□論 文□

교통망 평형 조건하에서 링크 교통량 자료를 이용한 기종점 통행표 추정방법에 관한 연구

Estimation of Trip Matrices from Traffic Counts : An Equilibrium Approach

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目 次

국문 요약

교통수요는 교통정책 및 교통시설 계획의 수립 및 평가에 중요한 영향을 미치게 되므로, 교통수요의 예측은 교통연구에서 중요한 부문을 차지하고 있다. 도로밑에 설치된 전자차량감지기(Electronic Vehicle Detector)로부터 자동 수집된 링크 교통량 자료(Traffic Counts)를 주요 입력자료로 이용하여 계획 지역의 기종점 통행표(Origin Destination Trip Matrix)를 작성할 수 있는 기법들이 최근 수년동안 많이 발달하게 되었다. 이러한 새로운 기법들은 가구조사(Home Inteview), 노변면접조사(Road－Side Interview)등을 통하여 조사된 자료를 기초로하는 전통적인 4단계 교통수요추정방법(Conventional 4－Stage Estimation Method)－통행발생(Generation), 통행분포(Distribution), 수단선택(Modal Split), 교통배분(Assignment)－과 비교하여 첫째로 정확도가 높은 링크 교통량 자료를 별도의 조사를 거치지 않고서도 수집이 가능하기 때문에 조사비용이 거의 들지 않아도 되어 경제적이고, 둘째로 전통적인 수요예측방법들에서 요구되어지는 복잡한 모형수립 및 계수조정(Parameter Calibration)이 필요하지 않아 간편하고, 셋째로 오래 전에 작성된 기종점 통행표를 단순히 링크 교통량 자료만을 이용하여 쉽게 보완할 수 있어 지속적인 자료의 축적(Data Ageing)이 가능하며 더 나아가서 소위 연속적인 교통계획 및 교통시설관리(Continuous Transport Planning and Management)를 가능케 하는 등의 여러 장점때문에 많은 주목을 받아오고 최근 몇년간 꾸준히 실무에 유용하게 적용이 되고 있는 실정이다.

본 연구는 링크 교통량 자료를 이용하여 기종점 통행표를 작성하기 위하여 개발된 기존의 여러 기법들 가운데, 특히 용량제약조건(Capacity－Restrained Condition)하에서 기존의 방법들을 상호 검토한 후

Wardrop의 교통망 평형 원칙(Wardrop's First Network Equilibrium Principle)을 만족하는 새로운 추정 기법을 제의하고 이의 시험 결과를 논의하는 것을 주요 내용으로 한다.

링크 교통량 자료를 이용하여 기종점 통행표를 작성하는 기법들의 근본 목표는 조사된 링크 교통량(Observed Traffic Counts)에 가장 근접한 교통망 통행 배정 링크 교통량(Assigned Link Volumes)을 재현(Reproducing)할 수 있는 기종점 통행표들 중에서 최적의 기종점 통행표를 발견하는 것이다. 따라서, 교통망에서 통행자의 여행 경로 배정을 가장 잘 반영할 수 있는 현실적인(Realistic) 교통망 통행 배정 모형(Network Traffic Assignment Model)의 선택은 중요한 요소가 되며, 특히 교통망에 교통체증(Traffic Congestion)이 심할 경우 교통망 통행자 평형 조건(Network Traffic Equilibrium Condition)을 고려하기 위한 특별한 처리가 요구되어진다.

본 연구는 Willumsen(Hall, Van Vliet and Willumsen, 1980)에 의하여 개발된 ME2(Maximum Entropy Matrix Estimation)기법에서 반복식 추정 방법(Sequential Estimation Method)을 사용할 경우 Wardrop의 평형조건을 만족하는 기종점 통행표를 구할 수 없다는 단점을 극복하기 위한 방안으로서 엔트로피 극대화 문제와 교통망 평형 조건(Entropy Maximisation and Network Equilibrium Condition)의 두 문제를 동시에 해결할 수 있는 새로운 수식모형과 이를 풀기위한 알고리즘(Simultaneous Solution Algorithm)을 제의하였다. 제의된 수식모형과 알고리즘은 예제 교통망(Example Network)을 이용하여 시험하고 그 결과를 ME2의 반복식 추정 방법으로부터 구한 기종점 통행표와 비교 검토하였다.

1. INTRODUCTION

In urban traffic management and planning, an important problem is how to obtain origin–destination trip matrices using the low–cost data such as traffic counts. Although the Conventional method, using the data from the direct Surveys can be used to estimate trip matrices, they appear to be inaccurate and expensive. By contrast, the use of traffic counts is attractive, as it is less expensive and more practical.

This paper examines the problem of estimating of trip matrices from observed link flows (traffic counts) when congestion effects in networks are considered. The method of estimating trip matrices from traffic counts by maximising a measure of entropy, subject to assigned link flows reproducing observed ones, is now well established [10]. The method (ME2) was initially developed by assuming fixed route choice proportions. Afterwards, Hall, Van Vliet and Willumsen [5] extended it to use equilibrium assignment. However, the sequential method devised to solve the extended problem is only a heuristic, as it solves the two sub–problems of equilibrium assignment and entropy maximization alternately. Willumsen [13] and Fisk [3] noted that the sequential solution method might fail either to converge or to estimate optimal solutions. As an alternative, Fisk [3] proposed a formulation to solve the two sub–problems simultaneously. It incorporated the equilibrium conditions as a constraint in the ME2 model by adopting the form of the variational inequality. However, Fisk's formulation might have no feasible solution due to inconsistencies in the observed flows.

Other equilibrium based approaches to the problem include Nguyen's model [7] and Fisk and Boyce's model [4]. Nguyen's model has the Same form of a traffic assignment problem with elastic demand and it uses a set of the interzonal travel costs as the input data which may be obtained from traffic counts Fisk and Boyce's model is an extension of a doubly constrained gravity model whose applications may not be suitable for urban transport studies. These two models are distingulshed from the ME2 model in a sense that they are based on the different level of the detail in the input data.

2. THE ESTIMATION PROBLEM

2. 1. Notation

I is the set of observed links in the network

P^a_{ij} is the proportion of trips from i to j using link a

$\underline{P}$ is the vector of P^a_{ij}

tij is the prior number of trips between zone i and zone j

Tij is the number of trips between zone i and zone j

$\overline{V}$ is the observed flow on link a

$V^*_a(\underline{T})$ is the modelled flow on link a using trip matrix $\underline{T}$

μ_n is the penalty parameter (n=1, 2, ·····).

A prior matrix $\underline{t}$ can be obtained from out-dated trip matrices(For further discussions, see Willumsen(1981)). When no prior information is available, one could plausibly set tij=1 for all i and j.

A gap function $G(\underline{T}, \overline{V})$ is used to messure the discrepancy between observed and modelled link flows :

$$G(\underline{T}, \underline{\overline{V}}) = \sum_{a \in I} (V^*_a(\underline{T}) - \overline{V}_a)^2$$

Two measures of entropy are used to evaluate a matrix :

$$S_0(\underline{T}, \underline{t}) = Log_e T, 1 - \sum_{i,j} T_{ij}(Log_e(T_{ij}/t_{ij}) - 1 + Log_e t_{..})$$

Where $T_{..} = \sum_{i,j} T_{ij}$ and $t_{..} = \sum_{i,j} t_{ij}$

$$S_1(\underline{T}, \underline{t}) = -\sum_{i,j} T_{ij}(Log_e(T_{ij}/t_{ij}) - 1)$$

2. 2. Trip matrix estimation from traffic counts

The problem of estimating trip matrices from traffic counts is to find a trip matrix that reproduces the observed link flows when reassigned to the network. However, the problem is normally under-specified and the solution set is infinite. Two main approaches to tackle this under-specification problem are known. The first one is to assume that a synthetic model —the gravity model or opportunity model—is capable of explaining the real trip making behaviour. In this case, observed link flows are used to calibrate the parameters of synthetic models. The second one uses entropy maximization (or information minimization) principles to explain real trip making behaviour. In this case, the most likely trip matrix is identified as the one having the greatest entropy value (or least additional information), For a comprehensive review, see [1], [8] and [12].

Traffic assignment methods can be used in order to associate observed link flows to estimated trip matrices. Traffic assignment methods may be classified into two main groups, i.e., proportional assignments and capacity restrained ones. Proportional assignment methods are simpler, but are unrealistic for congested networks. Capacity restrained assignment methods are more realistic but require more computing time. Equilibrium assignment satisfying the Wardrop's First Principle [11] is the prime example of this. However, modelled link flows calculated from traffic assignment may not be close to the real ones. Moreover, the observed link flows are not error free.

3. SIMULTANEOUS ESTIMATION OF TRIP MATRICES

3. 1. Formulation

A new formulation has been proposed to solve entropy maximization and equilibrium assignment simultaneously [9]. The proposed formulation was :

$$\text{P1} \quad \max_{\underline{T}} S_0(\underline{T}, \underline{t}) \tag{3.1}$$

$$\text{s. t. } V^{*}_{a}(\underline{T}) = \overline{V}_{a}, \ a \epsilon I \tag{3.2}$$

Problem P1 is a single optimization problem containing both matrix estimation and equilibrium assignment. It uses equilibrium link flows in the constraints rather than route choice proportions. Also, Problem P1 uses an entropy function $S_0(\underline{T}, \underline{t})$ Which does not assume that the total demand of the estimated trip matrix is fixed, whereas the ME2 model uses the entropy function $S_1(\underline{T}, \underline{t})$ Which does assume that the total demand is fixed.

3. 2. Solution method

Problem P1 is an optimization problem with a non-linear objective function and non-convex constraints. The equilibrium link flows, $V^{*}_{a}(I)$, are found only by solving equilibrium assignment problems.

3. 2. 1 Use of the penalty function method

The penalty function method approximates constrained optimization problems by solving a sequence of unconstrained ones. The approximation is accomplished by adding to the objective function a penalty term that prescribes a high cost for violation of the constraints. The use of the penalty function method is useful in solving problems such as Problem P1 where derivatives are not available [6].

Problem P1 is transformed into an unconstrained problem using the gap penalty function $G(\underline{T}, \overline{\underline{V}})$ and the penalty parameter $\mu_n(n=1, 2, ...)$, where μ_n is negative and decreasing in n:

$$\text{P2} \quad \max_{\underline{T}} S_0(\underline{T}, \underline{t}) + \mu_n G(\underline{T}, \overline{\underline{V}}) \tag{3.3}$$

where $G(\underline{T}, \overline{\underline{V}}) = \sum_{a\epsilon I} (V^{*}_{a}(\underline{T}) - \overline{V}_{a})^2$

The gap penalty function satisfies the properties required by the penalty function : (1) $G(\underline{T}, \overline{\underline{V}})$ is continuous, (2) $G(\underline{T}, \overline{\underline{V}}) \geq 0$ for all $\underline{T}$, and (3) $G(\underline{T}, \overline{\underline{V}}) = 0$ iff $\underline{T}$ is freasible. Thus, as the penalty parameter, μ_n, decreases sequentially, the solution points will converge to a solution which is also optimal for the original problem P1.

Problem P2 is one of sequential unconstrained non-linear maximization problems. It can be shown that the entropy function $S_0(\underline{T}, \underline{t})$is convex in each T_{ij} individually, although it is not convex for all dimensions simultaneously. It might be solved by uni-dimensional line search, but that would require many calculations of equilibrium assignment. Furthermore, it is not known whether the gap penalty function $G(\underline{T}, \overline{\underline{V}})$ is convex in each T_{ij}.

3. 2. 2 Linear approximation of equilibrium link flows

Let $\underline{V}^*(\underline{T})=\underline{P}(\underline{T})\cdot\underline{T}$ be the equilibrium link flows generated by the trip matrix $\underline{T}$. Let δT_{ij} be a non−negative element of $\delta\underline{T}$. Then, applying Taylor's formula, we have the following polynomial expression :

$$\underline{V}^*(\underline{T}+\delta\underline{T})$$
$$=\underline{P}(\underline{T})\cdot\underline{T}+(\underline{T}\cdot\frac{\partial\underline{P}}{\partial\underline{T}}+\underline{P}(\underline{T}))\cdot\delta\underline{T}+\underline{R}_2 \qquad (3.4)$$

where $\underline{R}_2$ is second order in $\delta\underline{T}$.

If δT_{ij} is small enough, the error term $\underline{R}_2$ can be ignored. Then, Equation (3.4) becomes :

$$\underline{V}^*(\underline{T}+\delta\underline{T})$$
$$=\underline{P}(\underline{T})\cdot\underline{T}+(\underline{T}\cdot\frac{\partial\underline{P}}{\partial\underline{T}}+\underline{P}(\underline{T}))\cdot\delta\underline{T} \qquad (3.5)$$

$$\text{so, } V_a^*\ (\underline{T}+\delta\underline{T})=\alpha_a+\beta_a\delta T_{ij},\ a\ \varepsilon\ I \qquad (3.6)$$

Equation (3.6) is a linear function in δT_{ij}. The coefficients, $\{\alpha_a\}$ and $\{\beta_a\}$, can be estimated by using the least squares estimation method over some pre−determined set of trip matrices $\underline{T}+\delta\underline{T}$ and modelled link flows $\underline{V}^*(\underline{T}+\delta\underline{T})$. This is done for each T_{ij}. It allows relatively few equilibrium assignments for evaluating the objective function for each T_{ij}.

Replacing the constraints of Problem P1 with Equation (3.6), we have the following modified problem :

P3

$$\underset{\delta\underline{T}}{\text{Max}}\ S_0(\underline{T}+\delta\underline{T},\underline{t}) \qquad (3.7)$$

$$\text{s. t. } \alpha_a+\beta_a\delta T_{ij}=\overline{V}_a,\ a\ \varepsilon\ I \qquad (3.8)$$

Applying the penalty function method, we have :

P4

$$\underset{\delta\underline{T}}{\text{Max}}\ S_0(\underline{T}+\delta\underline{T},\underline{t})+\mu_n G(\underline{T}+\delta\underline{T},\underline{\overline{V}}) \qquad (3.9)$$

where $G(\underline{T}+\delta\underline{T},\underline{\overline{V}})=\sum_{a\in I}(\alpha_a+\beta_a\delta T_{ij}-\overline{V}_a)^2$

Problem P4 is non−linear but is convex for unidimensional line search. It can be solved by performing a sequence of uni−dimensional line searches using methods such as the golden section search or direct search for roots of the partial derivatives.

3. 2. 3 Use of the extrapolation method

For the sake of further reducing the computing complexity two heuristic methods are adopted in the solution method. The first one is the use of an extrapolation method developed by Fiacco and McCormick [2]. This method is based on the existence of a unique trajectory of unconstrained local maxima when using the penalty function method. It uses the previous information on such a trajectory to estimate the solution to the next sub−problem and the final solution. In particular, the extrapolation method is used to eastablish the Starting point and the initial search interval for the local maximum of the next sub−problem.

3. 2. 4 Use of the perturbation method

The second heuristic is the use of a perturbation method in solving the equilibrium assignment problem. This method calculates freasible link flows, $\underline{V}(\underline{T}+\delta\underline{T})$, for a perturbation $\delta\underline{T}$ to the trip matrix $\underline{T}$ from the previous equilibrium link flows, $\underline{V}^*(\underline{T})$, rather than performing an equilibrium assignment from the beginning. The previous route choice proportions of trips between zone i and zone j using particular link a, P_{ij}^a, are used as follows.

$$\underline{V}(\underline{T}+\delta\underline{T})=\underline{V}^*(\underline{T})+\underline{P}\cdot\delta\underline{T} \qquad (3.10)$$

where $\underline{V}(\underline{T}+\delta\underline{T})$ is a vector of perturbed link flows.

Starting with perturbed link flows, some iterations of the usual Frank−Wolfe equilibrium, assignment procedure are performed to yield new equilibrium link flows, $\underline{V}^*(\underline{T}+\delta\underline{T})$.

3. 2. 5 Computational demand of the simultaneous method

In spite of the use of the extrapolation method and perturbation method, the simultaneous method still has an uncertainty about the practicality in computing for large network problems. However, computing capability is improving rapidly, and this advance in computing technology will permit the simultaneous method to be more practical in the near future.

4. TEST

A test using a small artificial network was designed in order to investigate the performance of the simultaneous estimation method. The main objective of the test is to see how accurately the simultaneous method estimates a known trip matrix using a prior trip matrix and a set of observed link flows. Also, the trip matrices estimated by the simultaneous methods are compared with those estimated by the sequential one.

The artificial network used in this test as shown in Figure 1 is based on the test network used by Nguyen [2]. A known trip matrix with elements−(1, 3), (1, 4), (2, 3) and (2, 4)−all equal to 700 was selected and observed link flows were generated from this by equilibrium assignment. As shown in Figure 1, five links−(6, 7), (6, 10), (9, 10), (9, 13), and (12, 8)−were observed. Three different prior trip matrices were used. The first prior trip matrix was uniformly scaled from the known trip matrix. The second one was selected to be close to the known trip matrix. The third one was chosen to be rather different from the known trip matrix.

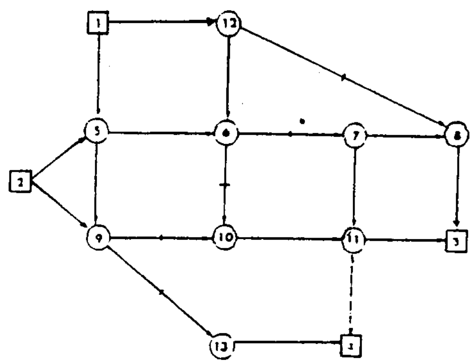

Figure 1 : A test network

Figures 2 and 3 show progress of the estimation process by the simultaneous method. Figure 2 shows that as the value of the Penalty parameter decreases, both gap values and entropy values gradually decrease. Figure 3 shows the trade−off between gap values and entropy values. The trade−off curve could be a useful practical tool, because it allows the selection of the estimated trip matrices to be controlled depending on the relative accuracy of the prior trip matrices and observed link flows that are input.

Table 1 summarises the results of estimated trip matrices, root mean square error(RMSE) values between the estimated and the known matrix, gap values and entropy values for each combination of the three prior trip matrices and three solution methods : SM−S_0 is the simultaneous method using the objective function $S_0(\underline{T}, \underline{t})$, SM−$S_1$ is the simultaneous method using the objective function $S_1(\underline{T}, \underline{t})$ and ME2 is the sequential method. It shows that with all three prior trip matrices, the simultaneous methods approximate the known trip matrix closely. It shows that the sequential method of ME2 estimates the known trip matrix reasonably with the second prior trip matrix. However, with the others, the sequential method gives solutions which are rather different from the known trip matrix.

Thr trip matrices estimated by the sequential method have lower entropy values and higher gap values than those estimated by the simultaneous ones.

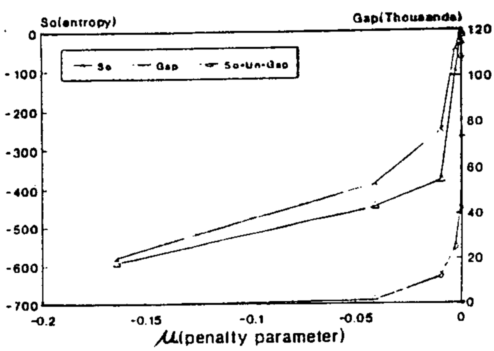

Figure 2 : Progress of gap and entropy values with decreasing penalty parameter values

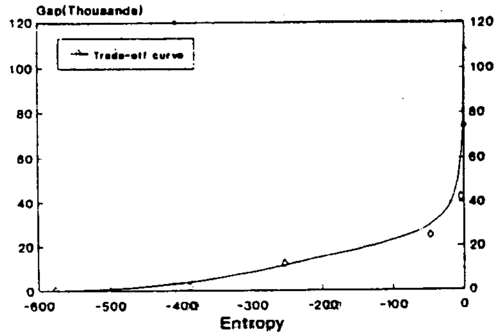

Figure 3 : A trade-off curve between gap and entropy values

5. CONCLUSIONS

This paper examined the problem of estimating trip matrices from observed link flows under equilibrium traffic assignment conditions. A new formulation and solution method has been presented which solves entropy maximization and euqilibrium, assignment simultaneously. The main difference between this method and the sequential method is that the estimation problem is solved

Table 1 : Estimates with different solution methods and different prior trip matrices

Solution Methods	Estimated Trip Matrices T_{13}	T_{14}	T_{23}	T_{24}	T..	RMSE	Gap Values	Entropy $S_0(\underline{T},\underline{t})$	Values $S_1(\underline{T},\underline{t})$
Prior 1	200	200	200	200	800	500	907539	0	800
SM-S_0	700	697	697	698	2792	2	278	0	-698
SM-S_1	757	647	655	741	2800	49	80	-7	-714
ME2	1049	181	271	1170	2671	446	5623	-649	-1198
Prior 2	600	550	800	650	2600	106	22806	0	2600
SM-S_0	726	666	702	704	2798	22	199	-25	2568
SM-S_1	735	658	674	728	2795	33	33	-32	2561
ME2	937	480	520	896	2833	209	460	-173	2417
Prior 3	350	250	170	50	820	507	900585	0	820
SM-S_0	733	694	678	694	2799	20	89	-577	-1215
SM-S_1	760	648	643	749	2800	55	63	-650	-1289
ME2	1087	363	455	946	2851	308	2325	-1043	-1745

without using route choice proportions.

Using a small artifical network, it has been shown that the simultaneous method estimates trip matrices which maximize the entropy measure as well as reproducing observed link flows to a good degree of accuracy.

On the other hand, the results showed that the sequential method fails to estimate a trip matrix which is close to the known trip matrix. This is due to the use of the inaccurate route choice proportions during the matrix estimation.

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