# Flow Characteristics for the Variation of Radius of Curvature in Open Channel Bends

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**ABSTRACT** / The flow characteristics varying with the rate of the radius of curvature to width (Rc/B) in open channel bends are investigated with a simplified numerical model. Secondary flow velocity and transverse bed slope are formulated from the equations of momentum and force balance analysis, respectively.

The conservation equations of mass and streamwise momentum are simplified by depth integration and its solution could be obtained from the explicit finite difference method. Three sets of computer simulation are executed. The rates of Rc/B adopted in simulations are 2.7, 5.4 and 8.1.

The terms analyzed in this paper are secondary flow velocity, streamwise velocity, the path of maximum streamwise velocity, deviation angle, and mass—shift velocity.

## 1. Introduction

The importance of river is increasing as civilization progresses. The shapes of meandering and braiding are common in natural rivers due to sucessive alteration of river channels. It is necessary to fix and stabilize river channels in order to construct hydrostuctures. The analysis of characteristics of flow and river bed variation is the basic solution for the river problems.

There are many physical and mathematical models to analyze the hydraulic characteristics in rivers bends. Recently, the mathematical models have been used widely. The mathematical models can be classified into three categories : the rivers bed variation type, the generation of meandering type, and the flow characteristics type.

The forecasting model of river bed variation are decomposed of two models: One is used to forecast the transverse bed slope in large curavature — shallow channel bends. In this case, the variation terms of streamwise velocity are neglected and it is assumed that the direction of depth—averaged streamwise velocity is parallel to the axis of river channel.

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This type of models are developed by Kikkawa (1976), Odgaad (1981,1988), and Nakato (1983). The other is the model which analyze the redistribution of flow and sedimentation with consideration of streamwise velocity transition. This type of model was developed originally by Rozovskii (1961), and then by Engelud (1974), and Struiksma (1985).

The models of generation of meandering can be classified into the linear stablity analysis model and the steady state analysis model except classic theory. Parker (1985) and Olsen (1988) developed this type of model.

The basic theory of flow characteristics was estabilished by Rozovskii (1961). This model was studied by De Vriend (1977), Ikeda (1975), Falcon (1979), Kalkwijk (1980), Rodi (1979), and other researchers.

These models mentioned above are used to analyze the flow characteristics in river channel, but they still have many restrictions. The characteristics of channel bends were studied by Ko (1975), Song (1981), Yoon (1986,1988), and Lee (1987) in Korea. But there is still a large room to be studied.

In this research, the river bed variation model and the flow characteristics model were combined to estimate secondary flow and transverse bed slope using the force equilibrium theory and the equation of momentum. The equations of continuity and momentum were integrated over depth in shallow channel condition, and assuming transverse bed slope is larger than transverse water surface slope, the transverse momentum equation was neglected. The flow characteristics in channel bends were analyzed using numerical simulation with a fixed channel width and variable raduis of curvature.

## 2. Basic Conception

In order to analyze the flow characteristics in channel bends, five assumptions were made:
1) flow is incompressible and steady–state,
2) river channel has a large width–to–depth (B/H) ratio,
3) free surface and channel bottom are parallel along channel centerline,
4) bed material is uniform sand,
5) wall effects are negligible.

### 2.1 Transverse Flow and Transverse Bed Slope

The transverse flow was separated into two components: a secondary flow velocity to account for the transverse flow circulation in channel bends and a mass– shift velocity having a uniform profile to account for the outward redistribution of flow that occurs in nonuniform bend flow (Vadnal, 1984).

In this case, two assumptions were made: first, the distribution of mass – shift velocity over depth is uniform; secondly, the vertical distribution of secondary flow velocity is linear relationship as shown in Eq.1. These assumptions are known to reasonable (Odgaard, 1981).

$$u = 2U\{(Z-Z_b)/h - 1/2\} \tag{1}$$

where, $U$ : transverse velocity of water surface
$h$ : water depth
$Z_b$: bed elevation
$Z$ : water level

In general, the transverse velocity of water surface equals approximately 10 percent of the depth averaged streamwise velocity. Mass-shift velocity exists in entrance section of meandering rivers and in the section where the radius of curvature changes rapidly. But, it decays as it moves downstream. The bed – shear stress is divided into streamwise component, Zo, and transverse component, Zr. The scale of Zr/Zo is important to calculate the transverse bed slope. The ratio of Zr/Zo is proportional to the ratio of the transverse velocity at the bed, $U_b$, and the streamwise velocity at the bed, $V_b$.
This relationship is shown in Eq.(2) (Rozovskii,1961).

$$Zr/Zo = U_b/V_b \tag{2}$$

Vadnal(1984) used the average velocity, V, instead of Vb, and adopted the proportionality constant, $\beta$, as shown in Eq.(3).

$$Zr/Zo = \beta(U_b/V) \tag{3}$$

Since it is difficult to calculate the value of $V_b$ and $\beta$, it is assumed that $V_b$ equals kV, where $V_b$ is the velocity at the height of median sand $D_{50}$.
The value of k was derived as Eq.(4) using power law.

$$k = n+1/n(D_{50}/H)^{1/n} \tag{4}$$

where, $n$ : power-law exponent
$H$ : water depth
$D_{50}$: the diameter of median bed matrials

The torque of transverse bed-shear stress was calculated from Zr estimated by using Eq.(2) and (4). The streamwise distribution of secondary flow , U(s), is presnted in Eq.(5).

$$U(s) = U(So)e^{-g_1/H(S-S_0)} + g2/g1(H\overline{V}/Rc)[1-e^{-g_1/H(S-S_O)}] \tag{5}$$

where, $g1 = f/8k(3n+1)(2n+1)/(2n^2+n+1)$,
$g2 = \{(n+1)(3n+1)(2n+1)\}/n(n+2)(2n^2+n+1)$,

f : Darcy–Weisbach's friction loss coefficient,

Rc : the radius of curvature,

So : the referance value of streamwise coordinate,

s : streamwise coordinate,

V : area averaged velocity.

Where the value of n is constant, transverse velocity distribution was suggested by Falcon (1983) as shown in Eq.(6).

$$U_b(r)/Uc=V/Vc(h/H)(Rc/Rc+r) \quad (6)$$

where, $U_b$: transverse velocity at centerline,

Vc: streamwise velocity at centerline,

h : water depth at centerline,

r : radial coordinate.

Transverse bed slope is represented as a function of transverse velocity. The force balance between the transverse–shear stress and movable bed–layer weight is presented in Eq.(7).

$$Zrdr=(1-\lambda)(\rho s-\rho)gy_b drsin\theta \quad (7)$$

where, $\lambda$ : porosity,

$\rho$ s: density of sand,

$\rho$ : density of water,

$y_b$: thickness of bed–layer,

$\theta$ : inclined angle of transverse bed.

Rearranging Eq.(7) with small $\theta$, then St becomes sin $\theta$. So, Eq.(8) presents the transverse bed slope.

$$St=g3U/\overline{V} \quad (8)$$

$$g3=(1/\alpha(1-\lambda)k)\sqrt{(f\Phi/8)}\times(V/\sqrt{(\rho s/\rho-1)gD_{50}})$$

where, $\Phi$: Shield parameter,

$\alpha$ : proportionality constant.

2.2 Governing Equation of Flow

The integral forms of the continuity and the momentum equation for flow through the control volume, as shown in Fig. 1 , are Eq.(9), (10), and (11), respectively.

$$F_1\overline{U}+F_2V+HF_4(\partial V/\partial S+\partial\overline{U}/\partial r)=0 \quad (9)$$

$$HF_4(\partial(V\overline{U}/\partial r) + F_1 V\overline{U} + 1/(2n+1)[HF_4(\partial V/\partial r) + F_1 V]$$
$$+((n+1)^2/n(n+2))HF_4(\partial V^2/\partial S)$$
$$+[((n+1)^2/n(n+2))F_2 + f/8]V^2 = gHSF_4 \quad (10)$$

$$F1\overline{U}^2 + HF_4\partial U^2/\partial r - ((n+1)/n(n+2))((HF_4/Rc+r))V^2 + V\overline{U}(F_2 + f/8k)$$
$$+(1/2n+1)HF_4\partial(V\overline{U})/\partial S) + F_3 V\overline{V} + gHF_4(\partial Zs/\partial r) = -(1/3)F1U^2 \quad (11)$$

where, $F1 = St + h/R_c + r$,

$F2 = [g2g3(r/R_c) - gSt(r/H)](Rc/Rc+r)$,

$F3 = (1/2n+1)(g2(H/Rc+r) - g1/g3(Rc/Rc+r)St)$,

$(h/H + g3(r/H)(U/\overline{V})) - (f/8k)(U/\overline{V})$,

$F4 = h/H$.

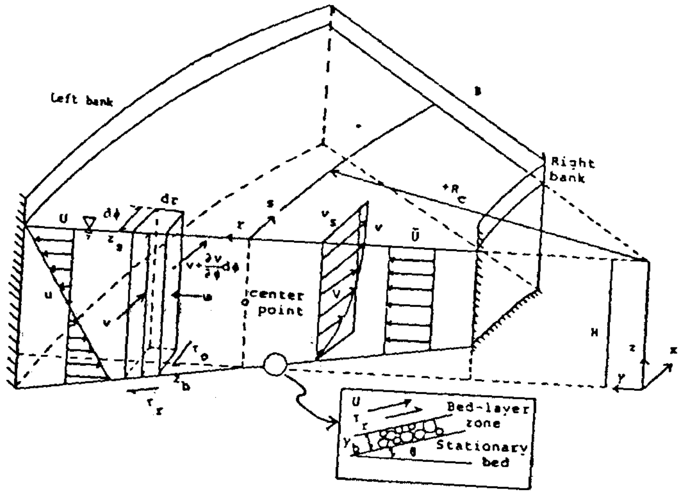



Figure 1. Control volume and coordinate

Because Eqs.(9), (10), and (11) are integrated over depth , the secondary velocity u, becomes zero. So u term is dropped and V ,U , and Zs are left as unknown variables. U was calculated from Eq.(5) and is independent of Eq(9), (10), and (11).

## 3. Analysis of Flow Characteristics with Various Radius of Curvature

### 3.1 Numerical Model

In general, the transverse bed slope is larger than the transverse water surface slope, the effect of transverse water surface slope was neglected. Therefore, the need for solving the radial momentum equation was eleminated. Because the continuity and streamwise momentum equations are used to determine the only two unknowns , U and V, computing time was saved significantly.

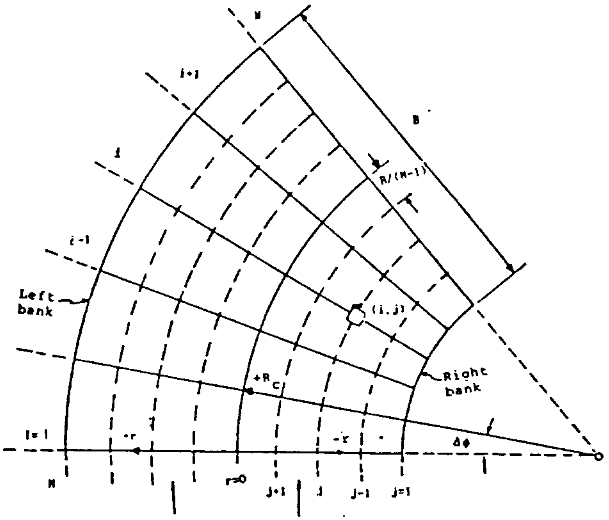



Figure 2. The coordinate − grid system.

The coordinate−grid system for this study is shown in Fig. 2. The models developed by Nakato(1983) and Vadnal(1984) were used to calculate U and V. They suggested that the value of $\beta$ in Eq.(3) is in the range of about 2−4 assuming $U/\overline{V}$ eqauls approximately 0.1. On the other hand, Zr was calculated directly from the value of k determined in this study , which is calculated from Eq.(4). V was calculated from the Darcy−weisbash equation ($V=\sqrt{8ghs/f}$ ), in which h was calculated from the equation of h= H+St r. Given the value of V and h, $\overline{U}$ was calculated from Eq.(12) as below:

$$\overline{U}(r_j)=\overline{U}(r_j-1)((h(Rc+r)\mid_{j-1}/(h(Rc+r)j- 3(\sqrt{(2gS/f)})g2g3e^{-g/h(S-So)}(Y/H(Rc+r)_j) \qquad (12)$$

where, $Y = 2(StRc-H)Rc^2(Y1+Y2+Y3)$,

$Y1= 1/4(StRc-H)\times t/(t^2-StRc)^{2j}{}_{j-1}$,

$Y2= ((5StRc-H)/(8StRc))\ (t\ (t2-StRc)\ j_{,-1}$,

$Y3= (H+3StRc/8StRc)\ \{(1/(\sqrt{-StRc}))\tan^{-1}(1/\sqrt{(-StRc)}\ {}^{j}{}_{j-1}\}\ StRc<0$,

$(H+3StRc/8)\ \{(1/\sqrt{2(StRc)})\ln\ t-\sqrt{(StRc)}/t-\sqrt{(StRc)}\ {}^{j}{}_{j-1}\}\ StRc>0$,

$t= (\sqrt{(Rc(H+Str)}/(Rc+r)$.

$\overline{U}(r_j)$ can be evaluated at each grid point across the transverse with Eq.12 if the boundary condition of $\overline{U}(r_{j-1})$ is specified. V can be calculated using the streamwise momentum equation, Eq.(10). Since the value of $\partial U/\partial r$ is an order of magnitude smaller than that of $\partial V/\partial r$, $\partial U/\partial r$ can be neglected from Eq.(10). Then, Eq.(10) is simplified as in Eq.(13), and Vij can be estimated in each grid point using Eq.(13).

$$AV_{ij}+BV_{ij}+C=0 \tag{13}$$

where ,

$A = HF_4/Si.j-S_{i-1}.j[1/(n(n+2)+1/2]+1/8k+F2/n(n+2),$

$B = (HF/rj-r_{j-1})\overline{U}i.j+Ui.j/2n+1[HF/rj-r_{j-1}+F],$

$C = -HF_4\{(U_{i,j}+\overline{U}i.j/2n+1)Vi.j-1/rj-r_{j-1}+[1/n(2n+1)+1/2]V^2_{i-1,j}/i.j-_{Si-1}.j+gs],$

After $\overline{U}$ and V are calculated from Eqs.(12) and (13), the iterative solution of $\overline{U}$ in Eq.(14) can be obtained using the continuity equation.

$$\overline{U}(rj)=\overline{U}(r_{j-1})h(Rc+r)_{j-1}/h(Rc+r)_j-((vh)_{i,j}-(vh)_{i-1,j}/2(Si.j-S_{i-1,j}))$$
$$\times(Rc+r)^2{}_{jj-1}/h(Rc+r)j \tag{14}$$

The new value of V was calculated from Eq.(13) using $\overline{U}(rj)$, which was calculated from Eq.(14), until a compatible set of velocities were obtained within error tolerance level.
As a boundary condition of $\overline{U}$ , it was assumed that $\overline{U}$ at inner bank is zero and transverse bed slope at entrance section is also zero.

3.2 Numerical Model Experiment

The transverse distribution of streamwise velocity, the path of maximum streamwise velocity, the vertical distribution of transverse velocity , deviation angle, and mass−shift velocity were calculated using the model described above, with varying ratios of Rc/B in 180 −degree channel bend with a constant channel width. Data used in this study is presented in Table 1.

The ratio of Rc/B is approximately 5.4 as shown in Table 1. If Rc is increased to 21.5 ft and 64.5 ft, the resulting ratio will also be changed to 2.7 and 8.1 , respectively.
It is known that the curvature of channel bends with the ratio of 2.7 is sharp, and the one with 8.1 is gentle. Begin(1981) suggested that the transverse movability of river banks is increased when the Rc/B ratio is in the range of 2−4.

Fig. 3, 4, and 5 show the transverse distribution of streamwise velocity V, nomalized with V, in section angle of 20, 100 , and 140 degree, respectively. The value of $\theta$ at the entrance section is zero, but it increases along the direction of the downstream. In Fig.3, when the ratios of Rc/B are 2.7 and 5.4 , the maximum streamwise velocity was found near the inner bank and the velocity slope ( slope of $\overline{V}/V$ ) was decreased gradually, approaching the outer bank. In contract, when the value of Rc/B is 8.1, the maximum streamwise velocity was found near the outer bank and the slope of velocity was increased gradually, approaching the inner bank. The steepest slope of velocity was found when the value of Rc/B was 2.7.

As flow progressed toward downstream, it developed slowly and the maximum streamwise velocity was found near the outer bank in all of the three cases. The magnitude and direction of velocity slope of each case was similar (Fig.4). As shown in Fig.5, even though the maximum streamwise velocity was found near the outer bank in each case, the slope of velocity was decreased with increasing Rc/B ratio. In other words, the slope of velocity is in inverse proportion to the ratio of Rc/B .

As shown in Fig.6, the maximum streamwise velocity was found near the outer bank when the ratio of Rc/B was 8.1 and 5.4 at 40 degree in section angle; however, when the ratio of Rc/B was 2.7 , it was found at 140 degree in section angle. This indicates that flow was developed at upstream part with high Rc/B ratio, and at downstream part with low Rc/B ratio.

Figure 7 shows the comparsion of the secondary velocity with various Rc/B ratio and assuming that the vertical distribution of secondary flow is linear.

Table 1 Basic data (Oakdale flume)

| | |
|---|---|
| V(ft/sec) | 1.56 |
| H(ft) | 0.505 |
| B(ft) | 8.0 |
| Q(cft) | 6.30 |
| Rc | 43 |
| $D_{50}$(mm) | 0.30 |
| $\alpha$ | 1.0 |
| $\phi$ | 0.032 |
| n | 4.24 |

The scale of secondary flow was largest when Rc/B ratio was 2.7 while it was smallest when it was 8.1. Even it is known that the value of $U/\overline{V}$ is about 0.1 in natural river, Fig.7 indicates that the value of $U/\overline{V}$ increases rapidly as the curvature of bends increase. In other words, the strength of secondary velocity is in inverse proportion to the ratio of Rc/B.

Fig.8 shows the streamwise variation of U/V ratio. The ratio of U/V was about 0.1 when Rc/B ratio is 2.7 while it was approximately 0.05 when Rc/B ratio is 8.1 in section angle of

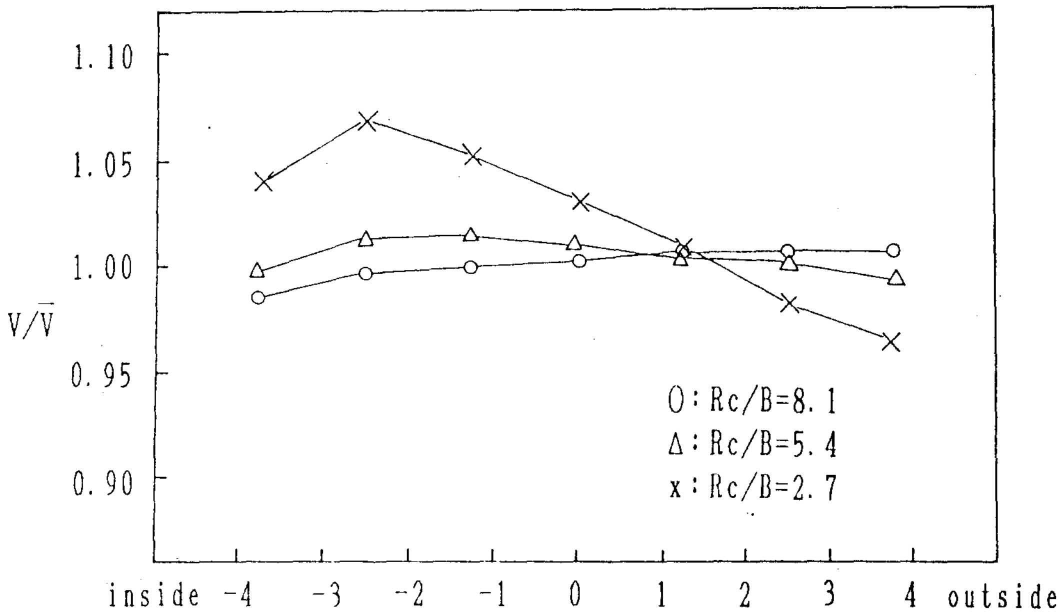



Figure 3. The transverse distribution of V in section angle of 20 degree

20 degree, and extremely small in downstream part . In general, the magnitude of mass-shift velocity is very small. It occurs at the entrance sections and the sections where the radius of curvature changes rapidly, and decays substantially in downstream direction as it can be verified through Fig.8. The ratio of $\overline{U}/\overline{V}$ was decreased with increasing Rc/B ratio.

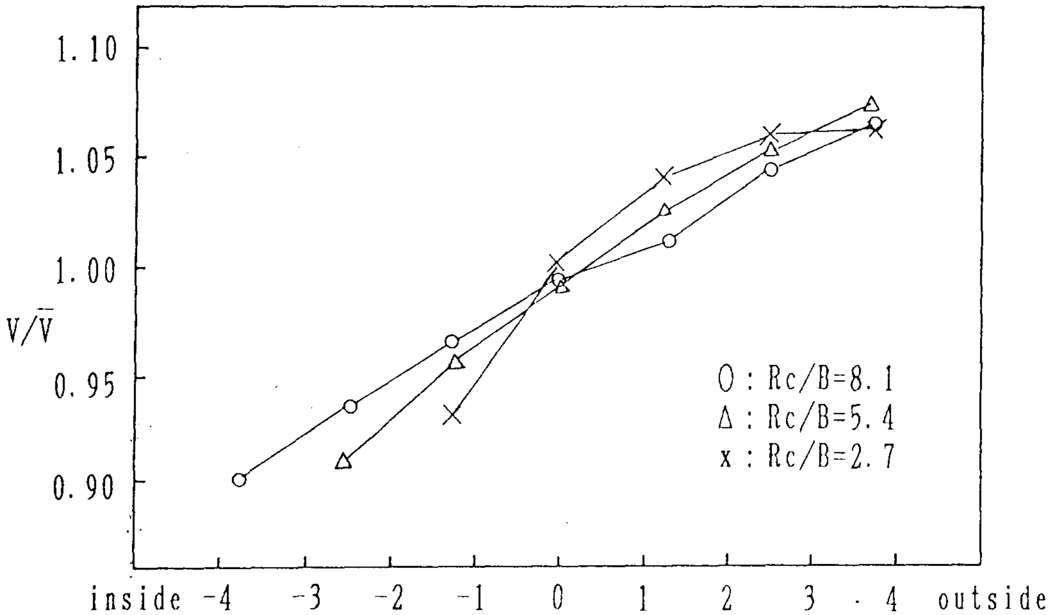



Figure 4. The transverse distribution of V in section angle of 100 degree

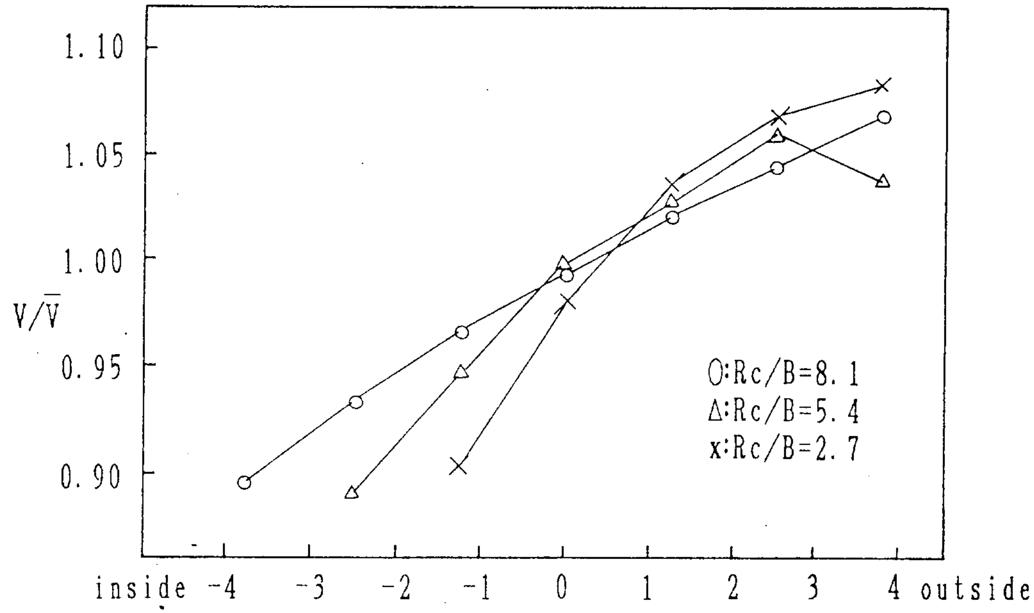



Figure 5. The transverse distribution of V in section angle of 140 degree

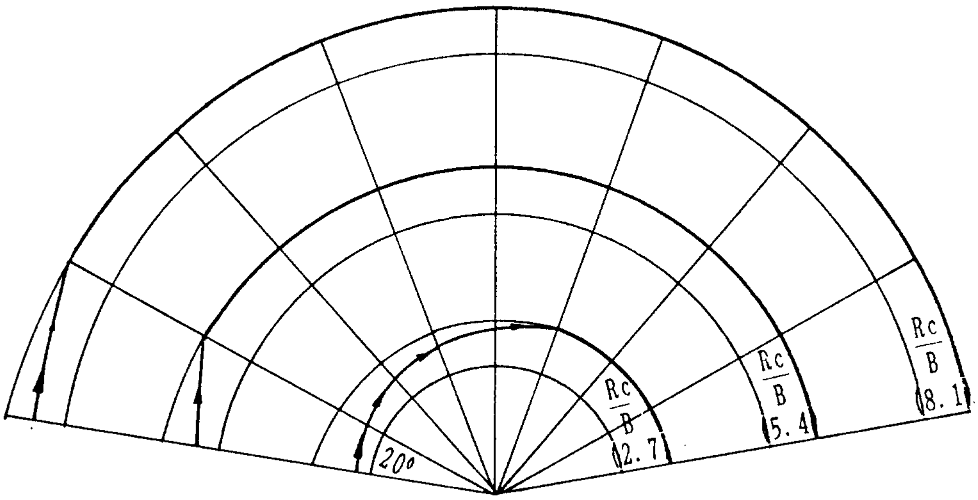



Figure 6. The path of the maximum streamwise velocity

Fig 9. shows the distribution of deviation angles along the centerline in channel bend. The deviation angle was calculated by the eqution, tan $\phi = (U + \dot{U})/\bar{V}$. It was found that $\phi$ was increased with decreasing Rc/B ratio, and it did not change substantially with constant Rc/B ratio.

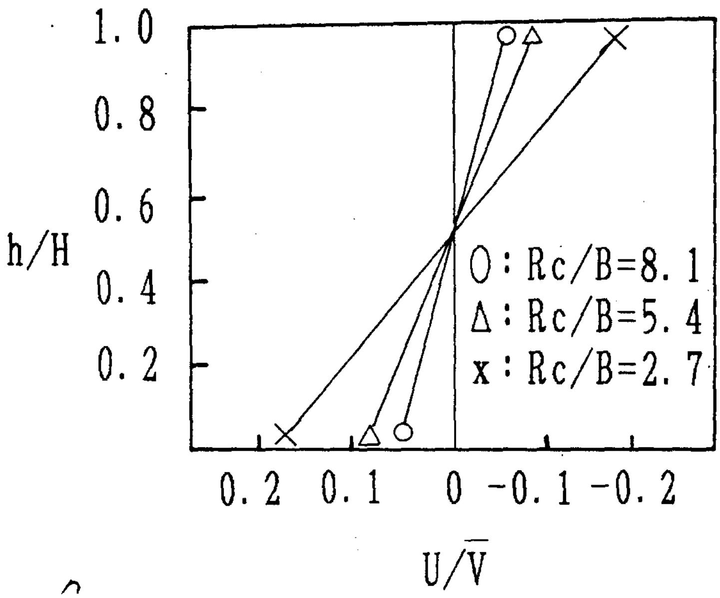



Fig 7. The secondary velocity in section angle of 100 degree

#### 4. Discussion

If the distribution of transverse velocity is assumed to be linear, the relationship between k in Eq.(4) and in Eq.(3) is reciprocal becauase $U_b$ in Eq.(2) and U in Eq.(3) are identical. The value of k calculated from Eq.(15) and (16) using the value of n , H , and Rc in Table 1 and assuming 0.1 for U/V was 0.2988(Vadnal, 1984).

$$U=8k(n+1)/(f(n(n+1))(HV/Rc) \tag{15}$$

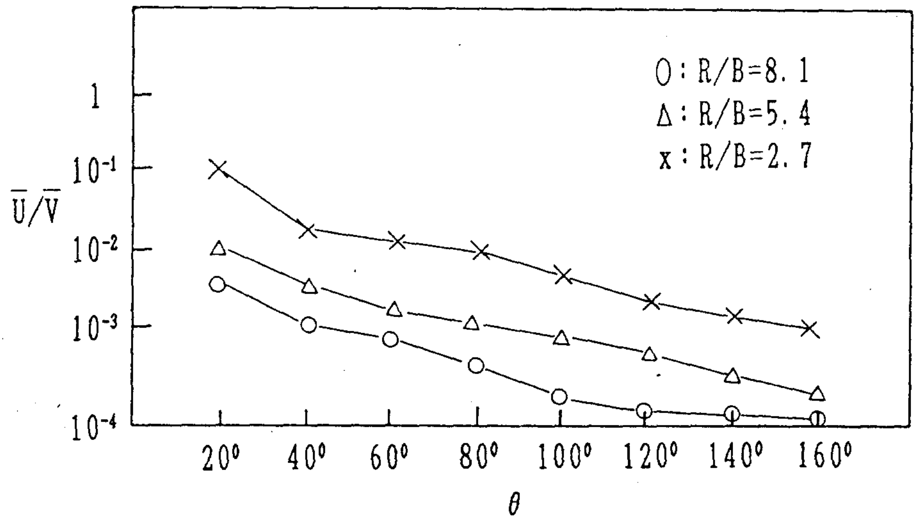
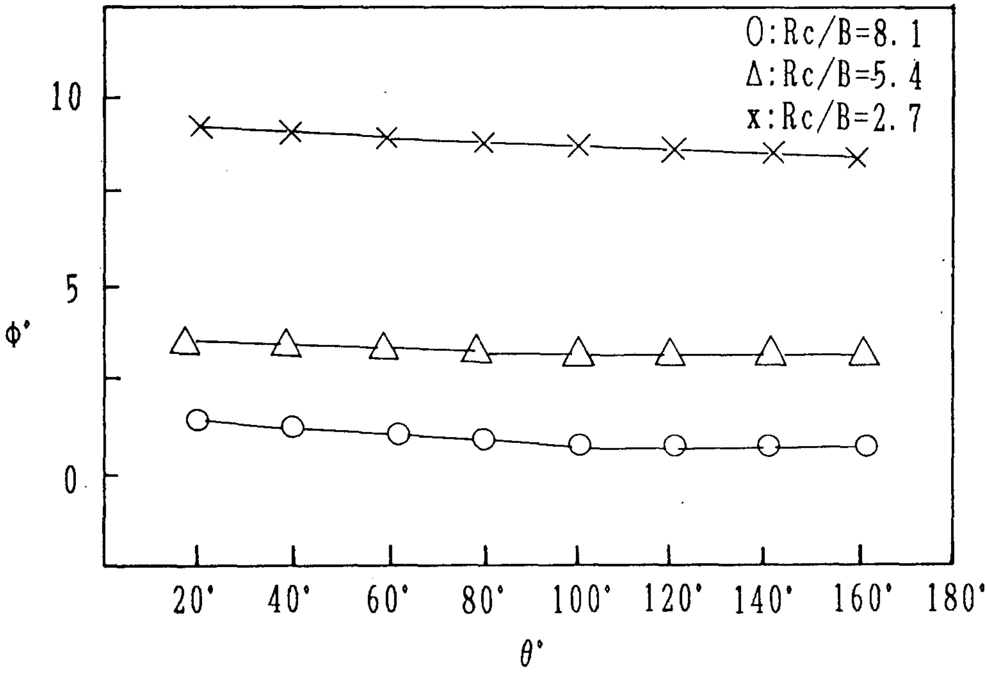



Figure 8. The streamwise distribution of U/V




$$n=1/\sqrt{(f)} \tag{16}$$

The calculted value of k from Eq.(4) using the value of H, n, and D50 in Table 1 was 0.2826. Therefore , the calculated values of k from Eq.(15) and Eq.(4) using the values in Table 1 are relatively well agreed.

If Rc/B ratio is decreased ,U/V ratio is increased, as shown in Fig.7; therefore, the value of k is increased. The value of U/V should be estimated to get the value of k using Eq.(15), but it is not required to get the value of k using Eq.4. Since the value of k is important parameter to estimate the strength of transverse flow and transverse bed slope, it is necessary to study k future.

The maximum streamwise velocity in channel bend is generated near the inner bank at entrance section and it shifts gradually toward the outer bank as flow progresses downstream (Lee,1987). A similar tendency can be found in Fig 3 ,4 ,5 and 6. When Rc/B ratio was 8.1 , the maximum streamwise velocity was found near the outer bank at the section angle of 40 degree. But when Rc/B ratio was 2.7, it was found near the outer bank at the section angle of 120 degree due to a strong secondary velocity and the transverse bed slope . In other words , the maximum streamwise velocity of the large Rc/B ratio occurs at upstream part, while that of the small Rc/B ratio occurs at downstream part in channel bends.

It is known that the transverse velocity is approximately 10 persent of area—averaged velocty in natural channel bends (Odgaad,1988).

But the ratio of $U/\overline{V}$ increases with decreasing Rc/B ratio. As shown in Fig.7, the ratio of $\overline{U}/V$ was 20% when Rc/B ratio was 2.7. Shukry(1950) suggested that the strength of transverse flow can be neglected if the ratio of Rc/B is above 3.0. This indicates that it is reasonable to ignore the effects of the transverse flow, in practical purpose, if $\overline{U}/V$ ratio is below 10 percent. Based on this study, it is known that the $\overline{U}/V$ ratio was below 10 percent when Rc/B ratio was above 5.0 (Fig.7).

The mass–shift velocity is generated in the zone where the radius of curvature changes rapidly. Since the magnitude of mass–shift velocity is small, it is ignored in practice (Vadnal, 1984). But it is verified through Fig.8 that U is affected by the ratio of Rc/B significantly.

When the ratio of Rc/B was 2.7, the $\overline{U}/\overline{V}$ ratio was about 0.1 at the section angle of 20 degree and the $U/\overline{V}$ ratio was same even the ratio of Rc/B was 5.4 . Therefore, the value of $\overline{U}$ can be ignored in channel bends which has small Rc/B ratio.

$$\phi = \tan^{-1} a(h/r) \qquad (17)$$

Rozovskii (1961) used Eq. (17) to get the deviation angle, $\phi$. Rozovskii (1961) and Engelund (1974) used the value of a as 11 and 7, respectively. Bridge (1982) suggested that the value of a was approximately 10 with measured data. The value of $\phi$ was calculated from Eq.(17) using 10 as the value of a for this study. The results are as fallows: $\phi$ was 13 degree for Rc/B ratio of 2.7 ; 7 degree for Rc/B ratio of 5.4 ; and 5 degree for Rc/B ratio of 8.1 . The calculated values of $\phi$ using Eq.(17) agree with the value in Fig.9 relatively well. It is worthy to note that the deviation of angle is increasing with decreasing Rc/B ratios.

## 5. Conclusion

The flow characteristics in channel bends with varying Rc/B ratios are investg ated with a mathematical model, which is composed of the continuity equation and the streamwise momentum equation with assuming that the transverse bed slope is much larger than the transverse water surface slope. The numerical simulation was performed with the Rc/B ratios of 2.7 , 5.4 , and 8.1, respectively.

The strength of secondary velocity , transverse bed slope, deviation angle, and mass–shift velocity were increased with decreasing Rc/B ratios. The location where the maximum streamwise veloctity occurred moved upstream with increasing Rc/B ratios.

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