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ON THE SPACING PATTERN OF PLANETS AND SATELLITES

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ABSTRACT

We show that spacing patterns of planets and satellites in the solar system are formulatable in a single form. It is suggested that a possible explanation for the rule might be the orbital resonance effect, which has existed at an earlier epoch of the solar (planet) system. By extrapolating the formulated spacing patterns beyond the sun-Pluto distance, we find the sun-Planet X distance falls in a range (46~79) A.U..

As manifested by the Titus-Bodet's "law", locations of major planets in the solar system are not completely random. Therefore, it would be an interesting question whether a common spacing rule could be formulated, which works for both the planets and satellites of Jupiter, Saturn, Uranus system. At first glance, this may sound an overly optimistic question, since as we are well aware, even the Titus-Bodet's law fails beyond the sun-Uranus distance. Also, there is an enormous mass difference between the solar system as a whole and satellite systems of major planets.

However, in this work, we present a new way of formulating the spacing pattern. This formulation takes the cruder form than the Titus-Bodet's law; nevertheless, it has an advantage that a single formulation works for both solar and satellite systems of major planets. For the major planets, we consider Jupiter, Saturn and Uranus. The Neptune system is excepted, since, one retrograding massive satellite, Triton, dominates her satellite system.

Let it be noted the following: For the solar system, see Table 1 (Allen 1973).
The third column, which is the distance from the sun to a planet divided by that of its nearest outer neighbor, falls in a relatively narrow range. If we exclude the noticeable exception of Pluto (which has a highly eccentric orbit), the distance ration falls in a range

$$f_{sol} = 0.5 \sim 0.72. \qquad (1)$$

Table 1.

Planet	Distance from the sun in Astronomical Unit	Percentage distance to the next planet
Mercury	0.39	0.54
Venus	0.72	0.72
Earth	1.00	0.66
Mars	1.52	0.56
Asteroids(Ceres)	2.70	0.52
Jupiter	5.20	0.55
Saturn	9.50	0.50
Uranus	19.20	0.64
Neptune	30.10	0.76
Pluto	39.44	*(Not meaningful)*

It is apparent that this is a manifestation of a familiar power-dependence of the sun-planet distance.

A question immediately following would be whether Eq. (1) carries any physical significances. In order to check this, let us apply the similar analysis to the satellite system of Jupiter. Considering only major massive ones, we find the following (Allen 1973):

Table 2.

Satellite	Distance from Jupiter in Jupiter Radii	Percentage distance to the next satellite
Io	5.95	0.63
Europa	9.47	0.63
Ganymede	15.10	0.57
Callisto	26.60	*(Not meaningful)*

[We neglected other satellites in that the next massive satellite, Amalthea, is $\sim 10^{-4}$ times less massive than the above four.] Compared with f_{sol}, the distance ration f_J of the system falls in the more narrower range

$$f_J = 0.57 \sim 0.63. \tag{2}$$

Now the proximity of f_J to Eq. (1) is apparent.

For the Saturn system, which has much larger number of satellites orbiting on a wide variety of separations, an appropriate grouping of the closer satellites becomes important.

The key physical property we will adopt in this grouping is the chemical composition (i.e., the mean density) and the mutual orbital proximity. [In this way, we would like to minimize the selection effect.] We find that Mimas and Enceladus can form the group. They are characterized by (a) isolated orbital locations from other major massive satellites, (b) mutual orbital proximity, and (c) identical density$\approx 1.2 g/cm^3$. Similarly, we find Dione and Rhea can form the second group. In this way, we group major massive satellites of Saturn as Table 3 (Allen 1973).

Table 3.

Satellite	Average distance from Saturn in Saturn radii	Percentage distance to the next satellite
Mimas,Enceladus,Tethys	3.97	0.53
Dione,Rhea	7.50	0.6
(?)	(12.5?)	(0.6?)
Titan	20.25	0.6
(?)	(37?)	(0.6?)
Iapetus	59.02	(*Not meaningful*)

Apparently, there are two missing satellites between orbits of Dione/Rhea and Iapetus. We presume that these satellites had existed in the past. Had these satellites somehow been disrupted, this could explain why (a) Titan, which now orbits between two hypothetical satellites, is noticeably massive ($\sim 10^2$ times massive) than others and (b) why Saturn, unlike other planets, has ample material in the ring. In this way, a rough estimate of the distance ratio for the Saturn system is

$$f_{sat} = 0.53{\sim}0.7(?). \tag{3}$$

For Uranus, despite her major satellites orbiting almost vertically to the solar equatorial plane, a similar distance ratio also exists. For major Uranian satellites (Allen 1973),

Table 4.

Satellite	Distance from Uranus in Uranus radii	Percentage distance to the next satellite
Miranda	5.2	0.68
Ariel	7.6	0.72
Umbriel	10.7	0.61
Titania	17.5	0.75
Oberon	23.4	(*Not meaningful*)

Therefore,

$$f_u = 0.61{\sim}0.75. \tag{4}$$

Compared with others, this system has a slightly higher range of f's: Whether this indicates a peculiar dynamical past of the Uranus is an open issue.

To summarize, what we have found thus far is that for the solar, Jupiter, Saturn, and Uranus systems, the orbital locations of planets (satellites) are found within $f{\sim}(50{\sim}75)$ percent distance of their nearest outer companion:

Table 5.

	f's
Solar System	(0.50-0.72)
Jupiter System	(0.57-0.63)
Saturn System	(0.53-0.7?)
Uranus System	(0.61-0.75)

Does the range of f's carry a "real" physical significance? Whether this is a mere coincidence or not, by extrapolating data of Table 1, one can predict the (possible) distance to the planet X from the sun d_{sx} as

$$d_{sx} \sim \frac{40}{f_{sol}} \tag{5.1}$$

For $f_{sol} = (0.50{\sim}0.76)$ for the solar system, the most likely distance of the planet X falls in a range

$$d_{sx} = 46{\sim}79 A.U.. \tag{5.2}$$

One possible explanation for the distance ratio might be the following: This is based on an interesting coincidence between f's of Table 5, and an orbital resonance distance determined by Kepler's 3rd law. From Kepler's 3rd law, for a solid body orbiting, its orbital semimajor axis is given by

$$a_* = \left(\frac{T_*}{T_m}\right)^{\frac{2}{3}}. \tag{6}$$

Here, the quantity T_m is the orbital period of a massive planet (satellite) in the system, and T_* is the orbital period of other solid bodies. [In this form of Kepler's 3rd law, we normalized the distances from the mass-center of the system to the given massive planet (satellite) as unity.] For $T_m = NT_*$, where $N > 0$, any minor solid bodies located at distance

$$a_N = \left(\frac{1}{N}\right)^{\frac{2}{3}} \tag{7}$$

are subject to the orbital resonance. This distance corresponds to the location where material is "pumped away" from their orbit. A very well known example is the Cassini Division of Saturn, produced by the shepherd moon, Mimas. Then the location where the larger amount of material is "pumped in" would be the region adjacent to the stronger orbital resonances. In particular, for N=2 and N=3, which occurs in the same side of the "shepherd" massive body,

$$a_2 \sim a_3 = 0.48 \sim 0.63. \tag{8}$$

Now the proximity of Eq. (8) to the f's of Table 5 is quite suggestive. [We note that Eq. (8) does not depend on the total mass of the system.] Thus in the region of Eq. (8), small orbiting solid bodies will undergo the more frequent encounter with the material "pumped in and out". In this way, the region of Eq. (8) would be the more likely place where the rapid-growth ("fattening") of major planets (satellites) can occur.

Of course, the orbital resonance picture may be an overly simplistic view. For the needed justification, many specific issues are to be answered: e.g., (a) whether the solar system has had an epoch in which most condensed solid bodies established well-defined rotation periods; (b) whether effects of orbital perturbation onto Eq. (8) are in/significant; and (c) how efficient is the "mass-accretion" effect in region of Eq. (8). [For example, for Saturn, the division at N=3 is much less conspicuous than that of the Cassini division (N=2).] Therefore, in this respect, we reserve a note that data of Table 5 may be a mere coincidence, but possibly of the orbital resonance origin, which should be checked by the more realistic planet (and satellite) growth model.

REFERENCE

Allen, C. W., 1973, *Astrophysical Quantities* (The Athlone Press, University of London)