Hyetograph Model for Reservoir Operation During Flash Flood

Lee, Jae Hyoung*/Sonu,Jung Ho**/Chung, Dong Kug***

ABSTRACT / Precise run—off forecasting depends on the ability to predict quantitative rainfall intensity. The purpose of this study is to develop a stochastic model for the short—term rainfall prediction. It is required for the model to predict rainfall intensities at all the telemetered rain—gauge locations simultaneously. All the model parameters, which are used in this work; velocity and direction of storm movement, radial spectrum, and dimensionless time distribution of rainfall, are the results of the previous study. We formulated the model and operated it, so that in this study was analyzed particularly the influence of 4 dimensionless time distributions on the prediction and the influence of the model on run—off.

1. Literature Review

In the literature, there are two practices to predict the rainfall for the real-time flood forcasting : meteorological methods for a short-term and hydrological methods for a long-term. This study is to suggest the hourly rainfall model based on the hydrological approach under the assumption that the total rainfall is given.

There are three types of hydrological methods for rainfall prediction; that is, to be based on the conditional probabilities, the covariance, and the time series analysis. Jamieson and Wilkinson (1972) suggested a first-order autoregressive model for short-term rainfall prediction to capture 45% of the variance of the original series. Other time series models could be suggested such as non-stationary autoregressive integrated moving average (ARIMA) models (Box & Jenkins,1970). However these methods needs a storm exterior characteristics such as total depth and duration of event, and rainfall interior structure including the time distribution of the total rainfall depth, the direction and velocity of the storm within each

* Prof. Dep. of Civil Eng., Chonbuk Nat. Uni., Korea

** Prof. Dep. of Civil Eng., Seoul Nat. Uni., Korea

*** Full-time Lecturer, Hannam Univ., Korea

event. Although the detailed informations about a storm were obtained, the parameter estimated statistically would be not usefull. Billuart and Tourasse (1980) pointed out two problems in the developement of the real—time flood prediction model. First, the telemetered raingage data is not enough to allow parameter estimation. Second, the current methods for short—term rainfall prediction are somewhat scarce. To overcome these problems, the stochastic model based on the generated instead of the synthesized rainfall was proposed by J.D. Creutin et. al (1980). But, in the Creutin's model the spatial struture of the storm is not considered.

Johnson and Bras (1980) developed a stochastic model for the short term (of the order of one hour) rainfall prediction. This model predicts both rainfall rates at multiple locations and multiple values of prediction lead simultaneously. Rainfall is assumed to be a non—stationary process and evolve in time according to a non—stationary Markov model.

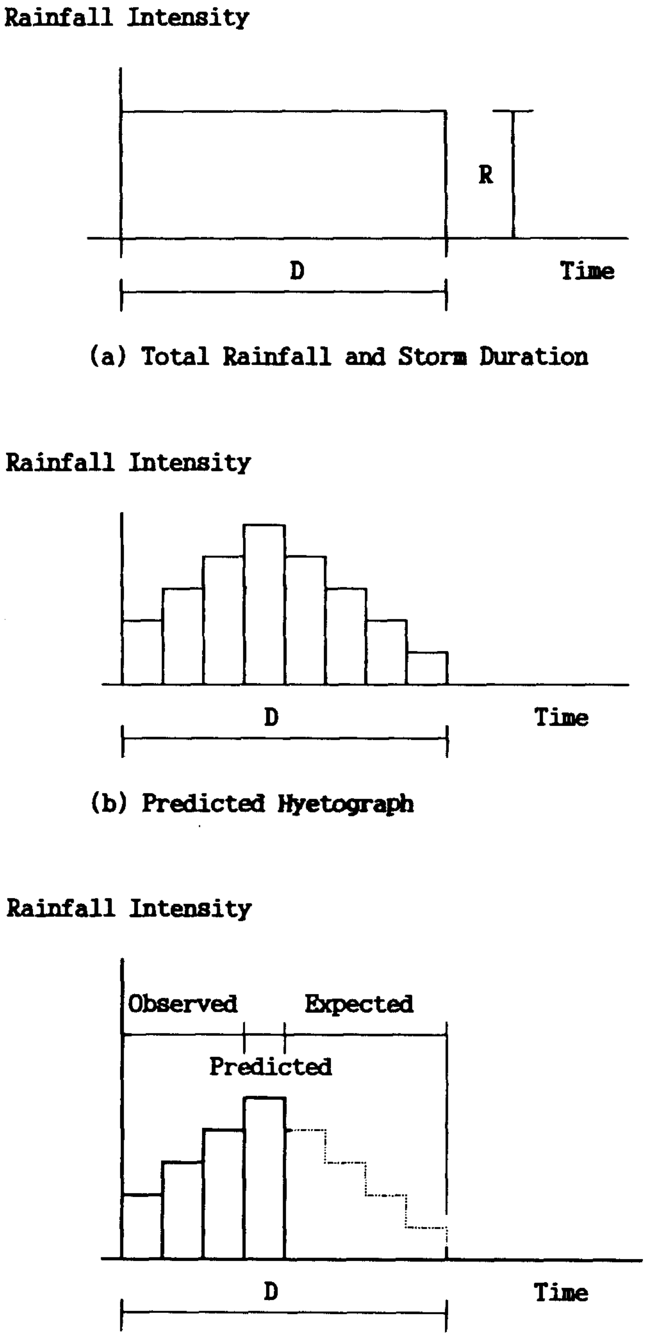
The purpose of this study is to develop a model for quantitative short—term rainfall prediction based on both the Creutin's and Bras's method. Predictions must be made at multiple points to provide some degree of spatial detail.

2. Approach and Assumption

Rainfall is a multidimensional process occurring in time and space. Spatial patterns of variation of rainfall phenomena can be studied by means of stochastic processes over the space considered.

Bras (1975) suggest that existing rainfall models can be classified as point, multivariate, and areal or multidimensional rainfall models. Point rainfall models are to generate time—sequences of rainfall depth at a single point. Multivariate rainfall models consider several raingages simultaneously and are intended to preserve the covariance structure of the historical rainfall data existing in those points. And multidimensional rainfall models characterize the rainfall phenomenon at every point over the area of interest. All of the above classifications may be subdivided into rainfall exterior models, which generate storm exterior characteristics like total depth, duration of event and time between events, and rainfall interior models, which generate the time distribution of the total rainfall depth, the direction and velocity of the storm within each event.

This study presents the multidimensional non—stationary model of rainfall interiors. Rainfall interior models are related to the motion of storm. As the phenomenon of rainfal is a very complex process, thus some assumptions are needed for model formulation. The development of the model is based on the following basic knowledge describing the behavior of storms: 1) each storm moves with an average velocity, u , over the area of interest and follows certain trajectory. Individual disturbances within the storm move about the same velocity. 2) Water falling at any instant is correlated to what happened at previous times. 3) Correlation rainfall of in space at any time is observed. 4) Spatial and time correlation are neither separable nor independent. 5) Rainfall is a non–stationary process. The mean and variance vary with time at all points in space. 6) It does not consider a storm aging.



(a) Total Rainfall and Storm Duration

(b) Predicted Hyetograph

(c) Reflection of Observed Values

Figure 1. Rainfall prediction model

At this point, it is assumed that the storm interior of an event with the given depth and duration can be modeled as:

$$i(\underline{x}_i, t) = i_u(\underline{x}_i, t) + \eta(\underline{x}_i + t) \tag{2.1}$$

where, $i(\underline{x}_i, t)$: rainfall intensity at a point with coordinate vector $\underline{x}_i$ at time t,

$i_u(\underline{x}_i, t)$: mean intensity at x and t ; where the mean value is taken over all possible storms of the same chracteristics,

$\eta(\underline{x}_i, t)$: noisy residual obeying a certain covariance function in time and space,

Usually, mean intensity and noisy residual are obtained from historical rainfall data. The predictions of future rainfall rates will be based on the generated values of rainfall from equation (2−1). Thus, the storm can be modeled as:

$$\hat{i}(\underline{x}_i, t) = i(\underline{x}_i, t) + r(\underline{x}_i, t) \tag{2−2}$$

where, $\hat{i}(\underline{x}_i, t)$ terms is the rainfall intensity and $r(\underline{x}_i, t)$ is residuals at time step t. The difference between equation (2−1) and equation (2−2) is how to estimate the random variable η and r.

Once the expected rainfall intensity from equation (2−1) is given, these residual rainfall intensity is calculated by the predicton model

3. Rainfall Generation Model

The undimensional mass curve is given in the interest area, and the mean temporal behavior of the storm at all points will be all the same. Also, it is assumed that storm duration is same in total everywhere. In order to represent all stations at the same time, an absolute time scale is defined within the area. Starting time is the moment when the moving storm hits the first point in the area (see Figure 2).

Assuming for simplicity that the storm moves parallel to the x, it is clear that :

$$iu(\underline{x}_i, t) = i_a(t - \underline{x}_i i/u) \tag{3−1}$$

where, ia(t) is average rainfall rate at time t where t is now the time it has been raining at a given point, u is storm average velocity in x direction, and xi is x coordinate of point i.

The next step is to hypothesize the form of the covariance function of the noisy residuals. In this work it is written as :

$$E[\eta(\underline{x}_i, t')\eta(\underline{x}_i, t'')] = \sigma(\underline{x}_i, t)\sigma(xj, t'')rc(\underline{x}_i, t'; \underline{x}_i, t'')$$

where E is expectation operator, σ is standard deviation of rainfall intensity at point $\underline{x}_i$ and time t, and $rc(\underline{x}_i, t'; \underline{x}_j, t'')$ is general functional form of normalized covariance. $\sigma(\underline{x}_i, t)$ corresponds to the variation around $iu(\underline{x}_i, t)$ and is obtainable from data in a similar way. The same time translation is applicable, so

$$\sigma(\underline{x}_i i, t) = \sigma a(t - \underline{x}_i i/u) \tag{3-2}$$

Equation (2−1) can then be expressed as:

$$i(\underline{x}_i, t) = ia[t - (\underline{x}_i i/u)] + R(\underline{x}_i, t)\sigma a(t - \underline{x}_i/u) \tag{3-3}$$

where $R(\underline{x}_i, t)$ is standardized residual at point xi, and time t with zero mean and unit variance.

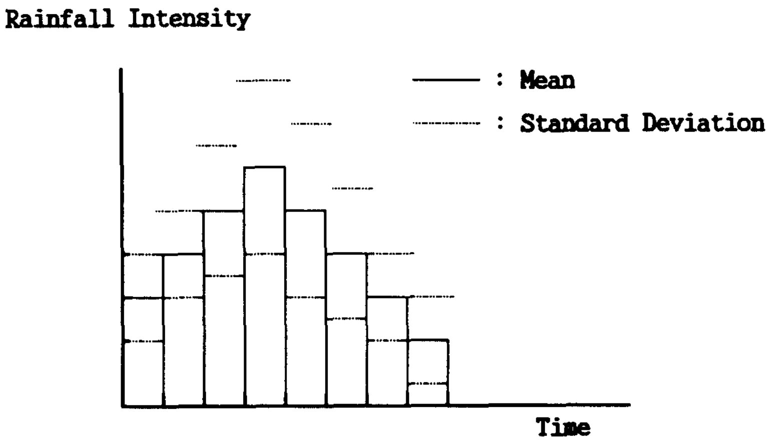


Figure 2 Mean temporal behavior

The statistical behavior of the residual $R(\underline{x}_i, t)$ embodies the spatial and time correlation of the rainfall process. The function $r(\underline{x}_i, t'; \underline{x}_j, t'')$ must then be defined. At this point, a basic assumption about the behavior of rainfall is introduced. It is assumed that Taylor's Hypothesis of turbulence is valid within a storm. The above implies that correlation in time is equivalent to that in space if time is transformed to space in the mean direction of storm movement. The second assumption used is that rainfall intensities (or depth per time interval) have isotropic spatial correlation functions at any instant of time. Therefore,

$$rc(xi, t'; xj, t'') = (\sqrt{(yj-yi)^2 + ((xj+ut') - (xi+ut''))^2}) \tag{3-4}$$

The spatial and time correlation of the generated values approaches equation (3−4) as the number of harmonics goes to infinity. The spatial correlation of total depths over the area are also preserved since

$$rc(v) = \int_0^D \int_0^D r(\sqrt{(yj-yi)^2+((xj+ut'')-(xi+ut'))^2}\, dt'dt'' \tag{3-5}$$

where D is the storm duration.

A set of commonly used isotropic correlation functions(3−5) are single exponential or quadratic exponential, Bessel form, etc.. The corresponding radial spectral densities and distributions are, for example , single exponential as follows :

$$G(w) = \frac{1}{\sqrt{1-w^2/\lambda^2}} \tag{3-6}$$

The simplicity of the radial spectral distribution corresponding to the given correlation functions allows the sampling of characteristic values by the inverse method. For example, inverting the equation (3−6) results in :

$$w = \lambda\left(\frac{1}{1-G(w)}-1\right)^{1/2} \tag{3-7}$$

Generation of uniformly distributed values between 0 and 1 and their substitution for G(w) in equation (3−7) results in a series of w values belonging to the population. Next, it is sometimes desirable to obtain estimates of rainfall averaged over particular areas. The areal process can be defined through the relationship (Bras,1976):

$$\varepsilon(\underline{x}) = \sqrt{2/N}\sum_{i=1}^{N} COS[(\underline{x}\; yi)wi + \theta i] \tag{3-8}$$

where, $\underline{x}$ represents a vector of coordinates(x1, x2) in R2; yi is a two dimensional random variable (yi1,yi2) on the unit circle; wi is a random variable whose distribution is the radial spectral distribution funtion G(w) corresponding to the isotropic correlation function of $\underline{\varepsilon}$ (x) ; θ i is a uniformly distributed random angle between 0 and 2 ; and N is the finite number of harmonics.

Mejia and Rodriguez−Iturbe (1974) have shown that the above process is homogeneous, isotropic, and asymptotically ergodic, as N goes to ∞, and multinormal. It also has zero mean, unit variance and, as N goes to correlation function with radial spectral distribution corresponding to G(w). Bras(1976) suggested that the effect on the synthesis of rainfall is not dominated as the number of harmonics N becomes larger than 50.

Since yi is equidistributed on the unit circle and x is a vector of coordinates, thus (3−8) becomes:

$$\varepsilon(xi,x2) = \sqrt{2/N}\sum_{i=1}^{N} COS[wi(x1COS\lambda i + x_2SIN\lambda i)+\theta i] \quad (3-9)$$

where, λi and θi are a uniformly distributed random variables, wi is a random variable whose distribution is the radial spectral distribution funtion G(w). The term R(xi,t) in equation (3−3) is the nomalrized residuals, and must be transformed to random fields ε (x1,x2) according to the above assumtion. Now, equation (3−9) is used in rainfall generation with mean depth and storm duration.

4. Rainfall Prediction Model

The non−stationary process of rainfall is described in the previous section. It described rainfall at all points in space(x,y) and time. The stated goal of this study is to produce predictions of future rainfall rates at specified locations. This naturally leads to a multivariate description of rainfall. In addition, time will also be discretized. In short,

$$i(t) = m(t) + r(t) \quad (4-1)$$

where, the term m(t) can be defined from i($\underline{x}$i ,t) of equation (2−1).
We can choose the dimensionless hyetograph obtained by historical data as m(t). Using a diagonal standard deviation matrix (t), r(t) is written as:

$$r(t) = \Sigma(t)\ \varepsilon(t) \quad (4-2)$$

It is notationally convenient to define a vector of residuals r(t) of equation (4−1) from equation (4−2). That is,

$$\begin{aligned} r(t) &= i(t) - m(t) \\ &= \Sigma(t)\ \varepsilon(t) \end{aligned} \quad (4-3)$$

At issue is the dynamics of the residuall term r(t). The residual will be assumed to evolve in time according to a non−stationary Markov model of the form:

$$r(t+\tau) = A(t,\tau)r(t) + B(t,\tau)W(t,\tau) \quad (4-4)$$

where , A(t, τ) = N x N state transition matrix at time step t for a transition steps into the future
W(t, τ) = N x 1 vector of disturbances with zero mean value
B(t, τ) = N x N matrix giving the effect of the noise terms at time step t on the

residuals at time step t+

It is to write only the one-step process:

$$r(t+1) = A(t,1)r(t) + B(t,1)W(t,1) \quad (4-5)$$

For the prediction points, a measurement equation is written:

$$\begin{aligned} z(t) &= q(t) - m(t) \\ &= r(t) + V(t) \end{aligned} \quad (4-6)$$

where, $q(t)$ = N x 1 vector of observed rainfall,
$r(t)$ = N x 1 vector of true values of residual,
$V(t)$ = N x 1 vector of measurement errors.

In addition to Eqs. (4-4) and (4-6), the following assumptions are made:

$$E[W(t,1)\ W^T(s,1)]=0 \qquad \text{for } t \neq s \quad (4-7)$$

$$E[W(t,1)\ V^T(s)] = 0 \qquad \text{for all } t,s \quad (4-8)$$

$$E[V(t)\ V^T(s)] = 0 \qquad \text{for } t \neq s \quad (4-9)$$

In other words, the state noise is uncorrelated in time and with the measurement noise, and the measurement noise is uncorrelated in time. Equations (4-4) - (4-9) form the classic framework for the discrete Kalman filter. The equations operate recursively; that is, the process forcasts only one time step of measurement at a time(Johson,1980). That is,

$$\hat{r}(t+1 \mid t) = A(t,1)\hat{r}(t \mid t) \quad (4-10)$$

$$\begin{aligned} P(t+1 \mid t) &= A(t,1)P(t \mid t)A^T(t,1) + B(t,1)\,B^T(t,1) \quad (4-11) \\ &= A(t,1)\ P(t \mid t)\ A^T(t,1) + Q(t,1) \end{aligned}$$

$$K(t) = P(t \mid t-1)\ \{P(t \mid t-1) + E[V(t)\ V^T(t)]\}^{-1} \quad (4-12)$$

$$\hat{r}(t \mid t) = \hat{r}(t \mid t-1) + K(t)\{z(t) - \hat{r}(t \mid t-1)\} \quad (4-13)$$

$$P(t \mid t) = \{I - K(t)\}\ P(t \mid t-1) \quad (4-14)$$

where $\hat{r}(t \mid t)$ denotes the linear minimum variance estimate of the true residual vector $r(t_1)$ based on all information available up to time step t2. It is necessary to define starting conditions $p(0 \mid 0)$ and $\hat{r}(0 \mid 0)$. Before it begins to rain, both the rainfall and mean value of rainfall are zero; therefore, the residual is zero and $\hat{r}(0 \mid 0)=0$.

The inital state error is taken to be the measurement error, i.e., $P(0 \mid 0)=E[V(0)V^T(0)]$. Here, $\{z(t)-\hat{r}(t \mid t-1)\}$ are innovation . In order to implement the prediction, it is necessary to estimate the terms A(t,1),B(t,1), $E[W(t,1)W^T(t,1)]$, and $E[V(t)V^T(t)]$.

5. Parameter Estimation

The model suggested above would be valid for the generation of rainfall interiors for a specified family of storms with a given duration, average speed, direction and statistical description of intensities such as time varying mean, and multidimensional correlation structure.

Estimating the mean are in two ways. One is that the rainfall for the moving storm is averged over space. The other is that a certain type for the total rainfall depth during the given duration is determined from historical rainfall data. The first method.needs a detailed information about storm. However, the current rainfall data is collected in increments of cumulative amounts over time scales of 1 hour, this method can not be applied. In this study, mean and variance is estimated from historical data (Eaglson,1970).

In the prediction model, error covariance matrices E[V(t)V(t)] can be estimated by the accuracies of raingage system. In order to implement the prediction scheme it is necessary to estimate two matrices of A(t) and B(t) describing the dynamics of the rainfall residuals. On the practical side, these matrices are calculated using the covariance of historical rainfall data. That is,

$$A(t) = S(t,t-1)^{S-1}(t-1,t-1) \qquad (5-1)$$

$$B(t) = S(t,t) - S(t,t-1)^{S-1}(t-1,t-1)S(t-1,t) \qquad (5-2)$$

where,

$$S(t,t) = E[r(t)r\ (t)],\ S(t,t-1) = E[r(t)r\ (t-1)]$$

And covariance function can be estimated as :

$$S(xi,t-1\ ;\ xi,t) = \sigma(t-1)\sigma(t)\hat{r}c(v)$$

$$v = \{(yj-yi)^2 + [(xj+u(t)) - (xi+u(t-1))]^2\}^{1/2} \tag{5-3}$$

where, $\sigma(t)$ is defined in equation(3−2), $\hat{r}c(v)$ are based on the previous study(Lee,1989). In this study, a single exponential function instead of the rc(v) is used for the operational purpose. That is,

$$\hat{r}c(v) = e^{-cv} \tag{5-4}$$

where constant c is obtained from sample covariance S. The S(t,t), S(t,t−1) matrix are derived from standard deviation and normalized covariance in the rainfall process, and then A(t) and B(t) are calculated by equation (5−1) and (5−2).

6. Case Study

The suggested model is applied to Namdaechon located in upstream area of Daechung basin. Namdaechon basin extends from latitude 35° 49′ N to latitude 36° 04′ N and from longitudes 127° 39′ E to longitudes 127° 56′ E. The slope of the basin is roughly one thirty as estimated by Horton's method. There are eight raingage stations in the Mujugun and one stage gage station is installed at Muju bridge. Figure 3 show the location of the raingage and the stage station.

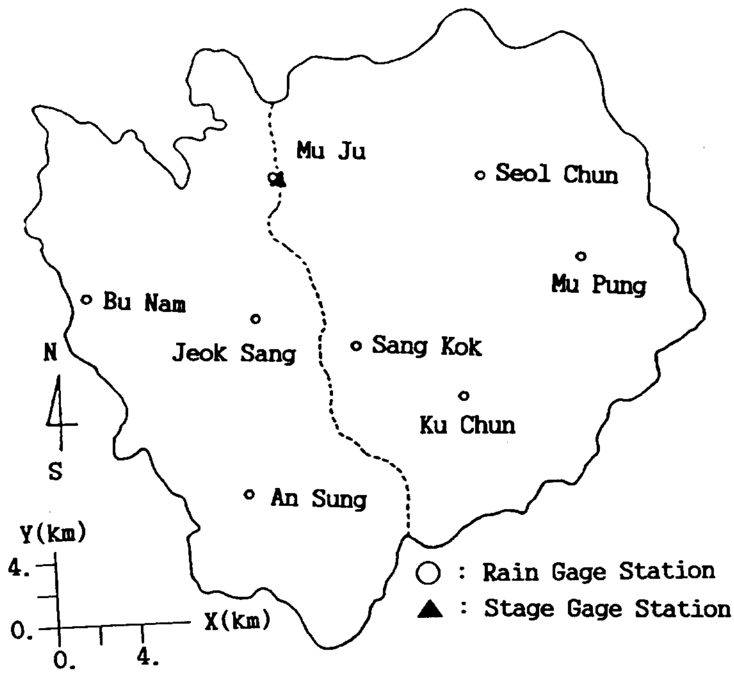


Figure 3 Raingage and stage station in the Namdae watershed

The model is tested in the two ways; one of them is to select the smallest among the standard deviations between the average intensity based on the observed rainfall and the forcasted rainfall intensity distributions based on the undimensional mass curves, the other is to select the rainfall pattern which is well suited to the observed runoff(①) among them; the calculated runoff with the rainfall predicted by Weather Bureau(②), with the hourly hyetograph predicted by the suggested model(④) and with the spatially distributed hyetograph derived from the given undimensional mass curve(③).

Table 1 Coordinates of raingage station

station	X(km)	Y(km)	remark
Muju	106.782	112.522	At a point
Muphung	119.478	109.043	
Sulchun	115.652	112.522	N 35° 51′ 37″
Chuksang	106.609	106.956	E 127° 34′ 02″
Ansung	106.434	100.000	
Bunam	100.000	107.826	X = 100km
Kuchon	114.608	103.478	Y = 100km
Sangkuck	116.609	105.913	

Before the numerical experiment, we must analyze a runoff characteristics of this watershed. Presently, there are many methods for the runoff calculation. Since this study do not concern with the exact flood discharge with the given rainfall, we used the time–area relationship instead of the distributed parameter methods. To ignore the lag effect and loss of rainfall, we chose the storm on the july 28, 1989 when the basin is fully saturated by antecedent rainfall. And in order to consider the velocity and direction of storm movement, the basin is divided by 0.677km x 0.677km grid using isochrones obtained from the previous investigation (Chonbuk MuJuGun,1989). The NE 80o direction and 12km/hr velocity of storm is adopted in the previous study (KWRC,1990). In our example, the duration of storm is 10 hour and total rainfall rate is 40mm. The observed hyetograph spatially averaged was applied to the runoff calculation. Figure 4 shows the calculated and observed hydrograph.

In the first case, rainfall intensity based on the undimensional hyetograph of four types (KWRC,1990) are forcasted by the suggested model. Table 2 gives the standard deviations between the observed and calculated rainfall for the four forecast of rainfall rate. The results of numerical experiments show that minimun standard deviations have occurred in the 2nd type, with its value of 2.262mm/hr, and maximun have occurred in the 4th type, with its values of 3.432mm/hr.

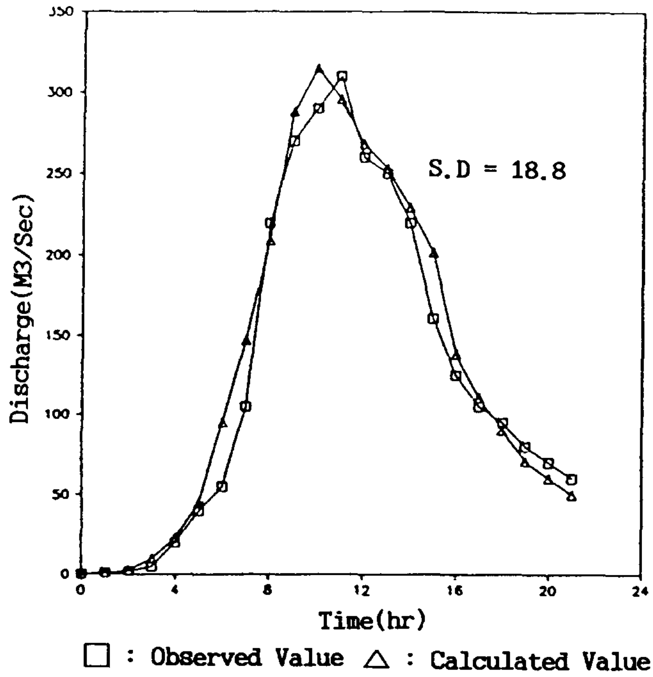


Figure 4 Hydrograph on the July 28, 1989.

Table 2 Standard deviations between calculated and observed rainfall

time	1st type	2nd type	3rd type	4th type
1	4.666	3.738	4.835	5.822
2	4.544	3.632	4.674	5.636
3	4.134	3.364	4.425	5.326
4	3.528	2.919	4.305	5.200
5	2.997	2.662	4.080	4.970
6	3.036	2.557	3.736	4.650
7	2.400	2.494	2.799	2.652
8	1.771	1.938	2.039	1.834
9	1.352	1.374	1.400	1.436
10	0.208	0.205	0.205	0.227
11	0.000	0.000	0.000	0.000
average	2.603	2.262	2.954	3.432

In the second case, we compared the observed runoff with the runoff calculated using the rainfall predicted by Weather Bureau as an input, with the hourly hyetograph predicted by the suggested model based on the third type, and with the spatially distributed hyetograph derived from the given undimensional mass curve. Figure 5 shows the runoff obtained by basin routing with each hyetograph.

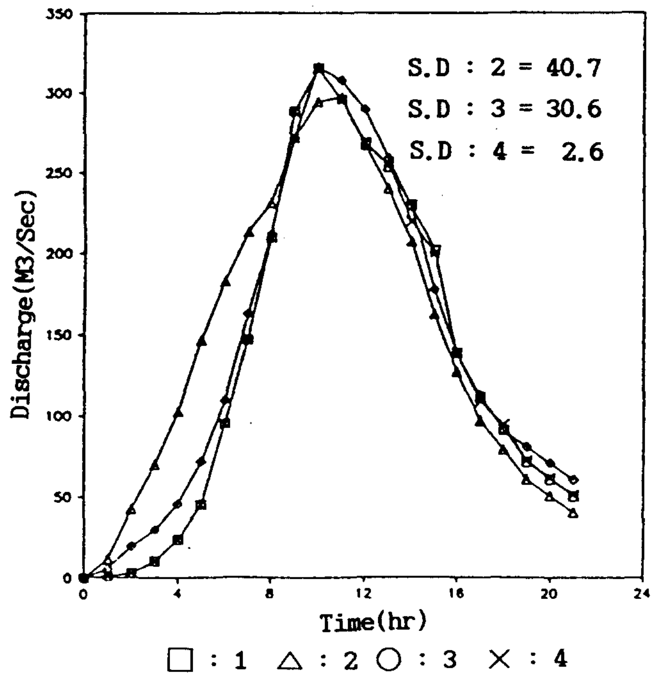


Figure 5 The runoff by each hyetograph.

In the figure 5, ①, ②, ③ and ④ is dotted by rectangle, triangle, diamond and cross, respectively . In the case of ②, 4mm/hr was constantly taken and it was uniformly distributed over the basin. The maximum standard deviations have occurred in the case ②. Its value was 40.7㎥/sec. In the suggested model, standard deviations varied from 20.1㎥/sec to 2.6㎥/sec, and the average of the variations was 11.3㎥/sec. And in the undimensional mass curve, it was 30.6㎥/sec.

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