OR/MS Society
Vol. 17, No. 1, April 1992

A Discount Price Schedule Based on Supplier's Profit Function+

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Abstract

It is discussed how a supplier should design a quantity discount pricing schedule for multiple buyers. It is emphasized that not only the supplier's surplus but also each buyer's surplus resultant from quantity discount should be considered in designing price schedule. It is shown that if the supplier's quantity pricing schedule is based on his/her profit function, each buyer's surplus may be maximized. And it is also shown that when the supplier's main benefit comes from the reduced number of setups, the incremental discount schedule satisfies the requirement. Formulas to determine values of parameters of the incremental discount schedule are provided.

1. Introduction

Recently, much attention was given on the problem how the supplier can formulate the terms of a quantity discount pricing schedule, assuming that the buyer always behaves opltimally for a given price schedule.

Larger customer orders give several economic advantages to the supplier such as fewer manufacturing setups, less frequent packaging and order processing, lower transportation cost, and less inventory holding cost.

On the other hand, the buyers expect decreases in purchasing cost by utilizing quantity discount offer in compensation for increase of order sizes. Thus, the supplier faces the problem of determining the quantity discount schedule which not only increases his/her profit by making buyer's order size larger, but also reduces each buyer's inventory cost by enough amount to entice him to accept the supplier's offer.

Similar problems have been discussed in the area of public utility pricing[2,3]. In the case of public utility, regulators in setting prices try to increase social welfare, which may be defined

+ This study has been supported by Research Fund of Pusan National University.

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as the sum of surpluses of supplier and customers, as much as possible.

As an effective device for accomplishing this objective, researchers in the area suggested nonuniform price shedule, which permits us to vary prices between customers who have different tastes for the goods in question, in place of a uniform price schedule on which total outlay is simply proportional to the amount purchased. And it has also been discussed that an appropriately designed nonuniform price schedule can make all the customers and the supplier better off relative to a uniform price schedule, in case of which we say that the nonuniform price schedule Pareto-domonates the uniform price. And much effort has been devoted to develop more efficient nonuniform pricing schedules from the viewpoint of social welfare.

From a broad point of view, the basic rationale behind public utility pricing problem coincides with the one behind discussions in this paper in that both try to design a pricing schedule considering not only the surplus of the producer's but also of the customer's.

But in this paper, the profit-making organization is dealt with instead of the nonprofit agencies whose behavior may be different, and the problem of this paper is more detail in describing behaviors of the supplier and the customer than in public utility theory.

Among several researches[1,4-13] related to the problem in this paper, Crowther[4] firstly discussed the possibility of improving the supplier's profit and reducing the buyer's cost simultaneously by utilizing a quantity discount system.

Monahan[12] analysed how a supplier can design an all-unit discount pricing schedule for a single customer to maximize his/her own profit. Rosenblatt and Lee[10,13] analysed a discount pricing model to determine not only the terms of a quantity discount but also the production lot-size of the supplier. They expressed the price as a linear function of order size with negative slope.

In two previous studies (Dada and Srikanth[5], Kim and Hwang[8]), it is analysed what effects price and order-size have on the inventory cost of a customer and a supplier and the range of price and order size is characterized over which the costs of both parties can be lowered simultaneously. They also presented a method for finding price and order size which minimize a joint cost function of a supplier and a customer and divide the savings in costs in a predetermined ratio between them. Kim and Hwang[8] also showed how the supplier can induce a buyer to a predetermined price and order-size level of mutual benefit by utilizing all-unit or incremental quantity discount system.

All the above studies considered the case that the supplier offer a quantity discount opportunity to a single customer. Kim and Hwang[7,9] extended Monahan's model to the case where the supplier deals with multiple buyers offering the all-unit or the incremental discount pricing schedule. But, in their papers, the objective functions were maximization of only the supplier's profit without considering each buyer's cost reduction.

In Lee's paper[11], a non-linear pricing schedule is studied for the case that the wholesaler knows only the probability distribution of each retailer's characteristics.

In the case with multiple buyers, since each buyer has his/her own cost parameters and the supplier offers a common quantity discount pricing schedule to all of them, it seems difficult for the supplier to induce every customer into the most beneficial price and order-size level to both parties which is possible for the case of single customer by the procedure of Kim and Hwang[8] or Dada and Srikanth[5]. Actually, in the numerical examples of Kim and Hwang's studies[7,9], where the objective functions were the supplier's profit, the resulting order size and price did not give each customer the cost as low as possible for a given level of supplier's profit.

To state this problem more clearly, we apply some terminologies used in the public utility theory. Let's define a term, surplus, as the improvement in cost or profit resulting from the revision of price and order size. Then, it is unreasonable to assume that the supplier will try to maximize the sum of surpluses of both customer's and his/her own, since we are considering profit-making organizations and the supplier may be worse off than before under the price schedule designed to maximize the sum of surpluses.

For a fixed level of of supplier's surplus, there are infinitely many combinations of price and order size each of which gives every different level of customer's cost.

Let's define deadweight loss in customer's cost as the difference between a his/her current cost and the minimum value of customer's cost for a given contribution to supplier's profit. Then, it will be reasonable to assume that the supplier basically wants to maximize his/her own profit and, at the same time, wants to reduce deadweight losses of customers as much as possible if it doesn't influence his/her profit too much.

In this paper, we suggest a property that pricing schedule should make deadwight losses of customers vanish. And it is shown that the incremental discount system can be designed to have the property for all customers who make use of the discounted price when the reduction in the number of setups is the largest benefit of the supplier resultant from quantity discount. And then, it is discussed how the supplier can specify the parameters of the incremental discount system with the above property to maximize his/her own profit.

And much simpler procedure, compared with the procedure in Kim and Hwang's study[9], is developed to determine the discount rate and the price break point and some examples are provided which illustrate the discussions in this paper.

2. Development of The Model

Notations and assumptions

The following notations will be used :

For supplier,

S = setup cost per order

C = variable unit cost

Hs = inventory carrying per unit of time expressed as a percentage of the value of the item

P_o = unit net price

For customer i,

n = the number of customers

D_i = quantity of demand per unit of time

Q_i = order size

A_i = ordering cost ($ /order)

H_i = inventory carrying cost per unit of time

P_i = average discounted price per unit

The following assumptions will be used:

(1) Each customer follows Economic Order Quantity(EOQ) inventory model which is based on deterministic demand, a single item, a single echelon, no stockouts, and deterministic lead times.

(2) The number of customers and the total demand per unit of time of each customer are fixed and do not depend on the discount pan.

(3) The number of units is treated as a continuous variable.

(4) Either Ai or Hi is known to the supplier. If either one is given, the other can be derived from the first assumption along with each customer's lot size and quantity of demand per unit of time.

(5) Discount price schedules with a single price break point are considered.

3. Price and Order Size without Deadweight Loss

The total annual inventory−related cost of customer i is expressed as follows :

$$E_i(P_i,\ Q_i) = A_i D_i / Q_i + H_i P_i/2 + P_i D_i \qquad (1)$$

With no discount available, the current order size becomes Q_{ai} from the EOQ formula, and

$$Q_{ai} = \sqrt{2A_i D_i/(H_i P_o)} \qquad (2)$$

The current order size is assumed to be determined optimally by the customer under the given price level, P_o.

The supplier's annual profit from customer i is given by

(revenue form selling at unit price P_i) − (order processing costs) − (variable costs) + (decrease in capital costs due to buyer purchasing the units),

which can be expressed as follows :

$$P_i D_i - S(D_i / Q_i) - CD_i + H_s P_i Q_i /2.$$

But since variable cost is independent on price and order size, we can set the profit function as follows :

$$F_i(P_i Q_i) = P_i D_i - S(D_i / Q_i) + H_s P_i Q_i/2 \qquad (3)$$

If the supplier's price discount gives economic advantages, customers may be willing to increase their order sizes. Conversely, if the order size affects the supplier's profit significantly, the supplier may want to increase the order size even if he/she accepts the price discount. Thus, it is possible to imagine another (P_i, Q_i) combination which gives extra benefits to both sides compared with the current level (P_o, Q_{ai}).

Now, suppose that the level of the supplier's surplus is fixed and let's try to find out price and order size to make the deadweight loss of customer's cost vanish (maximize the surplus of the customer's cost). The problem may be formulated as follows:

$$\max_{P_i,\ Q_i} J_i(P_i,\ Q_i) = E_i(P_o,\ Q_{ai}) - E_i(P_i,\ Q_i) \qquad (4)$$

subject to

$$F(P_i,\ Q_i) - F_i(P_o,\ Q_{ai}) = V \qquad (5)$$

where V is a positive constant value.

The problem (4)-(5) can be solved analytically and formulas of the solutions for a equivalent problem are derived in Kim and Hwang's paper [8].

Figure 1 illustrates solutions of the problem (4) − (5). F1, F2 and F3 represent the iso-profit curves of the supplier, and (profit of F1) > (profit of F2) > (profit of F3).

E1, E2, and E3 represent the iso-cost curves of the customer, and (cost of E1) > (cost of E2) > (cost of E3).

The point PT1 represents the original order size and price. So, both the supplier and the customer obtain surpluses by changing price and order size to any point (P_i, Q_i) in the area between E1 and F3. The piont which represents the optimal policy of the problem (4)−(5) moves from PT4 to PT2 on the path connecting the two points as V increases from zero to its maximum point within the area of mutual benefit. An arbitrary point PT3 on the path PT2−PT4 represents the intersecting point of a curve of F2, which passes through PT3, and the curve of E2, which has the least cost among those curves of $E_i(P_i, Q_i)$ intersecting F2.

In conclusion, we can characterize the point PT3 as price and order size without the deadweight loss for a fixed surplus of supplier's.

In Kim and Hwang's paper[8], it is also proved that when the supplier utilizes all-unit or incremental quantity discount schedule, he can entice a single customer to any point (P_i, Q_i) only if the point is included in the area between E1 and F3 which is the region of mutual benefit. And formulas are given for setting parameters of each discount system.

4. A Discount Price Schedule Based on Supplier's Profit Function

When a supplier offers a common price discount schedule to multiple customers, a question is whether the discount schedule can make every resulting price and order size coincide with the solutions of problem (4)−(5) on which each deadweight loss vanishes. The following property answers the question.

Property 1. *If the average unit price of a price schedule, which is expressed as a function of order size, coincides with the supplier's iso-profit function whose variables are price and order-size, every resulting deadweight loss vanishes.*

Proof. See figure 1. An arbitrary point PT3 on PT2−PT4 represents the intersecting point between the supplier's profit curve F2 and the customer's cost curve E2 which has the least cost among all the cost curves intersecting F2. The above statement means that when the supplier offers a discount price schedule which can be represented by F2, the buyer, who is also an optimal decision maker, will choose the point PT3 as price and order size to minimize his/her inventory−related cost. This holds for all the buyers simultaneously. Thus, the conclusion follows.
■

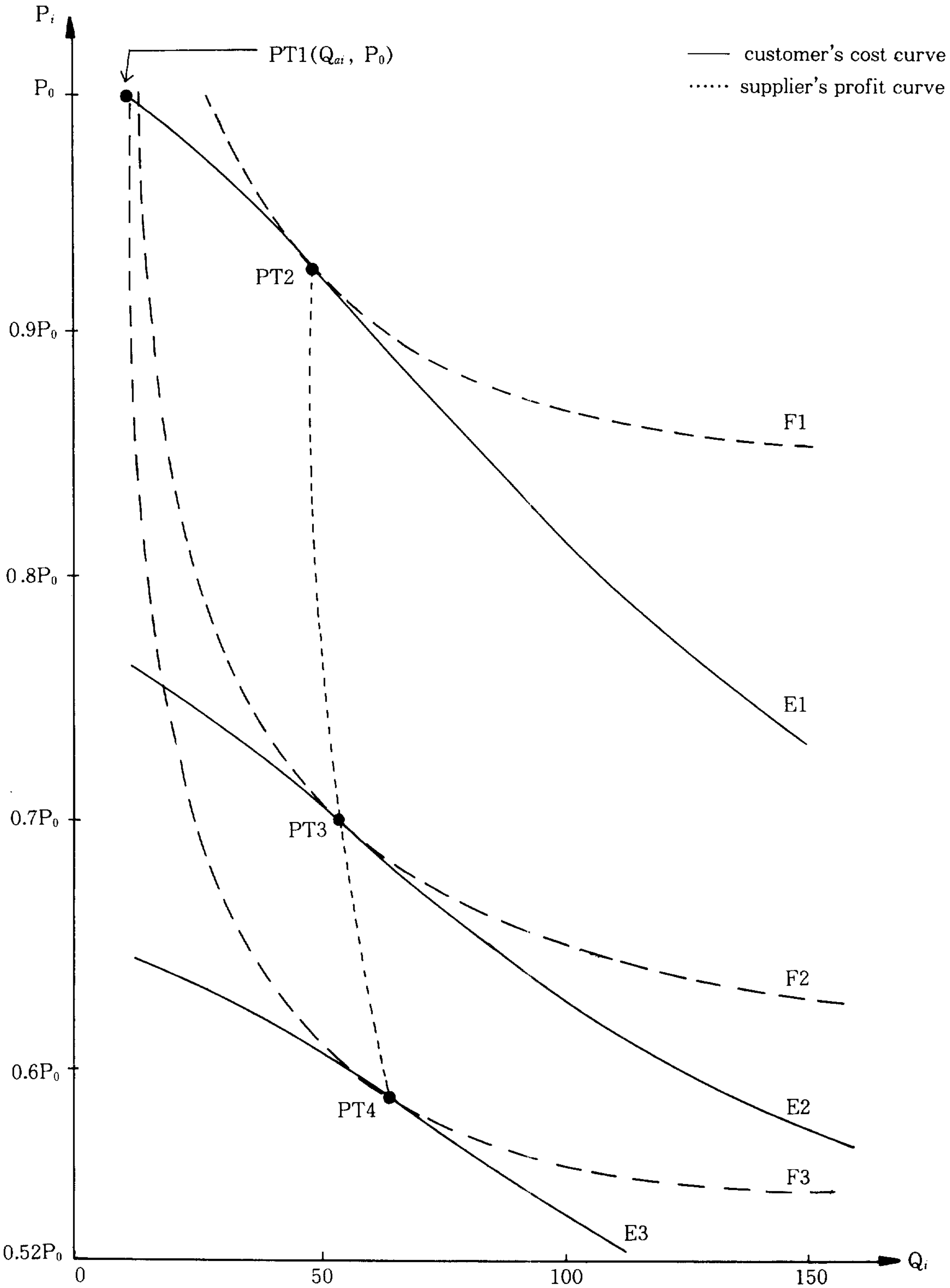

Figure 1. The iso-profit curves of a supplier and the iso-cost curves of a customer

5. When the Supplier's Main Benefit is Fewer Set-ups

Larger order sizes resulting from quantity discount give several benefits to the supplier. Suppose a supplier's benefit comes mainly from the reduction in the number of set-ups, then the supplier's iso-profit function contributed by customer i can be expressed as follows :

$$P_i D_i - SD_i / Q_i = K_i$$

where K_i is a constant value.

Then it follows that

$$P_i = K_i / D_i + S/Q_i \tag{6}$$

Which discount schedule has the average unit price function similar to the above iso-profit function? It's the incremental quantity discount schedule.

The average unit price of the incremental quantity discount system can be expressed as follows :

$$P_i = \begin{cases} P_o & \text{for } Q_i < B \\ P_o\{R + (1-R)\, B/Q_i\} & \text{for } Q_i \geq B \end{cases} \tag{7}$$

where B is the price break point and R is the discount coefficient and the unit purchasing price is P_oR for the amount exceeding B.

If we set

$$R = K_i / (P_o D_i) \tag{8}$$

and

$$B = S/\{(1-R)P_o\}, \tag{9}$$

the average unit price function in the range of $Q_i \geq B$ coincides with the supplier's iso-profit function, which guarantees that if the supplier offers the price schedule with parameters satisfying (9), every deadweight loss vanishes for the customer who pays the discounted price. Note that since K_i is an arbitrary constant value, it can be adjusted so that K_i / D_i has the same value for all the customers.

6. The Optimal Price Scheduling for Incremental Discount System

Two possible approaches may be assumed for the supplier to follow in designing a discount pricing schedule.

Scenario 1 : *The supplier tries to maximize his own profit without considering the reductions of customers' cost. In this case, the supplier engrosses most of the surplus resulting from the revision of prices and order sizes.*

Scenario 2 : *The supplier tries to maximize his own profit maintaining every deadweight loss at zero for customers who make use of the discounted price.*

Kim and Hwang[7,9] followed the first scenario. But the second scenario can be considered more reasonable since buyers can take more incentives to change their order sizes without so much loss of supplier's profit. Most of all, the total surplus from the revision of prices and order sizes can be increased more than in the case of the first scenario.

Now, we describe how the supplier can design an incremental quantity discount schedule based on the second scenario, which can be formulated as follows :

$$\max_{R,\ B} F(R,\ B)=\sum_{i=1}^{n} F_i(R,\ B) \tag{10}$$

where $F_i(R,\ B)=P_iD_i-S(D_i/Q_i)$ and P_i is expressed as (7).

The customers try to minimze their total inventory-related cost per unit time which is given by

$$\min_{Q_i} E_i(Q_i) \tag{11}$$

where $E_i(Q_i)=A_iD_i/Q_i+H_iP_iQ_i/2+P_iD_i$

In Kim and Hwang's paper[8], it is shown that for a given R, the optimal ordering quantity Q_i^* of customer i is dependent on the price break point B and

$$Q_i^*=\begin{cases} Q_{bi} \text{ for } B \le Q_{ci} \\ Q_{ai} \text{ for } B > Q_{ci} \end{cases} \tag{12}$$

where $Q_{bi}=\sqrt{(Q_{ai}^2H_i+2(1-R)BD_i)/(H_iR)}$ and $Q_{ci}=2\{(1-\sqrt{R})/(1-R)\}(Q_{ai}+2D_i/H_i)-2D_i/H_i$

To maintain the deadweight loss for each customer at zero, the equation (9) should be satisfied. Thus, replacing B in (7) with (9) to get P_i, it follows

$$\begin{aligned} F_i(R,\ B) &= (P_oR+S/Q_{bi})D_i-S(D_i/Q_{bi}) \\ &= P_oRD_i \text{ for } B \le Q_{ci} \end{aligned}$$

and $F_i(R,\ B)=P_oD_i-S(D_i/Q_{ai})$ for $B>Q_{ci}$.

Then, we explain how to find R^* under the condition that $B=S/\{(1-R)P_o\}$. Note that as R decreases from 1, the value of B decreases from infinity but Q_{ci} increases from Q_{ai}. Thus, B meets every Q_{ci} while R decreases from 1 to 0. Let the value of R at which B meets Q_{ci} be denoted as R_i. Then R_i can be derived by setting $B=Q_{ci}$ as follows :

$$R_i=[\{(Q_{ai}H_i+2D_i)-\sqrt{Q_{ai}^2H_i^2+2D_iH_iS/P_o}\}/(2D_i)]^2 \tag{13}$$

And the following property holds.

Property 2. *The optimal value of R, R^*, is one of R_i, $i=1,\cdots,n$.*

Proof. Let X_i, $i=1,\cdots,m(\leq n)$, be a sorted series of R_i in increasing order. Let G1 denote the set of customers whose ordering quantities are not affected by the discount schedule and G2 denote the set of customers who change their order sizes from Q_{ai} to Q_{bi} when price discount is available. Then on each (X_i, X_{i+1}) of R, G1 and G2 are uniquely determined. And as R increases in the range of (X_i, X_{i+1}), $F_i(R, B)$, $i\in G2$, increases, while $F_i(R, B)$, $i\in G1$, remains constant. $R=1$ may be excluded as a candidate since in the case of a single customer we can always find better price and order size different from the original ones, (P_o, Q_{ai}). Thus, the conclusion holds. ■

Comparing with the solution procedure of Kim and Hwang[9] where numerical search should be conducted more than ${}_nC_2$ times, it is much easier to find R^* in this study where no more than n candidates have to be evaluated to find the optimal solution for the same case.

7. A Numerical Example

To illustrate findings in the previous sections, a problem with 5 customers is considered with the relevant data given in Table 1. Table 2 shows that price schedule suggested in this study has the largest total surplus although the supplier's profit decreased a little compared with price schedules whose goals are to maximize only supplier's profit. The solution procedure for case 2 and 3 can be found in Kim and Hwang's papers [7, 9].

In Table 3, order size, inventory cost, contribution to supplier's profit and the total surplus for each customer are compared among four different cases. Note that the inventory cost and the total surplus of each buyer's in this study are improved by larger amounts compared with those in the other cases.

8. Discussions

In this paper, we considered the case where the reduction in the number of set-ups is supplier's main advantage of utilizing quantity discount. But, the supplier's profit may be influenced by other factors than the set-up frequency. In these cases, some different types of discount system may be better.

Generally, in order to determine the type of a discount system, the supplier's profit function should be analysed first and then we should find a discount system whose average unit price function has the same mathematical form as the supplier's profit function.

For example, all-unit discount system satisfies property 1 when the supplier's profit function can be expressed as the following :

$$F_i(P_iQ_i) = P_iD_i - C(Q_i)D_i$$

where $C(Q_i)$ equals to C_1 for $Q_i < B$, C_2 for $Q_i \geq B$ and $C_1 < C_2$.

That is, cost per unit decreases step by step as the order size increases. Examples can be found in some freight cost and purchasing cost with quantity discount.

Table 1. Data of the example before quantity discount is offered

customer	demand per unit time	order size	total inventory cost
1	50	10	265
2	200	20	1030
3	300	25	1538
4	250	50	1325
5	350	130	1945

unit price (Po) = $5

supplier's setup cost(S) = $25

inventory carrying cost(H_i) = 0.3 for every customer

Table 2. The optimal discount plans

cases	case 1	case 2	case 3	case 4
discount coefficient (R*)	–	0.972	0.942	0.906
price break point (B*)	–	73	40	53
no. of setups	35	18	18	14
total profit	4883	5151	5138	5109
sum of customer's cost	6103	6002	5980	5921
total joint cost	1220	851	842	812

case 1 : without discount plan

case 2 : all-unit quantity discount with scenario 1 (maximizing only the supplier's profit) case 3 : incremental quantity discount with scenario 1

case 4 : incremental quantity discount with scenario 2 adopted in this study (maximizing the supplier's profit while maintaining zero deadweight losses of customers)

Table 3. Order size, cost, contribution to supplier's profit, and joint cost of each customer

case \ customer		1	2	3	4	5
order size	1	10	20	25	50	130
	2	10	73	73	73	132
	3	10	61	75	82	154
	4	10	88	108	109	178
inventory cost	1	265	1030	1538	1325	1945
	2	265	1030	1519	1295	1894
	3	265	1030	1521	1296	1868
	4	265	1030	1510	1285	1831
contribution to supplier's profit	1	125	750	1200	1125	1683
	2	125	904	1356	1130	1635
	3	125	898	1360	1137	1619
	4	125	906	1359	1133	1586
the total surplus	1	0	0	0	0	0
	2	0	154	176	36	3
	3	0	148	177	41	8
	4	0	176	187	48	17

9. Conclusions

In this paper, the case is considered where a supplier offers a quantity discount pricing schedule to multiple customers. It is shown that if average unit price function of a price schedule coincides with supplier's iso-profit function, the price and the order size resultant from the optimal decision-making of customers give zero deadweight loss to every customer.

Utilizing the above property, the incremental discount schedule is suggested as a promising pricing regime in case that the supplier's main benefit of quantity discount comes from the reduction in the number of setups.

A procedure is suggested to determine values of parameters of the incremental discount schedule which make deadweight loss vanish for all the customers who make use of the discounted price. It is also shown that the procedure is much simpler than one developed in Kim and Hwang's study[9].

A numerical example is solved to illustrate the discussions of this study. Possibilities of applications of the same concept to other types of profit functions are discussed.

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