

# 레이레이 페이딩 채널에서 Trellis 부호를 적용한 $\pi/4$ shift QPSK

## ( $\pi/4$ shift QPSK with Trellis-Code in Rayleigh Fading Channel)

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### 요약

본 논문에서는  $\pi/4$  shift QPSK에 Trellis 부호화된 변조 기법(Trellis-Coded Modulation, TCM)을 적용시키기 위하여  $\pi/8$  shift 8PSK를 제안하고 위상차에 의한 신호 집합 확장과 신호 집합 분할을 수행하는 trellis 부호화된  $\pi/8$  shift 8PSK를 제안한다. 또한 BER(Bit Error Rate) 성능을 향상시키기 위하여 제1차 위상차뿐만 아니라 제L차 위상차의 자승 유클리드 거리를 메트릭(Branch Metric)으로 갖는 비터비 디코더(Viterbi decoder)를 설계한다. 그리고  $\pi/4$  shift QPSK, trellis 부호화된  $\pi/8$  shift 8PSK와 제L차 위상차의 자승 유클리드 거리를 메트릭(Branch Metric)으로 갖는 trellis 부호화된  $\pi/8$  shift 8PSK의 BER 특성을 AWGN과 다경로 페이딩 채널에서 Monte Carlo 시뮬레이션을 통해 알아본다. 제안된 알고리즘은 MDPSK에도 적용될 수 있다.

### ABSTRACT

In this paper, in order to apply the  $\pi/4$  shift QPSK to TCM, we propose the  $\pi/8$  shift 8PSK modulation technique and the trellis-coded  $\pi/8$  shift 8PSK performing signal set expansion and set partition by phase difference. In addition, the Viterbi decoder with branch metrics of the squared Euclidean distance of the first phase difference as well as the Lth phase difference is introduced in order to improve the bit error rate(BER) performance in differential detection of the trellis-coded  $\pi/8$  shift 8PSK. The proposed Viterbi decoder is conceptually the same as the sliding multiple detection by using the branch metric with first and Lth order phase difference. We investigate the performance of the uncoded  $\pi/4$  shift QPSK and the trellis-coded  $\pi/8$  shift 8PSK with or without the Lth phase difference metric in an additive white Gaussian noise (AWGN) and Rayleigh fading channel using the Monte Carlo simulation. The study shows that the  $\pi/4$  shift QPSK with the

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Trellis-code i.e. the trellis-coded  $\pi/8$  shift 8PSK is an attractive scheme for power and bandlimited systems and especially, the Viterbi decoder with first and Lth phase difference metrics improves BER performance. Also, the next proposed algorithm can be used in the TC  $\pi/8$  shift 8PSK as well as TC MDPSK.

## I. Introduction.

In 1982, the trellis-coded modulation(TCM) method was developed by Ungerboeck for power and bandwidth efficient digital communications[1]. Channel coding is well known as an effective scheme for combatting channel noise and other system impairments. In order to improve the bit error rate performance, we add redundancy bits by using error correcting codes such as the block or convolutional code. But the addition of bits causes the bit rate to increase and the bandwidth to expand. Unfortunately, this bandwidth expansion is not desirable, or even not possible for bandwidth limited channels. To compensate for the bandwidth expansion caused by the redundancy bits, one can use a larger signal constellation in the modulator. However, the larger the signal constellation, the more power of extra signal is required to obtain the same performance, or the lower performance is obtained at the same power. In the past, channel coding and modulation were treated as two separate operations. In 1982, Ungerboeck proposed the TCM to obtain the improved coding gain by combining the channel coding and modulation[1].

By using signal set expansion and signal mapping by set partitions, Ungerboeck showed that a 3dB (asymptotic) coding gain can be obtained with a 4-state code, and a 3.6dB (asymptotic) coding gain can be obtained with a 8 state code[1][2][3]. By [1], when the signal set is expanded from the  $2^n$  signal set to the  $2^{n+1}$  signal set, i.e. expanding the original signal set two times, we can obtain almost all coding gains. Hence we use the  $n/(n+1)$  convolutional encoder to expand the signal set from  $2^n$  to  $2^{n+1}$  and to maximize the Euclidean distance between the signals.

In this paper, in order to apply the  $\pi/4$  shift QPSK to TCM we propose the  $\pi/8$  shift 8PSK modulation technique and trellis-coded(TC)  $\pi/8$  shift 8PSK performing signal set expansion and set partition by phase difference. Also, the Viterbi decoder with branch metrics of the squared Euclidean distance of the first phase difference as well as the Lth phase difference is introduced in order to improve the bit error rate performance in differential detection of TC  $\pi/8$  shift 8PSK. We investigate the performance of the trellis-coded  $\pi/8$  shift 8PSK in an additive white Gaussian noise (AWGN) and multipath fading channel.

The study shows that the  $\pi/4$  shift QPSK with trellis-code i.e., TC  $\pi/8$  shift 8PSK, is an attractive scheme for power and bandlimited systems and especially, the Viterbi decoder with first and Lth phase difference metrics improves the BER performance.

## II. Viterbi decoder with first and Lth phase difference metrics.

In the  $\pi/4$  shift QPSK or the TC  $\pi/8$  shift 8PSK, information is transmitted by the phase difference between the current signal and the previous signal. At the receiver, only the phase difference between the two sampling instants is needed for detecting information. But by[4], the Lth order phase difference (phase difference between the current signal and the Lth previous signal) is capable of nonredundant error correction by operating as a syndrome, or by [5][6] one can perform multiple symbol detection to improve the performance of MDPSK. In this paper, we show that the BER performance can be improved by using the branch metric with first and Lth or-

der phase differences. The multiple symbol detection of TC MDPSK of [5] is conceptually the same as the block-by-block detection by using the multiple TCM(MTCM). But the next proposed Viterbi decoder is conceptually the same as the sliding multiple detection by using the branch metric with first and Lth order phase differences. Also, the next proposed algorithm can be used in the TC  $\pi/8$  shift 8PSK as well as the TC MDPSK.

Consider an N sequence of the TC  $\pi/8$  shift 8PSK signal to be transmitted

$$S = (s_0, s_1, \dots, s_{N-1}) \quad (1)$$

where each  $s_k$  can take any of the signals in Fig. 1 and  $k$  denotes the  $k$ th transmission interval. The transmitted signal in the interval  $kT \leq t < (k+1)T$ , has the complex form

$$s_k = \sqrt{2P} e^{j\phi_k} \quad (2)$$

where  $P$  denotes the signal power,  $T_s$  denotes the TC  $\pi/8$  shift 8PSK signal period and  $\phi_k$  denotes the transmitted phase. Then in the case of the TC  $\pi/8$  shift 8PSK

$$\begin{aligned} s_k &= s_{k-1} \cdot x_k = \sqrt{2P} e^{j(\phi_{k-1} + \Delta\phi_k)} \\ &= s_{k-L} \cdot x_{k-L+1} \cdot x_{k-L+2} \cdot \dots \cdot x_k \\ &= \sqrt{2P} e^{j(\phi_{k-L} + \Delta\phi_{k-L+1} + \Delta\phi_{k-L+2} + \dots + \Delta\phi_k)} \end{aligned} \quad (3)$$

If the channel effect is to add the zero mean complex gaussian  $n_k$  in the transmitted signal, the received signal sequences can be written in the form of  $r = (r_0, r_1, \dots, r_{N-1})$ . Here,  $r_k$  is the received signal in the  $k$ th interval.

Because information bits are transmitted by the phase difference of the transmitted signal, we must extract the phase difference of the received signal. The extracted first phase difference  $y_{1,k}$  is

$$y_{1,k} = r_k \cdot r_{k-1}^* = (s_k + n_k) \cdot (s_{k-1} + n_{k-1})^*$$

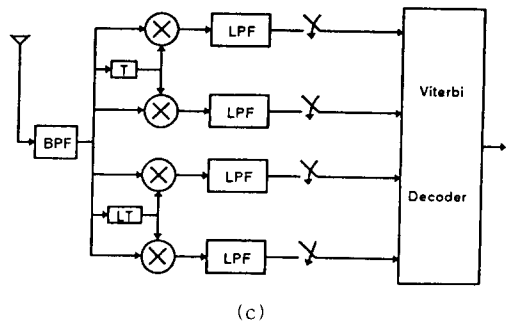
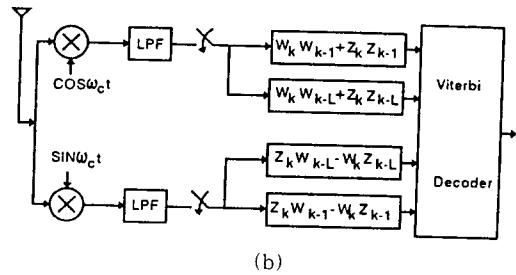
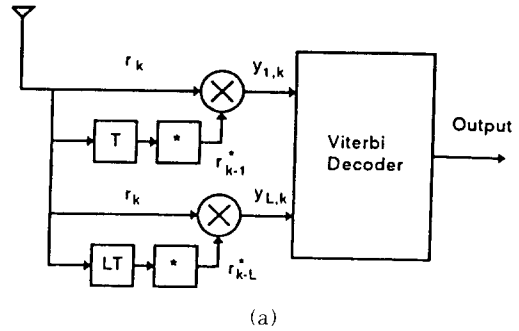


Fig. 1. Demodulation block diagram of TC  $\pi/8$  8PSK with Lth phase difference (continued)

$$\begin{aligned} &= s_k \cdot s_{k-1}^* + n_{1,k} \\ &= x_k + n_{1,k} \end{aligned} \quad (4)$$

where "\*" is the conjugate of complex signal. If we extract the Lth phase difference of the received signals, the extracted Lth phase difference  $y_{L,k}$  is

$$\begin{aligned} y_{L,k} &= r_k \cdot r_{k-L}^* = (s_k + n_k) \cdot (s_{k-L} + n_{k-L})^* \\ &= s_k \cdot s_{k-L}^* + n_{L,k} \\ &= x_{k-L+1} \cdot x_{k-L+2} \cdot \dots \cdot x_k + n_{L,k} \end{aligned} \quad (5)$$

The task of the demodulator is to process the phase difference signal sequence set  $Y$  in order to produce an estimate  $\hat{X} = (\hat{x}_0, \hat{x}_1, \hat{x}_2, \dots, \hat{x}_{N-1})$ . Assume that all information symbols are equally probable. In order to minimize the probability of error, we process  $y_k$  according to the so-called maximum-likelihood decision. This procedure compares the conditional probabilities of the received signal given each possible transmitted sequence and chooses the transmitted sequence corresponding to the largest probability among them. That is, the decoder chooses  $\hat{X}$  if

$$P_N(Y | \hat{X}) = \max_{all X} P_N(Y | X) \quad (6)$$

We assume that the noise is an additive white gaussian with a zero mean and the channel is memoryless, which means that the noise affects each code symbol independently of all other symbols. If we use the  $L$ th difference, then

$$P_N(Y | \hat{X}) = \max_{all X} \prod_{k=0}^{N-1} P(y_{1,k}, y_{L,k} | x_k, x_{k-1}, \dots, x_{k-L+2}, x_{k-L+1}) \quad (7)$$

Generally, it is computationally easier to use the logarithm of the likelihood-function since this permits the summation, rather than the multiplication of terms.

The log likelihood function[7] is expressed as :

$$\begin{aligned} P_N(Y | \hat{X}) &= \max_{all X} \prod_{k=0}^{N-1} \ln P(y_{1,k}, y_{L,k} | x_k, x_{k-1}, \dots, x_{k-L+2}, x_{k-L+1}) \\ &= \max_{all X} \prod_{k=0}^{N-1} \ln f(y_{1,k}, y_{L,k} | x_k, x_{k-1}, \dots, x_{k-L+2}, x_{k-L+1}) \\ &= \max_{all X} \prod_{k=0}^{N-1} \ln \frac{1}{2\pi\sigma_n^2 \sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)\sigma_n^2} \cdot (|y_{1,k} - \overline{y_{1,k}}|^2 - 2\rho|y_{1,k} - \overline{y_{1,k}}| \cdot |y_{L,k} - \overline{y_{L,k}}| + |y_{L,k} - \overline{y_{L,k}}|^2)} \end{aligned} \quad (8)$$

where

$$\begin{aligned} \overline{y_{1,k}} &= x_k \\ \overline{y_{L,k}} &= x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+1} \\ \sigma_n^2 &= E[(y_{1,k} - \overline{y_{1,k}})^2] \\ &= E[n_{1,k}^2] \\ &= E[(y_{L,k} - \overline{y_{L,k}})^2] \end{aligned}$$

$$\begin{aligned} &= E[n_{L,k}^2] \\ \rho &= \frac{E[(y_{1,k} - \overline{y_{1,k}}) \cdot (y_{L,k} - \overline{y_{L,k}})]}{\sigma_n^2} \end{aligned}$$

Therefore,

$$\begin{aligned} P_N(Y | \hat{X}) &= \max_{all X} \ln \sum_{k=0}^{N-1} \frac{1}{2\pi\sigma_n^2 \sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)\sigma_n^2} \cdot (|y_{1,k} - \overline{y_{1,k}}|^2 - 2\rho|y_{1,k} - \overline{y_{1,k}}| \cdot |y_{L,k} - \overline{y_{L,k}}| + |y_{L,k} - \overline{y_{L,k}}|^2)} \\ &= \max_{all X} [C - \sum_{k=0}^{N-1} A(|y_{1,k} - x_k|^2 - 2\rho|y_{1,k} - x_k| \cdot |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}| + |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}|^2)] \end{aligned} \quad (9)$$

where  $A$  and  $C$  are constants that can be disregarded in the maximization. In conclusion, the maximum-likelihood detector of a sequence of signal  $y_k$  is equivalent to the minimization of the metric defined as follows :

$$\begin{aligned} m_p(Y | X) &= \max_{all X} \sum_{k=0}^{N-1} (|y_{1,k} - x_k|^2 - 2\rho|y_{1,k} - x_k| \cdot |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}| + |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}|^2) \end{aligned} \quad (10)$$

We define the branch metric of the Viterbi decoder of the TC  $\pi/8$  shift 8PSK demodulation as follows :

$$\begin{aligned} m_b(Y | X) &= (|y_{1,k} - x_k|^2 - 2\rho|y_{1,k} - x_k| \cdot |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}| + |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+2} \cdot x_{k-L+1}|^2) \end{aligned} \quad (11)$$

In eq.(11), the first term is the Euclidean distance between the phase difference of the received sequence signal and the candidate signal  $x_k$ . The third term is the Euclidean distance between the  $L$ th phase difference of the received sequence signal and the phase of the candidate

signal  $x_k, x_{k-1}, \dots$  and  $x_{k-L+1}$ . The second term is the multiplication of the square root of the first and third term, which is then discarded as a common term.

Thus, the branch metric becomes

$$m_b(Y|X) = (|y_{1,k} - x_k|^2 + |y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+1}|^2) \quad (12)$$

The demodulation block diagrams of the TC  $\pi/8$  shift 8PSK are shown in Fig. 1. We will demonstrate how to construct a Viterbi decoder with the above metric. In this case, the decisions depend on not only the Euclidean distance of the present phase difference signal but also the Euclidean distance of the Lth previous symbol. Hence, the Viterbi decoder takes the branch metric as the sum of the first and second terms of eq. (12).

The first term is the Euclidean distance of the conventional Viterbi decoder of TCM. In order to compute the second term, the algorithm works as follows:

step 1: Find the candidate signal  $x_{k-j}$  (state num-

ber of  $t_{k-j}$ , state number of  $t_{k-j+1}$ ) between the states of  $t_{k-j}$  and  $t_{k-j+1}$  by backward searching the survivor path, where  $j = L-1, L-2, \dots, 1$ .

- step 2: Calculate  $|y_{L,k} - x_k \cdot x_{k-1} \cdot \dots \cdot x_{k-L+1}|^2$ .
  - step 3: Calculate the branch metric using eq. (12).
  - step 4: Repeat step 3 for the number of branches per state.
  - step 5: Repeat step 1,2,3,4 for the number of states.
  - step 6: Update and compare the path metric at  $t_{k+1}$  and find the survivor path at  $t_{k+1}$ .
  - step 7: Decode the information data.
- Here,  $L <$  decoding delay of Viterbi decoder.

For example, as shown in Fig. 2 consider when  $L=2$  and state number  $=2$ . At  $t_k$  and state 0, find  $x_{k-1}(1,0)$  by backward searching the survivor path. Then the branch metric is  $|y_{1,k} - x_k(0,0)|^2 + |y_{2,k} - x_k(0,0) \cdot x_{k-1}(1,0)|^2$ . By repeating this procedure, we obtain all of branch metrics, the path metric and the survivor path by comparing the path metric.

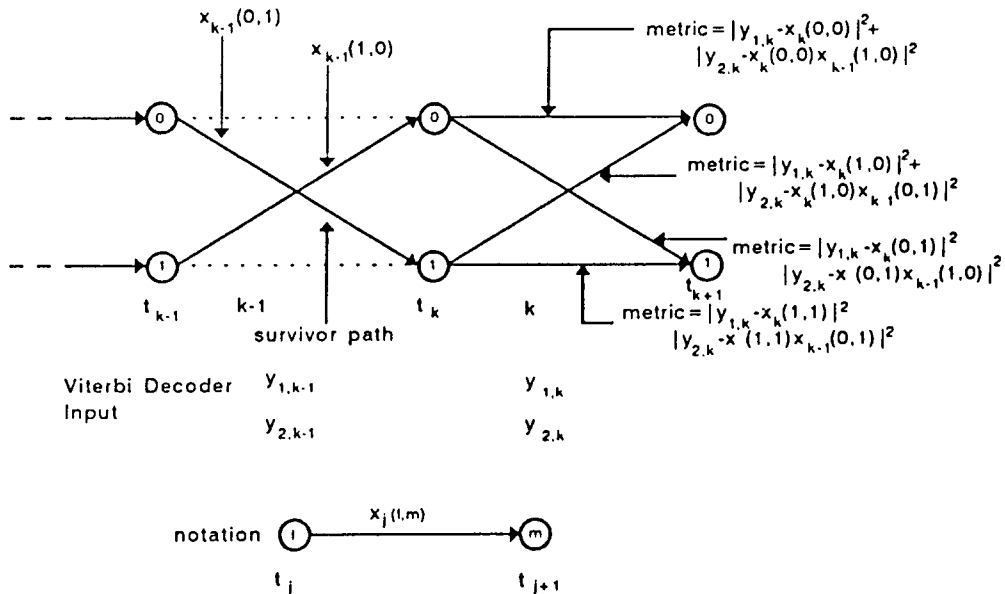


Fig. 2. Trellis diagram of Viterbi decoder with Lth difference metric.

### III. Simulation

In this section, we describe and present the results of a computer simulation of the uncoded  $\pi/4$  shift QPSK versus the TC  $\pi/8$  shift 8PSK. The performance improvement of the TC  $\pi/8$  shift 8PSK is checked through the computer simulation based on the Monte Carlo method. Assume that the transmitted signal is corrupted by AWGN and carrier is recovered perfectly at the receiver. The simulation calculates the SNR versus Bit Error Rate(BER) of the uncoded  $\pi/4$  shift QPSK and the TC  $\pi/8$  shift 8PSK with or without the second difference metric of 2 states, 4 states, 8 states and 16 states. The simulation block diagram is represented in Fig. 1.

The length of the path memory in the Viterbi decoding algorithm is  $M=5 \cdot v$  ( $M$ : decision delay,  $v$ : state number). An error event is counted when the transmitted data differs from the decoded received data bit. The results are shown in

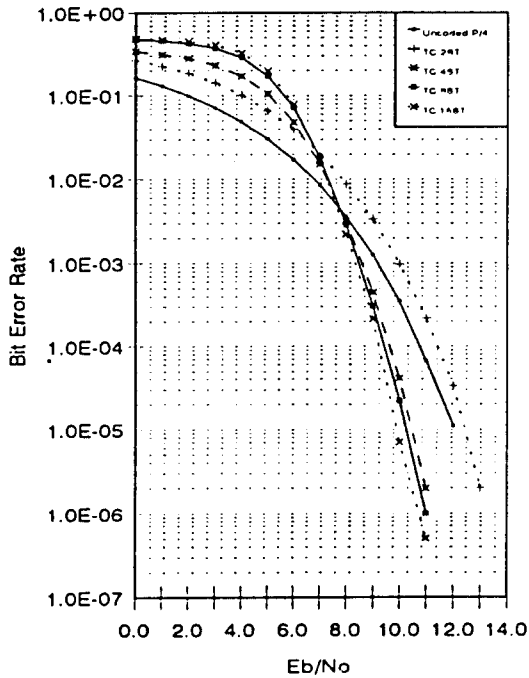


Fig. 3. Performance of the uncoded  $\pi/4$  shift QPSK vs. the TC  $\pi/8$  shift 8PSK.

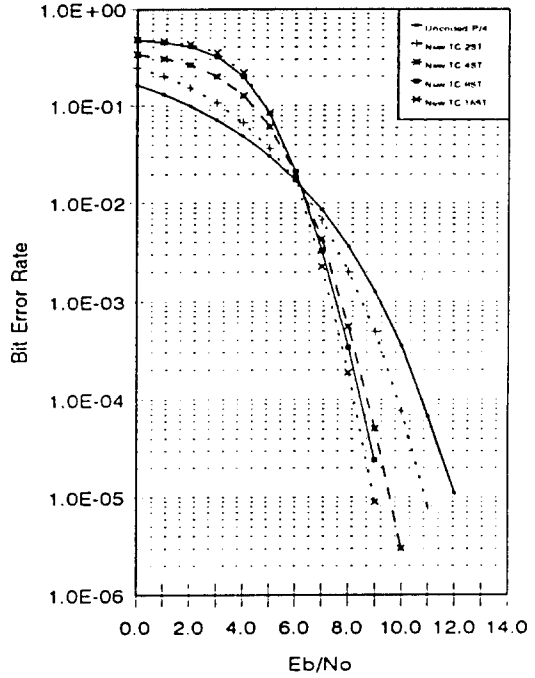
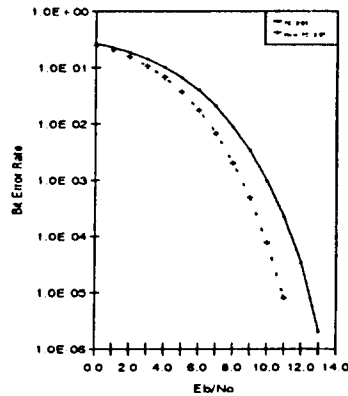


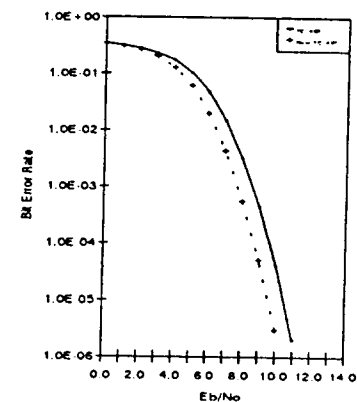
Fig. 4. Performance of the uncoded  $\pi/4$  shift QPSK vs. the TC  $\pi/8$  shift 8PSK with second phase difference metrics.

Fig 3,4 and 5. The simulation results are given with 90 percent confidence intervals.

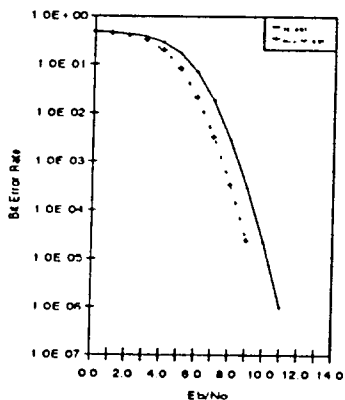
At the  $10^{-5}$  BER for data transmission, TC  $\pi/8$  shift 8PSK demonstrates an improvement of 1.5–2.0 dB over the uncoded  $\pi/4$  shift QPSK. The



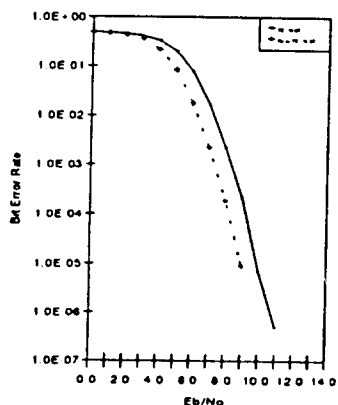
(a) 2 state



(b)4 state



(c)8 state



(d)16 state

Fig. 5. Performance of the TC  $\pi/8$  shift 8PSK v.s. the TC  $\pi/8$  shift 8PSK with second phase difference metrics.

TC  $\pi/8$  shift 8PSK with second phase difference metrics improves by 1.0–3.2 dB over the uncoded  $\pi/4$  shift QPSK. The trellis-coded  $\pi/8$  shift 8PSK with second phase difference metric demonstrates an improvement of 1.0–1.6 dB over the trellis-coded  $\pi/8$  shift 8PSK. At the  $10^{-5}$  BER, specific numerical results of various state are summarized in Table I.

Table I. Coding gains of the TC  $\pi/8$  shift 8PSK at  $10^{-5}$  BER.

states	$\pi/4$ shift QPSK v.s. TC $\pi/8$ shift 8PSK	$\pi/4$ shift QPSK v.s. TC $\pi/8$ shift 8PSK with 2nd phase difference metric	TC $\pi/8$ shift 8PSK v.s. TC $\pi/8$ shift 8PSK with 2nd phase difference metric
2	-0.6dB	1.0dB	1.6dB
4	1.5dB	2.5dB	1.0dB
8	1.8dB	2.9dB	1.1dB
16	2.0dB	3.2dB	1.2dB

The simulation block diagram of the multipath fading channel is shown in Fig. 6. The interleaving is used in order to randomize the burst error caused by fading. The order of interleaving is  $100 \times 30$  or  $32 \times 32$ [8]. The fading simulator based on a two-ray model was composited by [9][10].

Results of the computer simulation for a Rayleigh channel with the fading rate of 30Hz and 80Hz and 2,4,8,16 state code are shown in

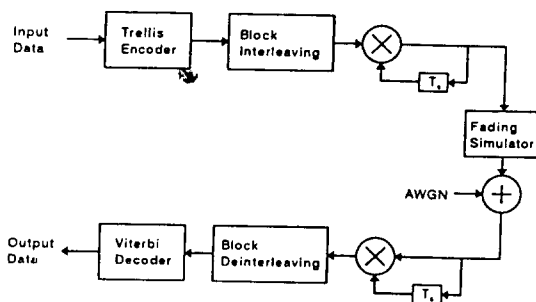


Fig. 6. Simulation block diagram of multipath fading channel.

Fig. 7. When the states number are 8 or 16, the improvement of measured coded  $\pi/8$  shift 8PSK

with respect to the error floor of the  $\pi/4$  shift QPSK is seen to be 11 ~ 18 dB at BER =  $10^{-1}$ .

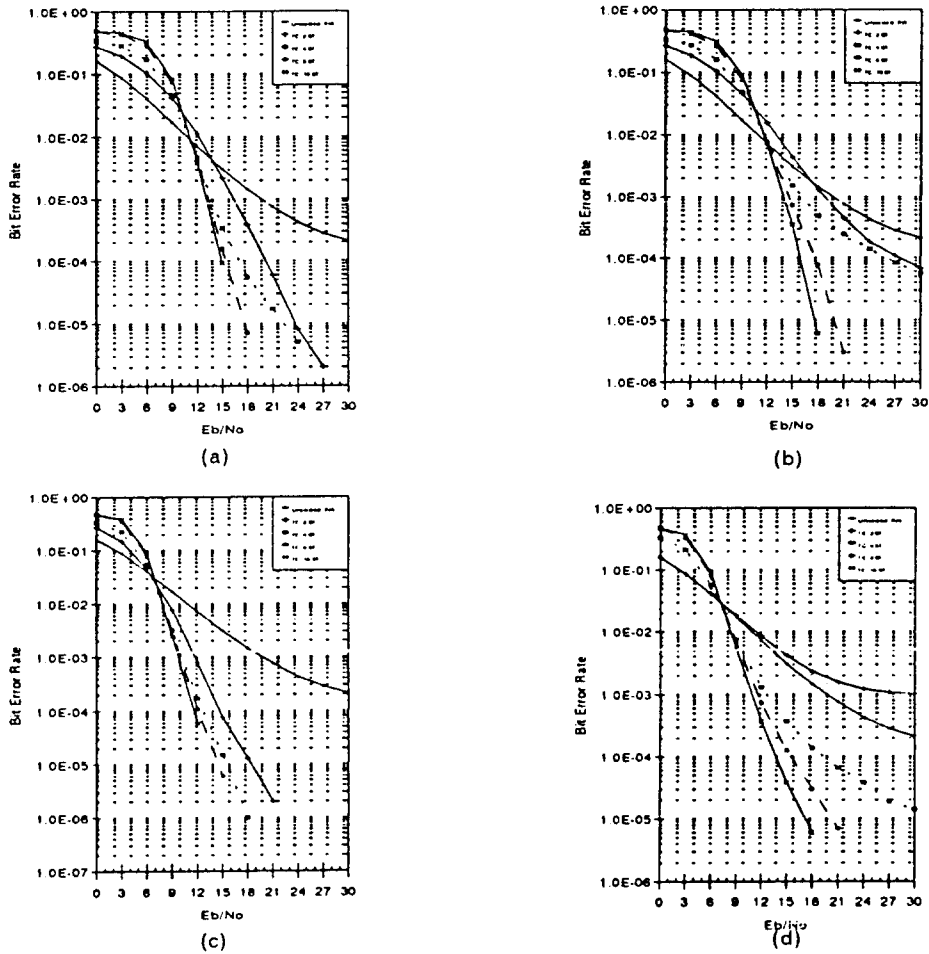


Fig. 7. Performance of TC  $\pi/8$  shift 8PSK in the Rician fading channel with undisred signal power = -10dB

- (a) Fading rate = 30Hz, Interleaving orders = 100×30
- (b) Fading rate = 80Hz, Interleaving orders = 100×30
- (c) Fading rate = 30Hz, Interleaving orders = 32×32
- (d) Fading rate = 80Hz, Interleaving orders = 32×32

## V. Conclusion

We have presented the  $\pi/4$  shift QPSK with Trellis coding. The Trellis code method that Ungerboeck proposed is the bandwidth and power

efficient modulation method. In order to apply TCM to the  $\pi/4$  shift QPSK, the phase difference expands from 4 to 8, and the signal number from 8 to 16. By the expanded phase difference the set partition is completed, and the encoder is



designed to attain this set partition. The output of the encoder is mapped by the phase difference, and the transmitted signal is derived through differential modulation. Through this method, a BER performance improvement of 1.5–2.0 dB at  $10^{-5}$  is obtained over the uncoded  $\pi/4$  shift QPSK. We also demonstrate that the BER performance can be improved by 1.0–3.2 dB with second difference metrics over uncoded  $\pi/4$  shift QPSK. We describe the Viterbi decoding algorithm with the Lth difference metric. This performance improvement is obtained without sacrificing any bandwidth efficiency (data rate) or power efficiency. For a Rayleigh channel, the improvement of measured coded  $\pi/8$  shift 8PSK with respect to the error floor of the  $\pi/4$  shift QPSK is seen to be 11–18 dB at  $\text{BER} = 10^{-4}$ .

The coding gains are 14–16 dB and 13–15 dB at  $\text{BER} = 10^{-4}$  with respect to uncoded  $\pi/4$  shift QPSK for the Rician channels with  $K = -5\text{dB}$  and  $-10\text{dB}$ .

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