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ALMOST COSYMPLECTIC MANIFOLDS WITH VANISHING CONTACT CONFORMAL CURVATURE TENSOR FIELD

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1. Introduction

The results of the study of a Sasakian manifold with vanishing contact Bochner curvature tensor had been listed in [5] and [6]. In [2] and [3], almost cosymplectic manifold with vanishing contact Bochner curvature tensor was discussed. In 1990, Kitahara, Matsuo and Pak [8,9] defined the so-called conformal curvature tensor field on a hermitian manifold which is conformally invariant, and nearly kaehlerian manifold with vanishing conformal curvature tensor field was discussed by present author [11]. Furthermore, Jeong, Lee, Oh and Pak [7] defined the so-called contect conformal curvature tensor field on Sasakian manifold, which is constructed from the conformal curvature tensor field by the Boothby-Wang's fiberation [1]. In the present paper, we shall study almost cosymplectic manifold with vanishing contect conformal curvature tensor field and prove the following theorem :

THEOREM A. *An almost cosymplectic manifold with vanishing contect conformal curvature tensor field does not exist.*

We shall be in C^∞-category. Latin indices run from 1 to $2n+1$. Einstein summation convention will be used.

2. Preliminaries

Let (M, ϕ, ξ, η, g)be a $(2n+1)$-dimensional almost contact metric manifold, that is, M is a manifold covered by a system of coordinate

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neighborhoods $\{U;x^h\}$ and (ϕ,ξ,η,g) an almost contect metric structure on M, formed by tensors of type $(1,1),(1,0)$ and $(0,1)$, respectively, and a Riemannian metric g such that

$$\begin{aligned}\phi_j^i\phi_i^h &= -\delta_j^h+\eta_j\xi^h, \phi_i^h=0, \eta_i\phi_j^i=0,\\ \eta_i\xi^i &= 0, g_{ts}\phi_j^t\phi_i^s = g_{ji}-\xi_j\xi_i, \xi_i = g_{ih}\xi^h\end{aligned} \tag{2.1}$$

On such a manifold we may always defined a 2-form by $\phi(X,Y)=g(\phi X,Y)$ [10]. (M,ϕ,ξ,η,g) is said to be *an almost cosympectic manifold* if the forms ϕ and η are closed, i.e., $d\phi=0$ and $d\eta=0$, where d is the operator of exterior differentiation (cf. [4]). Let M be an $(2n+1)(n\geq 1)$-dimensional almost cosymplectic manifold. Then we can consider the contact conformal curvature tensor field C_o on M (the same definition as the contact conformal curvature tensor field in [7]).

$$\begin{aligned}C_{o,kjih} = R_{kjih} &+ \frac{1}{2n}(g_{kh}R_{ji} - g_{jh}R_{ki} + R_{kh}g_{ji}\\ &- R_{jh}g_{ki} - R_{kh}\eta_j\eta_i + R_{jh}\eta_k\eta_i - \eta_k\eta_h R_{ji}\\ &+ \eta_j\eta_h R_{ki} - \phi_{kh}R_{ji} + \phi_{jh}S_{ki} - S_{kh}\phi_{ji}\\ &+ S_{jh}\phi_{ki} + 2\phi_{kj}S_{ih} + 2S_{kj}\phi_{ih})\\ &+ \frac{1}{2n(n+1)}(2n^2-n-2+\frac{(n+2)s}{2n})(\phi_{kh}\phi_{ji}\\ &- \phi_{ki}\phi_{jh} - 2\phi_{kj}\phi_{ih})\\ &+ \frac{1}{2n(n+1)}(n+2-\frac{(3n+2)s}{2n})(g_{kh}g_{ji}-g_{ki}g_{jh})\\ &- \frac{1}{2n(n+1)}(4n^2+5n+2-\frac{(3n+2)s}{2n})(g_{kh}\eta_j\eta_i\\ &- g_{ki}\eta_j\eta_h + \eta_k\eta_h g_{ji} - \eta_k\eta_i g_{jh}),\end{aligned} \tag{2.2}$$

where (ϕ,ξ,η,g) denotes the almost cosymplectic structure, R_{kjih}, R_{ji} and S are Riemannian curvature tensor, Ricci tensor and scalar curvature of M, respectvely, and $S_{ji}=\phi_j^t R_{ti}$.

LEMMA 2.1. ([10]) *If M is an (2n+1)-dimensional almost cosymplectic manifold, then it holds that*

$$R_{ji}\eta^j\eta^i + |\nabla_j\eta^i|^2 = 0, \tag{2.3}$$

where ∇_j *denote the operator of covariant differentiation with respect to* g_{ji}.

3. Proof of Theorem A

Assume that there is an $(2n+1)(n \geq 1)$ -dimensional almost cosymplectic manifold with vanishing contact conformal curvature tensor field. Then, from (2.2) we have

$$
\begin{aligned}
(3.1)\quad R_{ji} = &-\frac{1}{2n}\{(2n-2)R_{ji} + sg_{ji} - s\eta_j\eta_i + R_j^t\eta_t\eta_i \\
&+ \phi_j^t S_{ti} + S_j^t\phi_{ti} + 2\phi_{tj}S_i^s + 2\phi_{tj}\phi_i^t\} \\
&+ \frac{1}{2n(n+1)}\{2n^2 - n - 2 + \frac{(n+2)s}{2n}\}(\phi_j^t\phi_{ti} + 2\phi_{tj}\phi_i^t) \\
&- \frac{1}{n+1}\{n + 2 - \frac{(3n+2)s}{2n}\}g_{ji} \\
&+ \frac{1}{2n(n+1)}\{4n^2 - 5n - 2 + \frac{(3n+2)s}{2n}\} \\
&\times \{(2n-1)\eta_j\eta_i + g_{ji}\}.
\end{aligned}
$$

Thus we have

$$
(3.2)\qquad R_{ji}\eta^j\eta^i = 2n.
$$

Hence, Theorem A has been proved from Lemma 2.1.

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