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ON SEVERAL CONTINUOUS FUNCTIONS ON FUZZY CONVERGENCE SPACES

HYO IL CHOI

1. Introduction

The convergence function between the filters on a given set S and the subsets of S was introduced by D.C. Kent ([12]) in 1964 and it may be regarded as a generation of a topological space and further studied by many authors.

After Zadech created fuzzy sets in his classical paper ([14]), Chang ([5]) used them to introduce the concept of a fuzzy sets using metric defined as the Hausdorff metric between the supported endographs. Recently, B.Y. Lee and J.H. Park ([16]) defined a new structure, called by fuzzy convergence structure, using prefilter.

In this paper, we define new the several continuous functions between the fuzzy convergence spaces, that is, fuzzy super continuity, fuzzy δ-continuity, and fuzzy weakly δ-continuity, and introduce the relationships between them. And we introduce the concepts of initial fuzzy convergence structures and product fuzzy convergence spaces and investigate their properties.

2. Preliminaries

The reader is asked to refer to [14], [5], [19], [21] and [22], for fuzzy sets and fuzzy topological spaces.

Let X be a nonempty set and I the unit closed interval I=[0,1]. A fuzzy set A in X is an element of the set $F(X)$ of all functions from X into I and the elements of $F(X)$ are called fuzzy subsets ([14]). For fuzzy set A and B in X, $A \subseteq B$ if $A(x) \leq B(x)$ for all x in X. The symbol ϕ is used to denote the empty fuzzy set $\phi(x) = 0$ for all $x \in X$ and for X we have the definition $X(x) = 1$ for all $x \in X$. A fuzzy

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point p in X is fuzzy set in X defined by $p(x) = \lambda$ $(0 < \lambda \leq 1)$ for $x = x_p$ and $p(x) = 0$ for $x \neq x_p$. Then, we call x_p the support of p and λ the value of p. A fuzzy point $p \in A$, where A is a fuzzy set in X, if $p(x_p) \leq A(x_p)$.

A fuzzy point p is said to be quasi coincident with A, denoted by pqA, if $p(x_p) + A(x_p) > 1$ for a fuzzy point p and a fuzzy set A (see in [21]). A fuzzy set A is said to be quasi coincident with a fuzzy set B, denoted by AqB, if there exists some x in X such that $A(x)+B(x) > 1$.

A fuzzy topology is a family τX of fuzzy sets in X which satisfies the following conditions ([5]):
(1) $\phi, X \in \tau X$,
(2) $A, B \in \tau X$ implies that $A \cap B \in \tau X$,
(3) $A_i \in \tau X$ for each $i \in I$ implies that $\bigcup_i A_i \in \tau X$.
τX is called a fuzzy topology for X and the pair $(X, \tau X)$ is fuzzy topological space (fts in short). Every member of τX is called a τX-open fuzzy set. A fuzzy set is τX-closed if its complement is τX-open. A fuzzy set A is a Q-neighborhood of p (in short Q-nbd) if there exists some B in τX so that $B \leq A$ and pqB.

Let f is a function from a set X into a set Y and A, B be the fuzzy sets in X, Y, respectively. Then we define $f^{-1}(B)$ and $f(A)$ as follows:

$$f^{-1}(B)(x) = B(f(x)) \quad \text{and}$$
$$f(A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} A(x), & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{otherwise.} \end{cases}$$

A function f is said to be fuzzy continuous if $f^{-1}(B) \in \tau X$ whenever $B \in \tau Y$, where τX and τY are the fuzzy topologies on X and Y, respectively.

3. Fuzzy convergence spaces

In this section, we introduce fuzzy convrgence spaces using prefiters, and we define the set functions Γ_C, I_C and introduce their properties.

DEFINITION 3.1. ([4]) A *prefilter* on X is a nonempty *subset* Φ of the set I^X of functions from X into closed interval $I = [0, 1]$ with the properties:

(1) If $A, B \in \Phi$, then $A \cap B \in \Phi$.
(2) If $A \in \Phi$ and $A \subseteq B$, then $B \in \Phi$.
(3) $\phi \notin \Phi$.

If Φ and Ψ are prefilters on X, Φ is said to be *finer* than Ψ (Ψ is *coarser* than Φ) if and only if $\Psi \subseteq \Phi$. A prefilter Φ on X is said to be *ultra prefilter* if it is no other prefilter finer than Φ (i.e., it is maximal for the inclusion relation among prefilters).

A *prefilterbase* on X is the nonempty subset β of I^X with the properties:
(1) If $A, B \in \beta$, there exists $C \in \beta$ such that $C \subseteq A \cap B$.
(2) $\phi \notin \beta$.

If β is a prefilterbase then $< \beta >= \{A \in I^X : B \subseteq A$ for some $B \in \beta\}$ is a prefilter. If $< \beta >= \Phi$, we say that β is a prefilterbase for the prefilter Φ, or that β generates Φ.

We define convergence structure by prefilter, called *fuzzy convergence structure*. For nonempty universal set X, $P(X)$ denotes the set of all prefilters on X and $F(X)$ the set of all fuzzy sets on X. For each fuzzy point p in X, $\dot{p}$ is denoted by

$$\{A \in I^X : pqA\}$$

Let f be a function from X into Y. Then for a fuzzy point p in fuzzy set A in X, $f(p) \in f(A)$ and for two prefilters $\mathcal{F}, \mathcal{G}$ on X, $f(\mathcal{F} \cap \mathcal{G}) = f(\mathcal{F}) \cap f(\mathcal{G})$ and so $f(\mathcal{F} \cap \dot{p}) = f(\mathcal{F}) \cap f(\dot{p})$ and $\dot{f}(p) = f(\dot{p})$. For a fuzzy prefilter $\mathcal{F}$ on X, $f(\mathcal{F})$ is said to be the prefilter on Y generated by $\{f(A) : A \in \mathcal{F}\}$.

DEFINITION 3.2. ([4]) A *fuzzy convergence structure* on X is a function C_X from $P(X)$ into $F(X)$ satisfying the following conditions:
(FC1) For each fuzzy point p in X, $p \in C_X(\dot{p})$.
(FC2) For $\Phi, \Psi \in P(X)$, if $\Phi \subseteq \Psi$ then $C_X(\Phi) \subseteq C_X(\Psi)$.
(FC3) If $p \in C_X(\Phi)$, then $p \in C_X(\Phi \cap \dot{p})$.

Then the pair (X, C_X) is said to be *fuzzy convergence space*. If $p \in C_X(\Phi)$, we say that Φ *C_X-converges* to a fuzzy point p. The prefilter $\mathcal{V}_{C_X}(p)$ obtain by intersecting all prefilters which C_X-converge to p is said to be the *C_X-neighborhood prefilter* at p. If $\mathcal{V}_{C_X}(\mathrm{p})$ C_X-

converges to p for each fuzzy point p in X, then C_X is called a fuzzy pretopological structure, and (X, C_X) a *fuzzy pretopological space.* The fuzzy pretopological structure C_X is said to be *fuzzy topological structure* and (X, C_X) is said to be *fuzzy topological space*, if for each fuzzy point p in X, the prefilter $\mathcal{V}_{C_X}(p)$ has a prefilterbase $\beta_{C_X}(p) \subseteq \mathcal{V}_{C_X}(p)$ with the following property:

$$rq \sqcup \in \beta_{C_X}(p) \qquad \text{implies} \qquad \sqcup \in \beta_{C_X}(r)$$

Throughout this paper, let $C(X)$ be the set of all fuzzy convergence structures on X. Then we define that $C_1 \leq C_2$ for $C_1, C_2 \in C(X)$ if and only if $C_2(\Phi) \subseteq C_1(\Phi)$ for all $\Phi \in P(X)$. If $C_1 \leq C_2$ for $C_1, C_2 \in C(X)$, we say that C_2 is *finer* than C_1, also that C_1 is *coarser* than C_2.

DEFINITION 3.3. The fuzzy convergence space (X, C_X) is said to be fuzzy Hausdorff if each prefilter $\mathcal{F}$ C_X-converges to at most one fuzzy point p in X.

Let $F(X)$ be the set of all fuzzy sets in X and A a fuzzy set in X.The set function Γ_{C_X} (resp. I_{C_X}) from $F(X)$ into $F(X)$ is given by $\Gamma_{C_X}(A) = \{p : p$ is fuzzy point in X and $p \in C_X(\mathcal{F})$ for some ultra prefilter $\mathcal{F}$ with $A \in \mathcal{F}\}$ (resp. $I_{C_X}(A) = \{p : A \in \mathcal{V}_{C_X}(p)$ and p is a fuzzy point in $X\}$). Then $\Gamma_{C_X}(A)$ (resp. $I_{C_X}(A)$) is called *fuzzy closure* of fuzzy set A (resp. *fuzzy interor* of A).

For a prefilter $\mathcal{F}$ on X, $\Gamma_{C_X}(\mathcal{F})$ and $I_{C_X}(\mathcal{F})$ are the prefilters on X generated by $\{\Gamma_{C_X}(A) : A \in \mathcal{F}\}$ and $\{I_{C_X}(A) : A \in \mathcal{F}\}$, repectively.

DEFINITION 3.4. The fuzzy convergence space (X, C_X) is called fuzzy regular (resp. fuzzy semi-regular) if $\Gamma_{C_X}(\mathcal{F})$ (resp. I_{C_X} $(\Gamma_{C_X}$ $(\mathcal{F})))$ C_X-converges to p, whenever fuzzy prefilter $\mathcal{F}$ C_X-converges to fuzzy point p.

Let (X, C_X) be a fuzzy convergence space and A a subset of X. (A, C_A) is called a *fuzzy convergence subspace* of (X, C_X) if each prefilter $\mathcal{F}$ on A C_A-converges to p in A whenever the prefilter on X generated by $\mathcal{F}$ C_X-converges to p.

From definition of set functions Γ_{C_X} and I_{C_X}, we can obtain the followings: $\Gamma_{C_X} \supseteq A$ and $I_{C_X}(A) \subseteq A$ for each fuzzy set A in X.

4. Several continuous functions on fuzzy convergence spaces

In this section, we define super continuity, δ-continuity weakly δ-continuity on fuzzy convergence spaces and investigate the relationships among them.

And we introduce inital fuzzy convergence structure and product fuzzy convergence space.

Throught this section, let (X, C_X) and (Y, C_Y) be the fuzzy convergence spaces and p a fuzzy point in X.

DEFINITION 4.1. A function f from (X, C_X) to (Y, C_Y) is continuous at p if $f(\mathcal{F})$ C_Y-converges to $f(p)$, whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

DEFINITION 4.2. A function f from X to Y is said to be fuzzy super continuous at p in X if $\mathcal{V}_{C_Y}(f(p)) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$ whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

DEFINITION 4.3. A function f from X to Y is said to be fuzzy δ-continuous at p in X if $I_{C_Y}(\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$ whenever a prefilter $\mathcal{F}$ on X C_X-converges to p.

DEFINITION 4.4. A function f from X to Y is said to be fuzzy weakly δ-continuous at p in X if $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$.

If a function f is fuzzy continuous (resp. super continuous, δ-continuous, and weakly δ-continuous) at each fuzzy point in X, then f is said to be fuzzy continuous (resp. super continuous, δ-continuous, and weakly δ-continuous) on X.

THEOREM 4.5. *If a function f from (X, C_X) to (Y, C_Y) is fuzzy super continuous at fuzzy point p in X, then f is fuzzy weakly δ-continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to a fuzzy point p in X. Then $\mathcal{V}_{C_Y}(f(p)) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$ by Definition 4.2. Since $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq \mathcal{V}_{C_Y}(f(p))$, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$. Accordingly, f is fuzzy weakly δ-continuous at p in X by Definition 4.4.

THEOREM 4.6. *Let (Y, C_Y) be a fuzzy regular pretopological space. If a funtion f from (X, C_X) to (Y, C_Y) is fuzzy weakly δ-continuous at fuzzy point p in X, then f is fuzzy super continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to a fuzzy point p in X. Then, since (Y, C_Y) is fuzzy regular pretopological space, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))$ C_Y-converges to $f(p)$ in Y. Accordingly, by definition of $\mathcal{V}_{C_Y}(f(p))$, $\mathcal{V}_{C_Y}(f(p)) \subseteq \Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))$. Since f is fuzzy weakly δ-continuous at p in X, $\mathcal{V}_{C_Y}(f(p)) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$. Thus f is fuzzy super continuous at p in X.

THEOREM 4.7. *If a function f from (X, C_X) to (Y, C_Y) is fuzzy δ-continuous at fuzzy point p in X, then f is fuzzy weakly δ-continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ C_X-converges to fuzzy point p in X. Then $I_{C_Y}(\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$ by Definition 4.3. Since $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq I_{C_Y}$ $(\Gamma_{C_Y}$ $(\mathcal{V}_{C_Y}(f(p))))$, $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))\subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$. Thus f is fuzzy weakly δ-continuous at p in X by Definition 4.4.

THEOREM 4.8. *Let (X, C_X) be a fuzzy semi-regular convergence space. If a funtion f from (X, C_X) to (Y, C_Y) is fuzzy continuous at a fuzzy point p in X, then f is fuzzy weakly δ-continuous at p in X.*

Proof. Suppose that a prefilter $\mathcal{F}$ on X C_X-converges to a fuzzy point p in X. Then, $I_{C_X}(\Gamma_{C_X}(\mathcal{F}))\,C_X$- converges to p, as (X, C_X) is fuzzy semi-regular convergence space. Since f is fuzzy continuous at p in X, $f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))\,C_Y$-converges to $f(p)$ in Y. Accordingly, $\mathcal{V}_{C_Y}(f(p)) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$. But $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p))) \subseteq \mathcal{V}_{C_Y}(f(p))$. Thus $\Gamma_{C_Y}(\mathcal{V}_{C_Y}(f(p)))\subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))$, and so f is fuzzy weakly δ-continuous at p in X.

A fuzzy regular convergence space is fuzzy semi-regular,and so the follwing corollary is trivial.

COROLLARY 4.9. *Let (X, C_X) be a fuzzy regularly convergence space. If a function from (X, C_X) to (Y, C_Y) is fuzzy continuous at a fuzzy point p is X, then f is fuzzy weakly δ-continuous at p in X.*

Fuzzy δ-continuity and fuzzy super continuity are independent. But if (Y, C_Y) is the fuzzy regular pretopological space and the function

$f : (X, C_X) \to (Y, C_Y)$ is fuzzy δ-continuous then it is super continuous by Theorem 4.6 and 4.7.

THEOREM 4.10. *If A function f from (X, C_X) to (Y, C_Y) is fuzzy continuous and A is fuzzy set in X then $f(\Gamma_{C_X}(A)) \subseteq \Gamma_{C_Y}(f(A))$.*

Proof. If $q \in f(\Gamma_{C_X}(A))$, then there exists a fuzzy point p in $\Gamma_{C_X}(A)$ such that $f(p) = q$. By definition $\Gamma_{C_X}(A)$, there exists an ultra prefilter $\mathcal{F}$ such that $p \in C_X(\mathcal{F})$ and $A \in \mathcal{F}$. Since f is fuzzy continous, $f(\mathcal{F})$ converges to $f(p) = q$, and $f(A) \in f(\mathcal{F})$. By definition of $\Gamma_{C_Y}(f(A)), q = f(p) \in \Gamma_{C_Y}(f(A))$. Thus $f(\Gamma_{C_X}(A)) \subseteq \Gamma_{C_Y}(f(A))$.

THEOREM 4.11. *Let a f from (X, C_X) to (Y, C_Y) be a map, and p any fuzzy point in X and A any fuzzy set in X. Then the followings are equivalent*

(1) $f(\mathcal{V}_{C_X}(p)) \subseteq \mathcal{V}_{C_Y}(f(p))$.
(2) $f(I_{C_X}(A)) \subseteq I_{C_Y}(f(A))$.

Proof. (1) $\Longrightarrow$ (2) If a fuzzy point $q \in f(I_{C_X}(A))$, then there is a fuzzy point $p \in I_{C_X}(A)$ such that $f(p) = q$. Then $p \in A$ and $A \in \mathcal{V}_{C_X}(p)$ by defineition of $I_{C_X}(A)$. Accordingly, $f(p) \in f(A)$ and $f(A) \in f(\mathcal{V}_{C_X}(p)) \subseteq \mathcal{V}_{C_Y}(f(p))$ by (1). Thus, $q = f(p) \in I_{C_Y}(f(A))$ by difinition of $I_{C_Y}(f(A))$, and so $f(I_{C_X}(A)) \subseteq I_{C_Y}(f(A))$.

(2) $\Longrightarrow$ (1) If $B \in f(\mathcal{V}_{C_X}(p))$, then there is a fuzzy set A in $\mathcal{V}_{C_X}(p)$ such that $f(A) \subseteq B$. Accordingly, $p \in I_{C_X}(A)$ by definition of $I_{C_X}(A)$, and so $f(p) \in f(I_{C_X}(A))$. Since $f(I_{C_X}(A)) \subseteq I_{C_Y}(f(A)) \subseteq I_{C_Y}(B), f(p) \in I_{C_Y}(B)$, and so $B \in \mathcal{V}_{C_Y}(f(p))$ by $I_{C_Y}(B)$. Thus $f(\mathcal{V}_{C_X}(p)) \subseteq \mathcal{V}_{C_Y}(f(p))$.

THEOREM 4.12. *Let X be a nonempty set, $(X_\lambda, C_{X_\lambda})$ a fuzzy convergence spaces, and $f_\lambda : X \to (X_\lambda, C_{X_\lambda})$ a surjection for each $\lambda \in \Lambda$. If C_X is a map from the set $P(X)$ of all prefilters on X to the set $F(X)$ of all fuzzy sets in X satisfying the following condition $(*)$:*
$(*)$ for any fuzzy point p in X and $\Phi \in P(X)$,
$p \in C_X(\Phi)$ if and only if $f_\lambda(\Phi)$ C_{X_λ}-converges to $f_\lambda(p)$ for each $\lambda \in \Lambda$, then C_X is a fuzzy convergence structure on X.

Proof. Let p be any fuzzy point in X, then $\dot{f_\lambda}(p) = f_\lambda(\dot{p})\, C_{X_\lambda}$-converges to $f_\lambda(p)$ for each $\lambda \in \Lambda$. Thus $p \in C_X(\dot{p})$ by hypothesis. If Φ and Ψ are two prefilters on X with $\Phi \subseteq \Psi$ and $p \in C_X(\Phi)$, then $f_\lambda(\Phi) C_{X_\lambda}$-converges to $f_\lambda(p)$ for each $\lambda \in \Lambda$. Since $f_\lambda(\Phi) \subseteq f_\lambda(\Psi)$

for each $\lambda \in \wedge$, by definition of C_{X_λ}, $f_\lambda(\Psi)\, C_{X_\lambda}$-converges to $f_\lambda(p)$ for each $\lambda \in \wedge$. Therefore, $p \in C_X(\Psi)$. Finally, if Φ is a prefilter on X with $p \in C_X(\Phi)$, then $f_\lambda(\Phi \cap \dot{p}) = f_\lambda(\Phi) \cap f_\lambda(\dot{p}) = f_\lambda(\Phi) \cap \dot{f_\lambda(p)}\, C_{X_\lambda}$-converges to $f_\lambda(p)$ for each $\lambda \in \wedge$. Thus $p \in C_X(\Phi \cap \dot{p})$. The fuzzy convergence structure C_X, is given in Theorem 4.12, is called the *initial convergence structure* on X induced by the family $\{f_\lambda : \lambda \in A\}$.

THEOREM 4.13. *The initial fuzzy convergence structure C_X is the coarest convergence structure on X which allows f_λ to be fuzzy continuous for each $\lambda \in \wedge$.*

Proof. By Definition 4.1, it is clear that f_λ is continuous for each $\lambda \in \wedge$. Let C'_X be a fuzzy convergence structure on X relative to which f_λ is continuous for each $\lambda \in \wedge$. Suppose that a prefilter Φ on X C'_X-converges to the fuzzy point p in X. Since $f_\lambda : (X, C'_X) \longrightarrow (X_\lambda, C_{X_\lambda})$ is continuous for each $\lambda \in \wedge$, then $f_\lambda(\Phi) C_{X_\lambda}$-converges to $f_\lambda(p)$. Accordingly, by definition of the initial fuzzy convergence structure C_X, $\Phi\, C_X$-converges to p, that is, $C_X \leq C'_X$. Thus, C_X is the coarsest fuzzy convergence structure on X which allows f_λ to be continuous for each $\lambda \in \wedge$. Let $\{(X_\lambda, C_{X_\lambda}) : \lambda \in \wedge\}$ be a family of fuzzy convergence spaces and $P_\lambda : \prod_{\lambda \in \wedge} X_\lambda \rightarrow (X_\lambda, C_{X_\lambda})$ be canonical projection for each $\lambda \in \wedge$. Then, $\prod_{\lambda \in \wedge} X_\lambda$ endowed with initial fuzzy convergence structure C_X induced by $\{P_\lambda : \lambda \in \wedge\}$ is called the *product fuzzy convergence space* induced by $\{(X_\lambda, C_{X_\lambda}) : \lambda \in \wedge\}$. In this case C_X is denoted by $\prod_{\lambda \in \wedge} C_{X_\lambda}$, that is, $C_X = \prod_{\lambda \in \wedge} C_{X_\lambda}$.

THEOREM 4.14. *If $(\prod_{\lambda \in \wedge} X_\lambda, C_X)$ is the product fuzzy convergence space of family $\{(X_\lambda, C_{X_\lambda}) : \lambda \in \wedge\}$ of fuzzy convergence spaces, then*

(1) $\prod_{\lambda \in \wedge} \mathcal{V}_{C_{X_\lambda}}(p_\lambda) \subseteq \mathcal{V}_{C_X}(p)$,

(2) $P_\lambda(\mathcal{V}_{C_X}(p)) = \mathcal{V}_{C_{X_\lambda}}(p_\lambda)$ *for each* $\lambda \in \wedge$,

where $p = (p_\lambda)_{\lambda \in \wedge}$ is the fuzzy point in $\prod_{\lambda \in \wedge} X_\lambda$ for fuzzy point p_λ in X_λ and $P_\lambda : \prod_{\lambda \in \wedge} X_\lambda \rightarrow X_\wedge$ canonical projection for each $\lambda \in \wedge$.

Proof. (1) If $F \in \prod_{\lambda \in \wedge} \mathcal{V}_{C_{X_\lambda}}(p_\lambda)$, then there exists a $F_\circ = \prod_{\lambda \in \wedge} F_\lambda$ such that $F_\circ \subseteq F$, where $F_\lambda \in \mathcal{V}_{C_{X_\lambda}}(p_\lambda)$ for each $\lambda \in \wedge$ and $F_\lambda = X_\lambda$ for all but a finite number of induces. Suppose that $F_\circ \notin \mathcal{V}_{C_X}(p)$, then there exists a prefilter $\Phi\, C_X$-converges to p such that $F_\circ \notin \Phi$. Accordingly, $P_\lambda(\Phi)\, C_{X_\lambda}$-converges to $P_\lambda(p) = p_\lambda$ for each $\lambda \in \Lambda$, and so $\mathcal{V}_{C_{X_\lambda}}(p_\lambda) \subseteq P_\lambda(\Phi)$ for each $\lambda \in \Lambda$. Therefore $F_\circ \in \prod_{\lambda\in\Lambda} \mathcal{V}_{C_{X_\lambda}}(p_\lambda) \subseteq \prod_{\lambda\in\Lambda} P_\lambda(\Phi) \subseteq \Phi$. This contradicts that $F_\circ \notin \Phi$. Thus, $F_\circ \in \mathcal{V}_{C_X}(p)$, and so $F \in \mathcal{V}_{C_X}(p)$. Consequently $\prod_{\lambda\in\Lambda} \mathcal{V}_{C_{X_\lambda}}(p_\lambda) \subseteq \mathcal{V}_{C_X}(p)$.

(2) Since P_λ is continuous, $\mathcal{V}_{C_{X_\lambda}}(p_\lambda) \subseteq P_\lambda(\mathcal{V}_{C_X}(p))$ for each $\lambda \in \Lambda$. Now, we will show that $\mathcal{V}_{C_{X_\lambda}}(p_\lambda) \supseteq P_\lambda(\mathcal{V}_{C_X}(p))$. If F_μ is any element of $P_\mu(\mathcal{V}_{C_X}(p))$, then there is a $F \in \mathcal{V}_{C_X}(p)$ such that $P_\mu(F) \subseteq F_\mu$. Let $\mathcal{F}$ be an arbitrary prefilter which C_{X_μ}-converges to p_μ.

Take the prefilter $\prod_{\lambda\in\Lambda} \Phi_\lambda$ on $\prod_{\lambda\in\Lambda} X_\lambda$, where

$$\Phi_\lambda = \begin{cases} \mathcal{F}, & \text{if } \lambda = \mu \\ \dot{p}_\lambda, & \text{otherwise.} \end{cases}$$

Then $\prod_{\lambda\in\Lambda} \Phi_\lambda\, C_X$-converges to $p = (p_\lambda)_{\lambda\in\Lambda}$. Since $F \in \mathcal{V}_{C_X}(p) \subseteq \prod_{\lambda\in\Lambda} \Phi_\lambda$, $P_\mu(F) \in P_\mu(\prod_{\lambda\in\Lambda} \Phi_\lambda) = \mathcal{F}$. Therefore $F_\mu \in \mathcal{F}$, that is, $F_\mu \in \mathcal{V}_{C_{X_\mu}}(p_\mu)$. Consequently, $\mathcal{V}_{C_{X_\lambda}}(p_\lambda) = P_\lambda(\mathcal{V}_{C_X}(p))$.

THEOREM 4.15. *Let f_λ be a map from a fuzzy convergence space $(X_\lambda, C_{X_\lambda})$ to a fuzzy convergence space $(Y_\lambda, C_{Y_\lambda})$ and P_λ the canonical projection from $(\prod_{\lambda\in\Lambda} X_\lambda, \prod_{\lambda\in\Lambda} C_{X_\lambda})$ to $(X_\lambda, C_{X_\lambda})$ for each $\lambda \in \Lambda$. If f_λ is fuzzy super continuous for each $\lambda \in \Lambda$, then $f_\lambda \circ P_\lambda$ is fuzzy super continuous.*

Proof. Suppose that a prefilter $\mathcal{F}$ on $\prod_{\lambda\in\Lambda} X_\lambda$ $\prod_{\lambda\in\Lambda} C_{X_\lambda}$-converges to $p = (p_\lambda)_{\lambda\in\Lambda}$. Then, $P_\lambda(\mathcal{F})\, C_{X_\lambda}$-converges to $P_\lambda(p) = p_\lambda$ in X_λ. Since f_λ is super continuous,

$$\mathcal{V}_{C_{Y_\lambda}}(f_\lambda(P_\lambda(p) \subseteq f_\lambda(I_{C_{X_\lambda}}(\Gamma_{C_{X_\lambda}}(P_\lambda(\mathcal{F})))).$$

Since P_λ is continuous, $I_{C_{X_\lambda}}(\Gamma_{C_{X_\lambda}}(P_\lambda(\mathcal{F}))) \subseteq I_{C_{X_\lambda}}(P_\lambda(\Gamma_{\Pi C_{X_\lambda}}(\mathcal{F})))$ by Theorem 4.10. By Theorem 4.11 and 4.14,

$$I_{C_{X_\lambda}}(P_\lambda(\Gamma_{\Pi C_{X_\lambda}}(\mathcal{F}))) \subseteq P_\lambda(I_{\Pi C_{X_\lambda}}(\Gamma_{\Pi C_{X_\lambda}}(\mathcal{F}))).$$

Thus, $\mathcal{V}_{C_{Y_\lambda}}(f_\lambda(P_\lambda(p))) \subseteq f_\lambda(P_\lambda(I_{\Pi C_{X_\lambda}}(\Gamma_{\Pi C_{X_\lambda}}(\mathcal{F}))))$, and so $f_\lambda \circ P_\lambda$ is super continuous.

THEOREM 4.16. *Let f be a map from a fuzzy convergence space (X, C_X) into a fuzzy pretopological convergence space (Y, C_Y) and g a map from (Y, C_Y) into a fuzzy convergence space (Z, C_Z). If f is fuzzy super continuous and g is fuzzy continuous, then the composition $g \circ f$ is fuzzy super continuous.*

Proof. Let a prefilter $\mathcal{F}$ on X C_X-converges to a fuzzy point p in X. Since (Y, C_Y) is fuzzy pretopological convergence space $\mathcal{V}_{C_Y}(f(p))\, C_Y$-converges to $f(p)$. By definition of fuzzy super continuous, $\mathcal{V}_{C_Y}(f(p)) \subseteq f(I_{C_X}(\Gamma_{C_X}(\mathcal{F}))))$, and so $g(\mathcal{V}_{C_Y}(f(p))) \subseteq g(f(I_{C_X}(\Gamma_{C_X}(\mathcal{F}))))$. Since g is fuzzy coninuous, $g(\mathcal{V}_{C_Y}(f(p)))\, C_Z$-converges to $g(f(p))$. Thus, $g(f(I_{C_X}(\Gamma_{C_X}(\mathcal{F}))))$ C_Z-converges to $g(f(p))$. Consequently,

$$\mathcal{V}_{C_Z}(g(f(p))) \subseteq g(f(I_{C_X}(\Gamma_{C_X}(\mathcal{F})))),$$

that is, $g \circ f$ is fuzzy super continuous.

From above Theorem, we obtain the following corollaries.

COROLLARY 4.17. *Let $(X_\lambda, C_{X_\lambda})$ be the fuzzy pretopological convergence space for each $\lambda \in \wedge$ and $(\Pi_{\lambda\in\wedge}X_\lambda, \Pi_{\lambda\in\wedge}C_{X_\lambda})$ the fuzzy product convergence space of the family $\{(X_\lambda, C_{X_\lambda}) : \lambda \in \wedge\}$ of fuzzy convergence spaces. If $f : (X, C_X) \to (\Pi_{\lambda\in\wedge}X_\lambda,\ \Pi_{\lambda\in\wedge}\ C_{X_\lambda})$ is super continuous, then $P_\lambda \circ f$ is also super continuous for each $\lambda \in \wedge$, where $P_\lambda : \Pi_{\lambda\in\wedge}X_\lambda \to X_\lambda$ is the canonical projection for each $\lambda \in \wedge$.*

COROLLARY 4.18. *Let $g : (X, C_X) \to (Y, C_Y)$ be a function and $f : (X, C_X) \to (X, C_X) \times (Y, C_Y)$ given by $f(x) = (X, g(x))$ be its graph. If f is super continuous, then g is super continuous.*

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Department of Mathematics Education
Kyungnam University
Masan 631–701, Korea