□ 論 文 □

Rouphail의 교통신호시간 설정 및 좌회전처리의 동시최적화의 확장

An Extension of Rouphail's Simultaneous Optimization of Signal Settings and Left-Turn Treatments

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전통적인 신호제어모형은 다음의 두가지 결점을 내포하고 있다. 신호현시의 방법을 적정화하는 점이다. Rouphail(1990)은 전통적으로 행해온 신호주기와 녹색시간분할의 적정화에서 더 나아가 현시방법 (signal timing)과 좌회전의 문제를 동시적으로 적정화 시킬 수 있는 모형을 제시한 바 있다. 그러나 Rouphail의 모형은 4개 현시에 국한되었고 현시방법(또는 현시의 선택)도 대단히 제약적이었다. 본 연구는 이러한 Rouphail의 모형을 실제상황에 보다 일반성있게 적용할 수 있도록 확장하는 연구이다. 여기서는 현시방법(또는 선택하는 현시의 형태와 수)을 확대시키고 또 연속변수와 이산변수를 동시에 적정화시키는 혼합정수선형계획법을 적용한 컴퓨터 모형을 구성하고, 이를 사례지역에 적용하였다. 분석결과는 본모형이 Rouphail 모형과 비교할 때 현시방법의 선택문제에 있어서 뿐만 아니라 좌회전(비보호, 보호 또는 혼합적)현시의 처리에 있어서 훨씬 더 효율적임을 확인하였다.

1. Introduction

Most traffic signal timing models have two apparent weaknesses. One is the inability to determine the optimum number of phases, i.e., the number of phases cannot be optimized by those models. The other is the difficulty in optimizing left - turn phasing (e.g. protected,

permissive, or both).

The simultaneous optimization method of signal timing and left-turn treatments was developed by Rouphail (1990). The proposed method, which entails the optimization of cycle length and green splits, integrates these two elements mentioned above into the signal timing optimization process for isolated intersections. Rouphail's method directly considers the effect of minimum green times, practical cycle lengths, and permissive left-turn capacity models in an attempt to obtain an optimal decision.

As Rouphail's formulation has only 4 phases (phases 4, 5, 9, and 10 in Fig.2), phases selections are very restrictive. Thus, the signal timing solution from his approach may not be appropriate in some cases. Especially, this approach cannot treat the leading left - turn phase (e.g. phase1 - phase2 in Fig.2) and the lagging left - turn phase (e.g. phase2 - phase3 in Fig.2) which are common in real situation.

The purpose of this study is to extend Rouphail's formulation to the real situation for more general application. The six additional phases include exclusive through movement phases and the mixed protected left - turn and through movement phases. Since this

formulation both incorporates continuous variables (green splits and lost times) as well as discrete variables (number of phases and cycle length in increments of 5seconds), a mixed - integer linear programming (MILP) is used in formulating the problem.

2. Problem Definition

Two rather simple but basic principles of signal timing drive the problem formulation. That is:

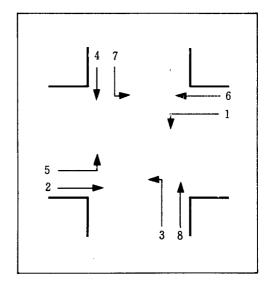
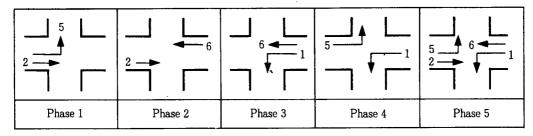
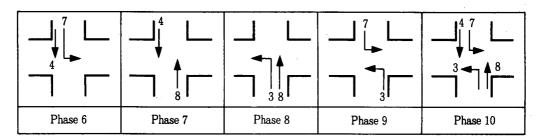


Fig. 1 Traffic Movements





Note: All left-turn movements are portected except those of phase 5 and phase 10

(b) North-South Bound

Fig. 2 Phase Designation

Table 1 Saturation Flow Matrix

Movement					Phase	(i)				
(i)	1	2	3	4	5	6	7	8	9	10
1	_	_	SU ₁₃	SUn	SO ₁₅	_	_	_	_	_
2	ST ₂₁	ST ₂₂	-	_	ST ₂₅	_		_		_
3	_		_	_	_	-	-	SU ₃₈	SU ₃₉	SO _{3.10}
4	-	-	_	_	-	ST46	ST47	_	_	ST _{4.10}
5	SU ₅₁	_	-	SU ₅₄	SO ₅₅	_	_	_	-	_
6		ST ₆₂	ST ₆₃	. —	ST ₆₅	_	-	_	_	_
7			_	_	_	SU ₇₆	_	_	SU ₇₉	SO _{7.10}
8	_	<u> </u>	_	_		_	ST ₈₇	ST ₈₈	_	ST _{8.10}

SU_{ij}=Unopposed left-turn saturation flow rate for movement i in phase i

ST_{ij}=Throug saturation flow rate for movement i in phase j

SO_{ij}=Opposed left-turn saturation flow rate for movement i in phase j

- i) selection of shortest possible cycle length,
- ii) selection of minimum sufficient number of phases.

Problem formulation involves determining cycle length, splits, and left - turn treatments that allow each movement to operate at or below a predetermined V/C ratio threshold.

Fig. 1 shows traffic movements in an intersection which are divided into through

movements (including right - turn movements) and left - turn movements. The 10 optional phases with protected or permissive left - turn are illustrated in Fig. 2. The scope of this study is limited to intersections with exclusive left - turn lanes on all approaches.

3. Rouphail's Formulation

Rouphail's formulation considers only 4

phase alternatives which are the dual left - turn phase (phase 4 and 9 in Fig. 2) and the dual through phase with permissive left - turn (phase 5 and 10 in Fig. 2).

Objective Function

The objective is to minimize the cycle length C subject to the given constraints. In order to maintain linearity throughout the formulation, cycle frequency K is maximized instead.

K=1/C, C= cycle length in seconds

Constraints

1) Minimum Green Constraints

Assume that the actual and effective greens to each phase are numerically equivalent then the minimum green constraint is expressed as

$$g_j \ge g^j \min (j=5 \text{ and } 10)$$
 (2)

where

g_i=effective green time for phase j (sec)
g'min=minimum effective green for phase j
(sec)

Multiplying both sides in constraint (2) by K yields

$$X_i - K g^i min \ge 0$$
 (3) where

 $X_i = green split (g_i * K)$ for phase j

Considering the case for portected left – turn phase 4 and phase 9, the minimum green constraint applies only when these phases become part of the optimum solution. Hence,

$$X_j \ge K g^j min - (1 - I_j)$$
 (j=4, 9) (4) ($I_j = 0$ or 1, if $I_j = 1$, phase j is selected, otherwise $I_i = 0$)

2) V/C Constraints

A. Through movements:

$$R_i ST_{ij} T_j \ge f_i \ (i=2, 4, 6, 8) \ (j=5, 10)$$
 (5) where

R_i=threshold v/c ratio for movement i

 ST_{ij} =through saturation flow rate for movement i in phase j

fi=flow rate for movement i

B. Left - Turn movements:

a. Protected phase capacity:

$$C_{pro} = SU_{ii} T_{i} \tag{6}$$

where

 C_{pro} =capacity of protected left-turn phase (vph)

 SU_{ij} =unopposed left turn saturation flow rate for movement i in phase j

b. Permissive phase capacity:

$$C_{per} = SO_{ij} \{ST_{opp} T_j - f_{opp}\} / \{ST_{opp} - f_{opp}\}$$
 (7) where

Cper=permissive phase capacity (vph)

 $SO_{ij} = opposed$ left turn saturation flow rate, generally expressed in terms of the opposing flow rate (f_{opp})

ST_{opp}=saturation flow rate for the opposing through movement

f_{one}=opposing through flow rate

T_i=permissive phase split (phase5 or phase10)

c. Clearance phase capacity:

$$C_{cle} = 3,600 \text{ Z K}$$
 (8)

where

C_{cle}=clearance phase capacity (vph)

Z=number of left turns allowed per cycle in the clearance interval

Then, the v/c constraint for left – turn is: $R_{if}SU_{ii} X_{iu} + SO_{ij}(ST_{opp}X_{io} - f_{opp})/(ST_{opp} - f_{opp})$

$$+3,600 \text{ Z K}] \ge f_i$$
 (9)
i=1, 3, 5, 7, and j=4, 5, 9, 10

where

X_{ju}=unopposed green split

X_{io}=opposed green split

3) Cycle Length Constraints

In most traffic signal applications, the length of the cycle is typically set in 5 second multiples. For a feasible cycle range $C_{\text{min}} \leq C \leq C_{\text{max}}$, this constraint can be expressed as :

$$K = \sum_{m=0}^{(C_{max}-C_{min})/Cd} \{1/(C_{min}+mC_{d})\} j_{m+1}$$
 (10)

$$\sum_{j} J_{m+1} = 1 \tag{1}$$

 $(m=0, 1, ..., (C_{max}-C_{min})/C_d)$

where

C_{max} = maximum cycle length (sec)

C_{min}=minimum cycle length (sec)

 C_d =allowable cycle length increment (sec) J_{m+1} =0 or 1

....

4) Phase Split Constraints

$$\sum_{i} X + 2IK + \sum_{i=4,9} IK_{i} = 1$$
 (12)

$$K_iK - (1 - I_i) j = 4 \text{ and } 9$$
 (13)

$$K_i \leq I_i j = 4 \text{ and } 9$$

where

I=lost time per phase (sec)

 $I_i = 0$ or 1

4. Extended Formulation

One of limitations of Rouphail's formulation is that his formulation cannot produce various phasing combinations. Optimal phasing pattern can be combined in various ways. Thus, 10 optional phases are used for phase selection in this formulation.

Objective Function

$$Z=Max K$$
 (1')

Constraints

1) Minimum Green Constraints

$$X_j \ge K g_{min}^j - (1-I) (j=1,2,...10)$$
 (4')
($I_j=0$ or 1, if $I_j=1$, phase j is selected, otherwise $I_i=0$)

2) V/C Constraints

A. Through movements:

$$R_i ST_{ij} X_j \ge f_i (i=2, 4, 6, 8, j=1,...,10) (5')$$

B. Left-Turn movements

$$R_{i}[SU_{ij} X_{iu} + SO_{ij} (ST_{opp} X_{jo} - f_{opp}) / (ST_{opp} - f_{opp}) + 3,600 Z K] \ge f_{i}$$

$$i=1, 2, 3, 5, 7, \text{ and } j=1, 2,...,10$$

3) Cycle Length Constraints

$$k = \sum_{m=0}^{C_{max}-C_{min}/Cd} \{1/(C_{min}+m\ C_d)\}\ J_{m+1} \ (10')$$

$$\sum_{j} j_{m+1} = 1 \tag{11'}$$

$$(m=0, 1, ..., (C_{max}-C_{min})/C_d)$$

4) Phase Split Constraints

$$\sum_{i} X_{i} + \sum_{i} I K_{i} = 1$$
 (12')

$$K_j \ge K - (1 - I_j) \tag{13'}$$

$$K_{i} \leq I_{i} \tag{14'}$$

(i=1.2....10)

5) Additional Constraints

$$\sum I \le 6 \ (i_i = 1, 2, ..., 10)$$
 (15)

Constraint (15) restricts the maximum

number of phases to 6. It is not practical to have more than 6 phases in a cycle because lost time increases with the number of phases.

The binary—mixed—integer linear program was solved with the branch—and—bound solution algorithm available in the Linear Interactive Discrete Optimizer (LINDO) package.

5. Application

The intersection (Iocated in Austin, Texas) of 26th street and Red River street was selected for application of the extended formulation. The data obtained from Traffic Control Center of City of Austin containing traffic volume counts by movement, phasing patterns, and green splits was used. As a matter of chance the results show exactly the same phasing scheme as in the existing condition. The only difference lies in the cycle length and the green splits. The existing cycle length is 90 seconds long. whereas the cycle

length obtained from the extended formulation is 60 seconds. The signal timing plan for the existing condition and the one obtained from the extended formulation are shown in Fig. 3. This phaseing pattern cannot be selected by Rouphail's formulation because of ti's limitation in the number of optional phases.

To compare the results from the extended formulation with the existing signal timing plan, the TRAffic Network Study Tool (TRANSYT-7F), which is one of the most widely used traffic signal timing models in the world, was used. The average delay was used as a criterion for comparing the two signal timing plans. According to the 1985 Highway Capacity Manual, level of service for signalized intersections is defined in terms of delay. Delay is a measure of drivers discomfort, frustration, fuel consumption, and lost travel times. The average delays obtained from TRANSYT-7F for the two signal timing plans are shown in Table 2. Comparison of the average delays by movement indicates a

	Phase 1	Phase 2	Phase 3	Phase 4
Phasing pattern			4	-
Existing timing	19(4)	19(4)	6(4)	30(4)
New timing	10(3)	13(3)	10(3)	15(3)

^{*}Phase 4 includes permissive left-turn

Fig 3. Existing Signal Timing and Proposed One

^{() :} yellow time

decrease in the delays for 5 of the 8 movements and an increase in the delays for the remaining 3 movements. As a result of using signal timing plan obtained from the extended formulation, there is an overall decrease in the average delay by 5 seconds per vehicle for the whole intersection

Table 2. Comparison of the Two Signal Timing

movement #	Exising (A)	Proposed (B)	difference (A-B)
1	119.1	25.9	-93.2
2	18.2	14.9	-3.3
3	34.3	44.4	+10.1
4	21.8	20.5	-1.3
5	31.0	26.5	-4.5
6	24.7	12.4	-12.3
. 7	71.8	97.7	+25.9
8	25.8	31.8	+6.0
overall	26.3	21.3	-5.0

6. Conclusion

Although Rouphail's formulation partially remedied the weaknesses in the conventional traffic signal timing models, it is not appropriate in some real-situation-cases. Especially, his formulation cannot treat the leading left-turn phase and the lagging left-turn phase which are common in real situation.

In an attempt to extend Rouphail's formulation, the extended formulation for the simultaneous optimization of signal timing and left—turn treatments is presented. Conclusions drawn from this study are described as

follows:

- The extended formulation is more flexible in the selection of the number of phasing patterns in comparison with Rouphail's formulation
- 2) The signal timing plan obtained from the extended formulation shows better performance than the existing plan.
- 3) One major feature of this formulation is a more efficient treatment for left-turn phases (protected, permissive, or both) when compared with Rouphail's formulation.
- 4) This study is limited to intersections with exclusive left—turn lanes on all approaches. Thus, further study on the left—turn treatments with no exclusive left—turn lane is recommended.

7. References

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- 2) M. Sakita, Timing Design of Traffic Signals, Transportation Research Record 1069, TRB, National Research Council, Washington, D.C., 1986.
- L. Schrage, User's Manual for Linear and Quadratic Programming with LINDO, 3rd ed., The Scientific Press, Redwood City, California, 1986.
- 4) Special Report 209: Highway Capacity Manual, TRB, National Research Council, Washington, D.C., 1985.

5) C. Wallace., et al, TRANSYT-7F User's Manual, Transportation Research Center,

University of Florida, Gainesville, Florida, 1988.

APPENDICES

APPENDIX 1. Extended MILP Formulation

MAX K

SUBJECT TO

- 2) X1-10K-I1>=-1
- 3) X2-10K-12>=-1
- 4) X3-10K-I3>=-1
- 5) X4-5K-I4>=-1
- 6) X5-10K-I5>=-1
- 7) X6-10K-16>=-1
- 8) X7-10K-I7>=-1
- 9) X8-10K-18>=-1
- 10) X9-5K-19>=-1
- 11) X10-10K-I10>=-1
- 12) X1 I1 < = 0
- 13) X2 I2 < = 0
- 14) X3 I3 < = 0
- 15) X4 I4 < = 0
- 16) X5 I5 < = 0
- 17) X6 I6 < = 0
- 18) X7-I7<0
- 19) X8- I8<=0
- 20) X9 19 < = 0
- 21) X10 I10 < = 0
- 22) 1440X3+1440X4+114X5-3115+3240K >=101
- 23) 114X5-3115>=0
- 24) 4080X1 + 4080X2 + 408X5 > = 1308
- 25) 1260X8 + 1260X9 + 1082X10 107 I 10 + 3240K > = 127

- 26) 1082X10-107 I 10>=0
- 27) 2720X6 + 2720X7 + 2720X10 > = 316
- 28) 1260X1 + 1260X4 + 680X5 108I5 + 3240K>=110
- 29) 680X5-108 I 5>=0
- 30) 4080X2+4080X3+4080X5>=764
- 31) 1260X6 + 1260X9 + 833X10 174 I 10X+3240K>=57
- 32) 833X10-174 I 10>=0
- 33) 2720X7+2720X8+2720X10>667
- 34) X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + 3K1 + 3K2 + 3K3 + 3K4 + 3K5 + 3K6 + 3K7 + 3K8 + 3K9 + 3K10 = 1
- 35) K1-K-I1>=-1
- 36) K2-K-I2>=-1
- 37) K3-K-I3>=-1
- 38) K4-K-I4>=-1
- 39) K5-K-I5>=-1
- 40) K6-K-I6>=-1
- 41) K7-K-I7>=-1
- 42) K8-K-I8>=-1
- 43) K9-K-I9>=-1
- 44) K10-K-I10>=-1
- 45) K1-I1 < = 0
- 46) K2-I2 < = 0
- 47) K3-I3 <= 0
- 48) K4-I4 < = 0
- 49) K5-I5 < = 0
- 50) K6-I6 < = 0
- 51) K7 I7 < = 0
- 52) K8-I8 < = 0
- 53) K9-I9 < = 0
- 54) K10-I10<-0

- 55) K-0.01667J1-0.01538J2-0.01429J3-0. 01333J4-0.0125J5-0.01176J7-0. 01053J8-0.01J9-0.00952J10-0.00909J11-0.00870J12-0.008333J13=0
- 56) J1+J2+J3+J4+J5+J6+J7+J8+J9+ J10+J11+J12+J13=1
- 57) I1 + I2 + I3 + I4 + I5 + I6 + I7 + I8 + I9 + I10 ≤ 6

END

INTE I1-I10

INTE J1-J13

Appendix 2(A) Result from Existing Signal Timing

(PERFORMANCE WITH INITIAL SETTINGS)

LINK NO.	101	103	104	53	106	107	108	32.7
PHASE LENGTH (SEC)	34	75 P	34	46	ន	æ	9	PI:=
FUEL CONSUM (GA/H)	6.41	1.40 2.61	1,13	9.81	1.19	7.05	2.98	32.66
QUEUE CAPACITY (VEH/LK)	0	>	0	0	0	0	0	
MAX BACK OF QUEUE (VEH/LK)	14 >	^ ^ ~ c		^ 83	5	15 >	7 >	
UNIFORM STOPS (VEH/H:%)	525.9(79%)	218.8(69%)	54.3(95%)	899.8(69%)	89.0(81%)	579.7(76%)	(%96)6.96	2571.2(75%)
AVERAGE DELAY (SEC/VEH)	25.8	21.8	71.8	18.2	31.0	24.7	119.1	26.3
TOTAL	4.79	7 6	1.14	9.90	왕	5.25	3.34	25.19
DELAY ANDOM H/H)	92; 53	3.5	55.	87	%	.12	2.18	3.58
UNIFORM RANDOM (VEH-H/H)	4.53	3.8	3 6	6.39	6	5.13	1.16	21.61
TOTAL TOTAL VEH-H/H)	4.79	121	1.14	9.90	Ŗ	5.25	3.34	25.19
TOTAL TRAVEL (VEH-MI/H) (8.8	3.8	8.	8	8	8.	8;	00:
DEGREE OF SAT (%)	ខេ	25 65	92	22	æ	49	* 36	* 98
SAT 1 FLOW (VEH/H)		3200	0	4800	1600	4800	1600	MAX=
FLOW (VEH/H) (299	316	27	1308	110	764	101	3450
LINK NO.	101	32	2	105	99	107	108	
NO.	 ,	,				-		끋

(SYSTEM WIDE TOTALS INCLUDING ALL LINKS)

H	(M/H)	.00(TOTALS)
PERFORMANCE	S	32.66
OPERATING	3	98.74
TOTAL FUEL	(GA/H)	32.66
TOTAL	(%-H/HA)	2571.21(75%)
AVERAGE DELAY	(SEC/VEH)	26.28
TOTAL DBAY	(VEH-H/H)	25.19
TOTAL RANDOM	(VEH-H/H)	3.58
TOTAL	(VEH-H/H)	21.61
TOTAL TRAVEL	(VEH-H/H)	25.19
TOTAL	(VEH-MI/H)	00.

NOTE: PERFORMANCE INDEX IS DEFINED AS:

PI=DELAY+STOPS

NO. OF SIMULATIONS=3 NO. OF LINKS=16 ELAPSED TIME=291.1 SEC.

Appendix 2(B) Result from Extended Formulation

(PERFORMANCE WITH INITIAL SETTINGS)

LINK NO.	101	102	103	104	55	901	107	8	29.6
PHASE LENGTH (SEC)	<u>«</u>	2 ∞	281	18	83	:E	53	13	PI≔
FUEL CONSUM (GA/H)	7.67	1.79	2.70	1.43	9.78	1.12	4.63	1.01	29.62
QUEUE CAPACITY (VEH/LK)	0	0	0	0	0	0	0	0	
MAX BACK OF QUEUE (VEH/LK)	^ =	. ^ ~	4	^-	17 >	. ^	Оп	· ^	
UNIFORM STOPS (VEH/H:%)	605.7(91%)	115.7(91%)	251.4(80%)	52.9(93%)	963.4(74%)	94.7(86%)	487.9(64%)	86.8(86%)	2658.4(77%)
AVERAGE DELAY (SEC/VEH)	31.8	44.4	20.5	97.7	14.9	26.5	12.4	25.9	21.3
TOTAL	5.90	1.57	8:1	1.55	5.41	æ:	2.64	.73	20.39
DELAY RANDOM H/H)	1.77	.73	89.	1.11	:3	01.	99.	89.	4.22
UNIFORM R (VEH-I	4.13	\$.	1.72	44	5.10	.71	2.58	:65	16.16
TOTAL TIME (VEH-H/H)	2:30	1.57	1.80	1.55	5.41	∞;	2.64	.73	20.39
TOTAL TRAVEL (VEH-MI/H)	8:	8.	8.	8.	8:	8.	8.	8.	90:
DEGREE OF SAT (%)	88	8	45	82	92	46	œ	45	68
SAT FLOW (VEH/H)	3200	0	3200	0	4800	1600	4800	1600	MAX=
		~	9,	<u>-</u>	∞ :	요 :	79	5	_
FLOW (VEH/H)	299	15	₩,		<u> </u>	- '	_		3450
	101 667								3450

(SYSTEM WIDE TOTALS INCLUDING ALL LINKS)

H	(M /H)	.00(TOTALS)
PERFORMANCE	2	29.62
OPERATING	<u> </u>	93.43
TOTAL FUEL	(GA/H)	29.62
TOTAL	(%-H/H-N)	2658.4(77%)
AVERAGE DELAY	(SEC/VEH)	21.27
TOTAL	(VEH-H/H)	20.39
TOTAL RANDOM	(VBH-H/H)	4.22
TOTAL UNIFORM DELAY	(VEH-H/H)	16.16
TOTAL TRAVE	(VBH-H/H)	20.39
TOTAL DISTANCE TRAVE ED	(VEH-M/H)	00.

NOTE: PERFORMANCE INDEX IS DEFINED AS:

PI=DELAY+STOPS

NO. OF SIMULATIONS=3 NO. OF LINKS=16 ELAPSED TIME=290.4 SEC.