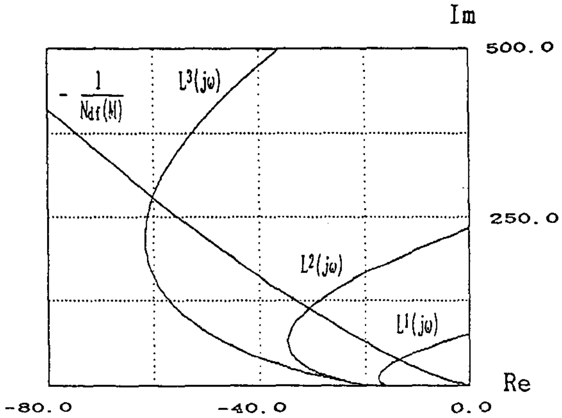


Fig. 8 Loci of $-1/N_{df}(M)$ and $L(j\omega)$

of $L(j\omega)$ is moved up, for the magnitude becomes smaller as the intersection point moves up through the locus of the describing function. Computer aided plotting of $L(j\omega)$ shows that the locus of $L(j\omega)$ is moved up if the feedback parameters K_1 and K_6 are increased and the parameter K_5 is decreased. Of course, one must be careful not to modify the locus of $L(j\omega)$ so much that the resulting locus encircles the critical point (-1, 0), for this will result in the system being unstable.

To reduce the limit cycle, the feedback parameters K_1 and K_6 are first increased from their original values of $K_1=2.83$ and $K_6=8.67$ to the values of $K_1=4$ and $K_6=12$. Then, to further reduce the magnitude, K_1 and K_6 are increased to $K_1=4.5$ and $K_6=15$, and K_5 is

Table 1 Magnitude and Frequency of Limit Cycle for L^1, L^2 and L^3.

				Describing Function			Actual System	
	K_1	K_5	K_6	ω rad/sec	M N·M	M_1 volts	ω rad/sec	M_1 volts
L^1	2.83	8.84	8.67	1.80	0.95	0.04	1.09	0.045
L^2	4.00	8.84	12.0	1.63	0.93	0.023	0.09	0.024
L^3	4.50	6.00	15.0	1.35	0.92	0.014	-	0.0

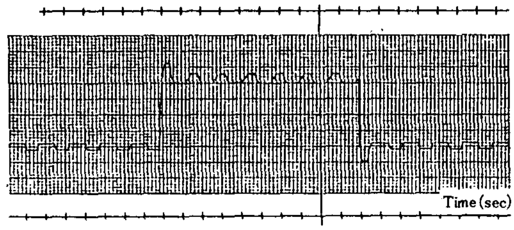
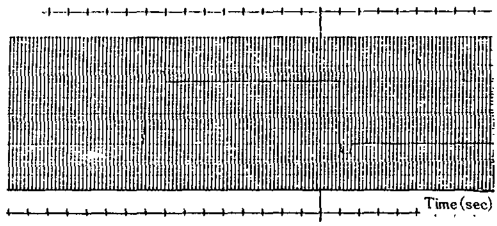
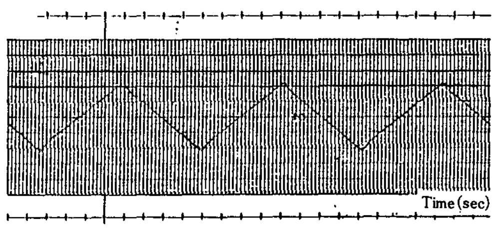


Fig. 9 Step Response (10 mV/div or 0.12°/div, 1 sec/div) for L^2 ($K_1=4$, $K_5=8.84$, $K_6=12$)



(a) Step response (10 mV/div or 0.12°/div, 1 sec/div)



(b) Ramp response (50 mZV/div or 0.6°/div, 1 sec/div)

Fig. 10 System Responses for L^3 ($K_1=4.5$, $K_5=6$, $K_6=15$)

decreased to $K_5=6$. The resulting loci are given in Fig. 8 as $L^2(j\omega)$ and $L^3(j\omega)$, respectively. The corresponding limit cycle data as well as the data for the original $L^1(j\omega)$ are given in Table 1. The actual system responses are given in Fig. 9 and Fig. 10, respectively. As can be seen from the figures, the limit cycle is indeed reduced in the case of L^2, and it is completely eliminated in the case of L^3. Thus, for the feedback controller

$$u(kT)=-\{ 4.5x_1(kT) + 1.21x_1((k-2)T) - 0.045x_1((k-4)T) + 0.27x_1((k-6)T) + 6x_5(kT) + 15x_6(kT) \} \quad (18)$$

the system response follows the given step or ramp reference input with zero steady state error. The experimental results also showed that the controller is able to cancel the effects of all step or ramp disturbances.

V. CONCLUSIONS

A satisfactory robust digital controller has been designed and implemented for a robotic manipulator. The objective is to maintain zero steady state error for a step or ramp reference input under constant or ramp disturbance. The method is based on augmenting the system with two additional digital integrators and feeding back delayed values of the observable state variables. The feedback parameters are obtained by applying the optimal control theory and then readjusting the parameters using the describing function analysis in nonlinear sampled-data system. Although the method is developed for a particular geared manipulator, it is also applcable to other similar manipulators.

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