Monte-Carlo simulation of earthquake sequence in the time and magnitude space
시간 및 규모 영역에서 지진 발생의 몬테-카를로 가상 수치 계산

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Abstract/요 약

A computer simulation of earthquake sequence in the time and magnitude space was done using random number generation. The theory of the simulation are based on the two statistical models of earthquake events. Those models are Stationary Poisson Process for independent earthquakes and Branching Markov Process for aftershocks. The generated earthquake sequnces resemble the actual earthquake catalogs.

난수 발생 방법을 이용하여 시간과 규모의 영역에서 일련의 지진 발생에 대해 가상 수치 계산을 수행 했다. 가상 수치 계산의 이론은 지진 발생에 관련된 두 종류의 통계적 모델에 기초 하였다. 이들 모델들은 각기 독립적으로 발생하는 지진에 관련된 고정 포아송 처리와 여진에 관련된 분기 마르코프 처리등이다. 계산된 일련의 지진 발생들은 실제 발생한 지진의 목록과 유사함을 보여 주었다.

INTRODUCTION

In a given seismic region, for example, the region confined in one or two degrees of distances, after some strain energy is absorbed, there will be radiation of seismic energy, i.e., earthquake. The information such as travel time, magnitude, hypocenter of the seismic energy can be gathered from the earthquake event. If we make a magnitude-time plot of earthquake sequence in a given region, two kinds of patterns are found.
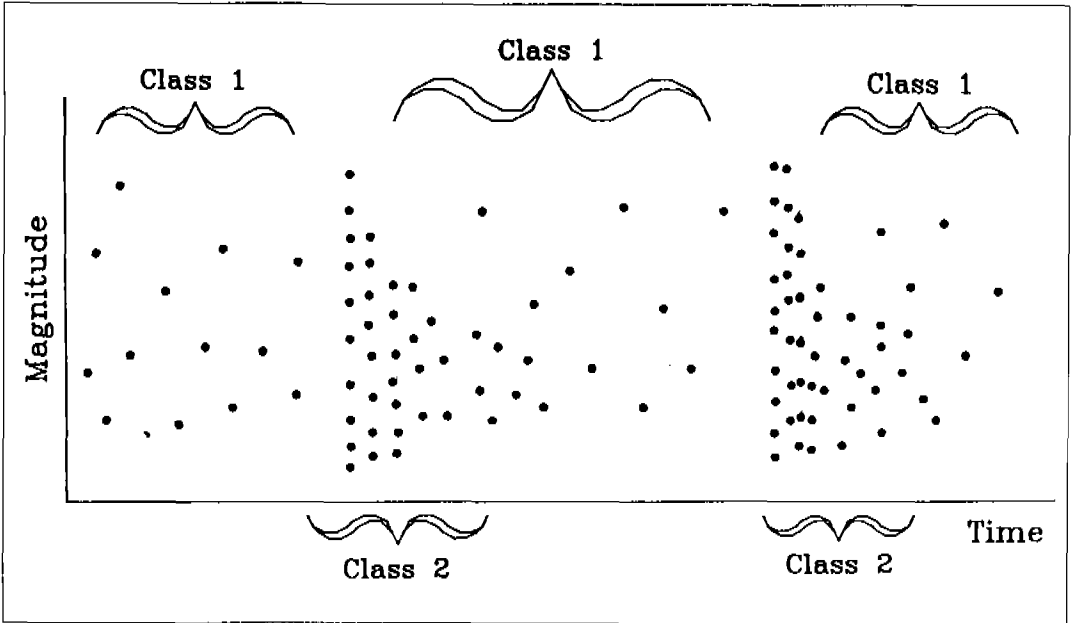
Fig. 1　Two kinds of patterns of earthquake sequence in time-magnitudespace.
Class 1 and 2 represent independent and dependent events, respectively.

The typical classes are shown in Fig. 1.

If we calculate the interevental time for class 1, the distribution of it follows exponential one (Utsu, 1972). It seems that there is no magnitude relation among each event. Therefore they are called independent shock. Due to the earthquake exponential distribution of interevental time, this process of earthquakes belongs to Poisson Process (Parzen, 1962 ; Vere-Jones, 1970).

With careful inspection of class 2, we can find some magnitude or energy relations among events. These shocks are dependent earthqukes, i.e., aftershocks. Some energy level transitions are involved in the process . The transition probability may depend on the current energy state of the system (Vere-Jones, 1966). Therefore

we could guess a Markov Process (Shilien and Toksoz, 1970)

Now assuming that the processes of series of earthquakes be Poisson and Markov Processes, we can simulate real earthquake sequence in the time and maginitude space by random number generation.

THEORY

The occurrences of the independent earthquakes are governed by Poisson Process. This indicates that the probability of N events occurring in the time interval $\varDelta t$ is given by

$$Pn\ (\varDelta t) = \frac{(\lambda \varDelta t)^n\ exp(-\lambda \varDelta t)}{n!}$$
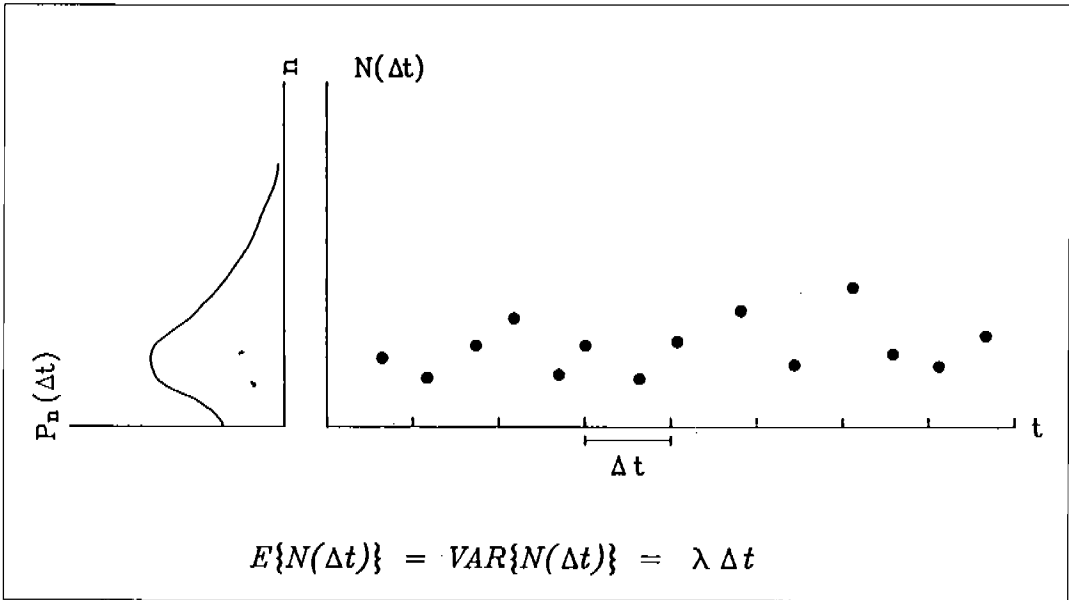
Fig. 2   Poisson distribution of independent earthquakes.
Pₙ (𝛥 t) is the probability of N events occurring in the 𝛥 t

Mean and variance are the same and equal to $\lambda \Delta t$. λ is time rate of events (Fig. 2). Exponential distribution of interevental time can be derived from Poisson Process (Parzen, 1962),

$$f(t) = \lambda \, exp(-\lambda t)$$

Aftershock process is more complex than independent shock process. Let us assume that just before a mainshock, there was accumulated strain energy ε_0 in a seismic region. When the main shock occurs, there will be some energy drop of $\varepsilon_0 - \varepsilon_1$. From now on we get shower of small earthquakes here and there. The system enters into Markov Process session. In each shock, some energy drop is accompanied(Fig. 3).

In each discrete energy drop, the assumption that there is some transition probabilty function $T(X/\varepsilon)$ is provided. $T(X/\varepsilon)$ indicates the probability of transition occurring from an energy state ε to another energy state X. Let the probability of transition occurring at state ε into any other state be $\lambda(\varepsilon)$. This is the rate of events. $P(\varepsilon, t)$ is the probability of being in state ε at a time state t. If we assume that future evolution depends only on it's present state and does not depend on how it had reached the state, we get the following integro-differential equation.

$$\lambda(\varepsilon)P(\varepsilon, t) + \frac{\partial P(\varepsilon, t)}{\partial t}$$
$$= \int P(X, t) \, \lambda(X) \, T(X/\varepsilon) \, dX$$

Now if we know $\lambda(\varepsilon)$ and $T(X/\varepsilon)$, everything about the system can be derived. In order to find $\lambda(\varepsilon)$ and $T(X/\varepsilon)$, some known seismological relations can be used.

First, for the determination of transition probability $T(X/\varepsilon)$, seismologists Shlien and Toksoz (1975) used Gutenberg-Richter's frequency-magnitude relation and magnitude-energy relation.

$$log\ N(M) = a - bM$$
$$log\ E = 11.8 + 1.5\ M$$

Combining these relations, the number of shocks having energy E or greater become

$$N(E) = A\ E^B$$

where

$$B = b/1.5$$
$$log\ A = a + 11.8\ b\ /1.5$$

By differentiation, we get the probability density function of energy released.

$$P(E) = \frac{d}{dE}\ N(E) = C\ E^{B-1}$$

In order to get boundness of the probability at zero E value, the function should be truncated at small E. This procedure results in ignoring earthquakes below a certain magnitude. By normalization of $P(E)$, the constant becomes
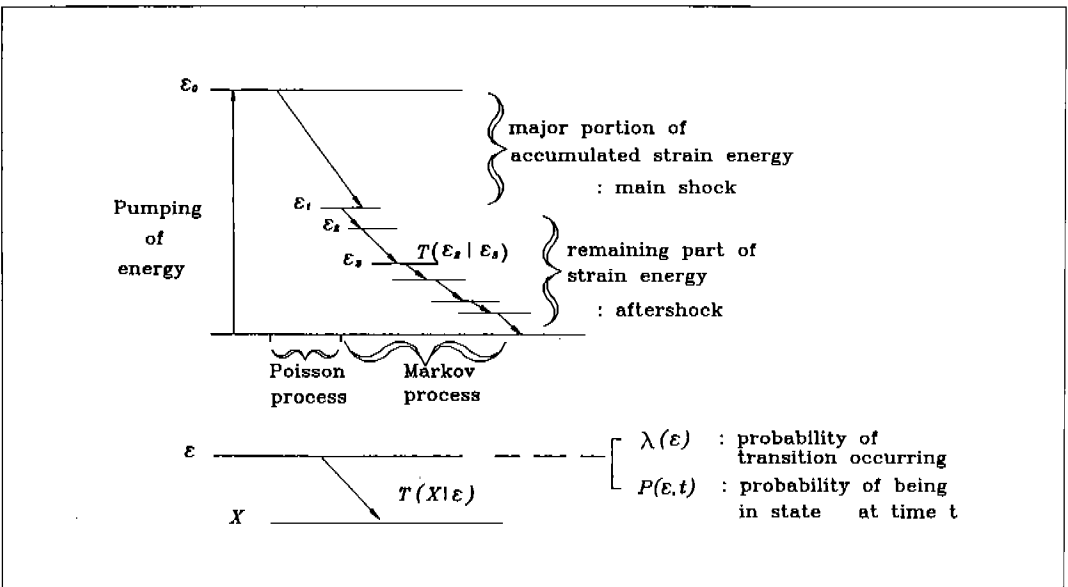

Fig. 3   Strain energy transition scheme following the Poisson and Markov process.
$T(X/\varepsilon)$ indicates the probability of transition occurring from an energy state at ε to another state at X.

$$C = B E_0^B$$

The mean energy is

$$\overline{E} = \int_0^\infty E\,P(E)\,dE = \frac{B\,E_0}{B\text{-}1}$$

But this mean energy is unreasonable for the value $B \leq 1$ $(b \leq 1.5)$. Therefore we need limitation of maximum magnitude, and the function should be truncated at some upper limit, e.g., no greater magnitude than 9. By normalization of $P(E)$, constant C becomes

$$C = \frac{B}{E_0^B - E_{max}^B}$$

Now we get mean energy E by integration of $P(E)$ from E_0 to E_{max}.

$$\overline{E} = \frac{B}{B\text{-}1}\,\frac{E_{max}^{-B+1} - E_0^{-B+1}}{E_{max}^{-B} - E_0^{-B}}$$

If we assume that frequency-magnitude relation is invariant with the state energy, i.e., assuming constant b value during aftershocks (Lomnitz, 1966 : Hamilton, 1966), the energy drop distribution function $P(E)$ become transition probability $T(X/\epsilon)$ since E indicates dropped energy, $E = \epsilon - X$.

$$T(X/\epsilon) = P(E) = C\,E^{-B-1}$$

$$= \begin{cases} C\,(\epsilon - X)^{-B-1} & E_{max} \rangle \epsilon - X \rangle E_0 \\ 0, & \epsilon - X \langle E_0,\ \ \epsilon - X \rangle E_{max} \end{cases}$$

Next, for determination of the time rate of event $\lambda(E)$, Shlien and Toksoz used Omori's law that during an aftershocks sequence the rate of earthquake decays with time t.

$$\Delta n(t) = \frac{r}{t^P}\,\Delta t$$

where $\Delta n(t)$ is expected number of aftershocks in time interval Δt, and r,p are given constants. The energy state becomes

$$\epsilon = \overline{E} \int_t^\infty \frac{dn}{dt}\,dt$$

$$= \frac{r\,\overline{E}\,t^{p+1}}{r-1}$$

$$t = \left[\frac{\epsilon(p-1)}{r\,\overline{E}}\right]^{-1/(p-1)}$$

where $\overline{E}$ is mean energy, and previous result can be used for it. The time t can be expressed in terms of energy state ϵ and mean energy $\overline{E}$. The time rate is E

$$\lambda = \frac{dn}{dt} = r\,t^{-P}$$

$$= r^{-1/(P-1)} \cdot \left[\frac{p-1}{\overline{E}}\right]^{p/(p-1)} \epsilon^{p/(p-1)}$$

Now, since we know $\lambda(\epsilon)$ and $T(X/\epsilon)$ for aftershock, we know everything about the system, and computer simulation is

possible for Markov Process.

COMPUTER SIMULATION AND DISCUSSION

Now we are ready to simulate earthquake sequences in a region with given values of time rate λ and transition probability T, where T is related with mean energy $\bar{E}$.

First, for the Poisson Processes we have to generate two series of random numbers. By inserting one of the random number series into the inverse of exponential function, we get interevental time of each event. This means Poisson Process. Another set of random numbers is plugged into the frequency-magnitude relation to get magnitude of each event (Fig. 4).

Next, for the aftershock simulation we need to calculate total energy of all aftershocks from the fraction of main shock energy.

Energy for aftershock : $\varepsilon = E f$

Here, f is a fraction.

Using the calculated total energy, the time rate of events becomes

$$\lambda = r^{-1/(p-1)} \left[\frac{p-1}{E} \right]^{p/(p-1)} \varepsilon^{p/(p-1)}$$

By generating a random number inserting it into the inverse exponential time function, we get interevental time of

one aftershock. This means branching Poisson Process. Generation of another random number produces magnitude and energy drop by use of the following equations,

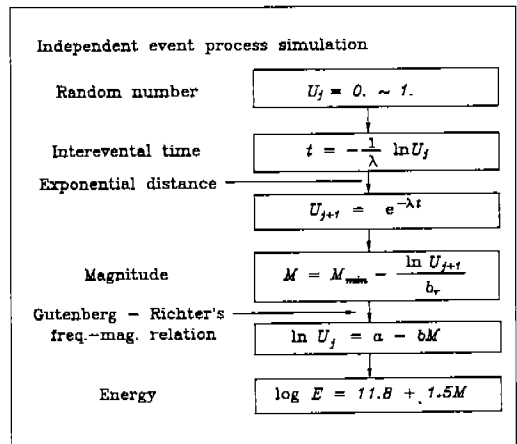$$M = M_{min} - \frac{\ln U_{i+1}}{ba}$$

$$logE = 11.8 + 1.5M$$

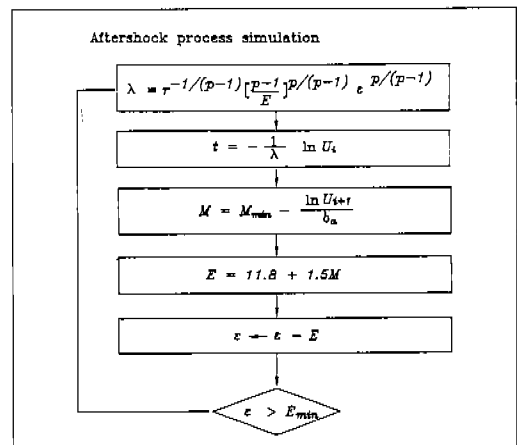Fig. 4   Procedure to generate independent events of the Poisson Process.

Fig. 5   Procedure to generate aftershocks of the Markov Process.

The calculation of new energy state is done for next aftershock. The repeitition of the same calculation produces interevental time and magnitude of new aftershock(Fig. 5).

Fig. 6 and 7 show the results of computer simulation for both independent and aftershocks. In Monte-Carlo simulation of these processes the following parameters are used : Omori's parameter $p = 4/3$, $r = 2$. : fraction of energy going into aftershocks $f = 0.4$; b value for the frequency-magnitude relationship for aftershocks $b_a = 1.1$, for independent events $b_i = 0.9$: maximum and minimum magnitude $M_{max} = 0.8$, $M_{min} = 4.0$; Poisson rate $\lambda = 0.05$ for the first plot (Fig. 6), and $\lambda = 0.01$ for the next plot (Fig. 7).

The magnitudes of events are shown as a function of time. The aftershock sequences after large independent earthquakes are quite clear. The sequences resemble the actual earthquake catalogs, but better plots can be produced by changing parameter and initial number of random numbers.
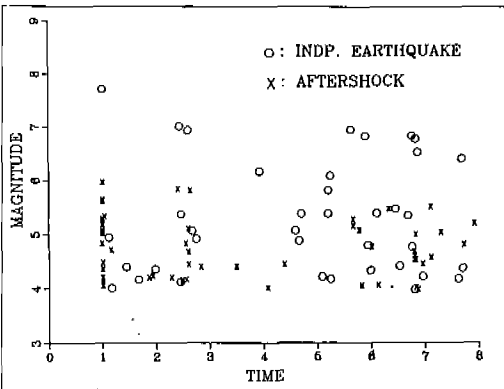

Fig. 6 A result of computer simulation for both of independent and aftershocks. Parameter used in the computation are : $p = 4/3$, $r = 2$, $b_a = 1.1$, $b_i = 0.9$, $M_{max} = 8.0$, $M_{min} = 4.0$
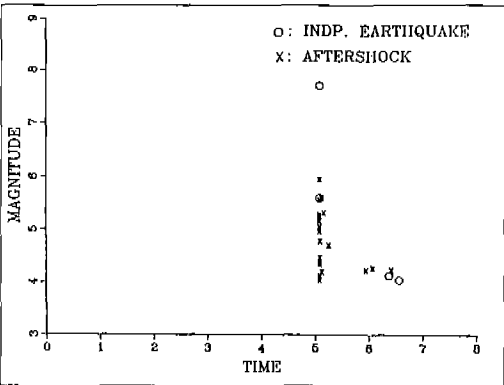

Fig. 7 A result of computer simulation for both of independent and aftershocks. All parameters except λ are equal to those of Fig. 6. λ for this plot is 0.01.

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