PUBLICATIONS OF THE KOREAN ASTRONOMICAL SOCIETY
7:19~23, 1992

CAN THE UNIVERSE BE "TILTED"?

LA, DAILE
Center for Particle Astrophysics and Astronomy Department
University of California at Berkeley*
(received December 1, 1991)

ABSTRACT

We investigated the "tilting" of the Universe, i.e., a non-Doppler origin of the dipole moment of the cosmic background radiation (CBR). Superhorizon-sized isocurvature, rotational and true vacuum bubble perturbations are considered. We show that the more natural way of the "tilting" the Universe is via the true vacuum bubble perturbation. Nevertheless, due to the small filling fraction of the bubbles of viable extended inflationary models, we find that the probability of the real occurrence in the Universe is quite insignificant.

I. INTRODUCTION

The "tilted" universe is the Universe where the dipole moment of the CBR is not induced by the motion of galaxies in our local neighborhood, but originated by the matter perturbation on superhorizon-scales (Turner 1991). The perturbations referred to here include adiabatic, isocurvature, rotational (Rebhan 1991) and true vacuum bubble perturbations (La and Steinhardt 1989; La 1991).

At first glance, it may sound counter-intuitive in that we are discussing information from superhorizon-sized perturbations. Are we not violating the causality condition? Of course, the causality condition is not violated; since, what we are interested in is the dipole and the higher multipoles of the CBR induced by the perturbations at their horizon crossing scales.

The strength of multipole moments of the CBR can be compuited via the Sachs-Wolfe formula. Hence, in this paper, we will consider a flat ($k = 0$), matter-dominated Universe $H^2 = \frac{8\pi\rho_\sigma(t)}{3M_p^2}$. Here, $H(t)$ is the Hubble parameter, which is determined by the scale factor $R(t)$ of the Friedman-Robertson-Walker (FRW) Universe ($H \equiv \frac{\dot{R}}{R}$); and the quantity $\rho_\sigma(t) \propto R^{-3}$ is the energy density of the pressureless cosmic matter. Then, for a given density/velocity perturbations ($\delta\rho/\rho$), the relative deviation of the CBR temperature seen in the direction $\mathbf{n}$ by an observer at $\mathbf{r} = 0$ today, is given as (Sachs and Wolfe 1967; Turner 1991)

---

* Submitted to the Cosmology conference held in honour of Professor J. J. Hyun, Seoul, Korea, March 1992

$$\frac{\delta T(\mathbf{r};\mathbf{n})}{T} = \left[\sqrt{R}(\mathbf{x}.\nabla)\, \delta_H e^{-i(\mathbf{k}.\mathbf{x})} + \delta_H e^{-i(\mathbf{k}.\mathbf{x})}\right] |^{e}{}_{r}. \tag{1}$$

Here, δ_H is the horizon crossing amplitude of the density perturbation $(\delta\rho/\rho)$; "e" denotes the emission event; "r" denotes the reception event; $\sqrt{R_r} = 1; \sqrt{R_e} \sim 10^{-3}$; $\mathbf{r}_e = (1 - \sqrt{R_e})\mathbf{x}$; $2\pi\mathbf{k}^{-1}$ is the wavelength of the perturbation; and the quantity $\delta\rho(\mathbf{r})$ is measured at the observer's position. Now the physical meaning of two terms in the right side of Eq. (1) should be made clear. Due to the presence of the gradient operator, the first term in the right side of Eq. (1) is of a kinetic origin. The second term represents the contribution from the presence of the density gradient in the direction $\mathbf{k}$. As viewed from a Newtonian perspective, the second term represents the gravitational-potential difference between the last-scattering surface and an observer here ($\mathbf{r} = 0$).

II. SUPERHORIZON-SIZED ADIABATIC PERTURBATIONS (SAPs)

For the SAPs of $(\delta\rho/\rho) \sim e^{-i\mathbf{k}.\mathbf{x}}$, where $\mathbf{k}.\mathbf{x} \ll 1$, via Eq. (1), it is easy to show that the dipole moment of the CBR temperature vanishes. This implies that for the SAPs, the rest frame of the isotropic expansion of the Universe coincides with the frame which would make the CBR temperature as isotropic as possible. Thus for the SAPs, there is no "tilting" of the Universe. Geometrically, this cancellation is a manifestation that the adibatic perturbation can possess a coordinate frame where the dipole component can be transformed away. In this way, for the SAPs, it is the quadrupole component (i.e., $O[(\mathbf{k}.\mathbf{x})^2]$-term), which becomes the first non-trivial moment of the CBR temperature. This feature was first observed by Grischuk and Zel'dovich (1978).

III. SUPERHORIZON-SIZED VECTOR PERTURBATIONS (SVPs)

Superhorizon-sized rotational perturbations can induce a non-vanishing dipole moment of the CBR temperature. This effect was studied by Collins and Hawking (1973) within the context of homogeneous anisotropic models, and recently by Rebhan (1991) in the Friedman Universe. Therefore, it is possible to "tilt" the Universe via the SVPs. The problem, however, is that unlike the dipole moment induced by the Doppler effect of motion of the observer, the vector perturbations yield the higher magnitude multipole moments. Therefore, the observational quadrupole fluctuation data of the CBR disprove the vector perturbations as a possible origin of the observed dipole moment of the CBR. [As we well aware, observed upper limits of the quadrupole moment are significantly smaller than the dipole data.]

IV. SUPERHORIZON-SIZED ISOCURVATURE FLUCTUATIONS (SIPs)

Recently, it wqas claimed that a single model superhorizon-sized isocurvature perturbation can "tilt" the Universe (Turner 1991). The key argument is the following: For

$\delta\rho_{total} = \delta\rho_\gamma + \delta\rho_a$, where δ_γ and ρ_γ denotes the photon and axion fluctuations, respectively, $\delta\rho_{total} = 0$. (This follows from the very definition of the isocurvature perturbation.) It is apparent that due to the causality condition, the condition $\delta\rho_{total} = 0$ strictly holds for any superhorizon-scales. Thus, in effect, any change in the axion-component of cosmic matter is to be "compensated" by the corresponding photon fluctuations $\delta\rho_\gamma$. It is this "compensating" fluctuation which can produce an intrinsic dipole moment of the CBR temperature.

However, the model has not escaped criticisms: First, due to inflation, unless we control the e-foldings of inflation to a certain degree, it is extremely hard that any pre-inflationary isocurvature perturbations can survive the expenential inflationary redshift (Note that the same argument also holds for the SVPs.) Second, as we shall show shortly, the strongest signal of multipoles results from perturbations entering the horizon. The problem is, should this happen, the magnitude of the dipole and the higher moments fell in a comparable range. Thus the upper limits of quadrupole data, which are orders of magnitude smaller than that of the dipole moment, implies that we have to prevent the SAP modes which already become subhorizon perturbations from responsible for the observed multipole moments of the CBR. This is an (ad hoc) initial condition. (Further, in the model, why we should only consider a single mode fluctuation greater than the present horizon as responsible for observed multipole moments of the CBR is not answered.)

V. SUPERHORIZON-SIZED VACUUM BUBBLE PERTURBATIONS (SVBPs)

The true vacuum bubbles produce an alternative form of the superhorizon-sized perturbations (La 1991): And we will show that the SVBPs can "tilt" the Universe. How the SVBPs produce the dipole moment of the CBR is the following: Let us be noted that during the matter-dominated epoch, the very presence of the matter-empty void, triggers a "motion" of cosmic matter surrounding the bubbles. Its general relativistic counterpart is the void expansion: during the matter-dominated epoch, a matter-devoid bubble expand faster $\propto t^{\frac{2}{3}+\frac{2}{15}}$ than the global expansion $\propto t^{\frac{2}{3}}$. Thus, over superhorizon scales, what becomes "dynamic", is the 3-space. Accordingly, the memory of the 3-space expansion, no matter how large-scale the expansion occurs, is recorded in CBR photons. This is the way how the superhorizon-sized bubble induce fluctuations of the CBR temperature.

In the discussion of the SVBPs, the key physical quantity, we are going to make a wide use, is the filling fraction of the bubbles. For most extended inflationary models, the filling fraction f of the true vacuum bubble is found as (Weinberg 1989; La and Steinhardt 1989)

$$f(> x) \;=\; [1 \;+\; H(t_e)x]^{-\frac{4}{5}}. \tag{2}$$

[The fraction of a volume $V \equiv x^3$ engulfed by bubbles of sizes greater than x, is f.] Here,

$H(t_e)$ denotes the Hubble parameter at the end of inflation. Now the false vacuum decay process is a random quantum tunneling process (Coleman 1977; Coleman and de Luccia 1987). Therefore, the spatial locations of the true vacuum bubbles are not correlated. Consequently, the probability of finding two bubbles of the same radii $H_e x + \Delta[H_e x]$ separated by a distance D is

$$P(H_e x;\, D) \;=\; \left[1 - \left(\frac{x}{D}\right)^3\right]\, \Delta^2[H_e x] \left(\frac{df}{d[H_e x]}\right)^2 . \tag{3}$$

Hence, the probability that the second bubble is found at distance $H_e x$ is

$$P(H_e x\;;\; D \to 2x) \;=\; \frac{7}{8}\;\Delta^2[H_e x]\;\left(\frac{df}{d[H_e x]}\right)^2 \;\sim\; (H_e x)^{-\frac{8}{5}} . \tag{4}$$

Consequently, for $M_F \sim 10^{15} GeV$, and x ranging $H^{-1}(t_{eq})$ to 10^n times of the present horizon scale, $H_o^{-1} \sim 3000\ Mh^{-1}Mpc$, the probability $10^{20} < [H_e x] < 10^{25+n}$ (Kolb and Turner 1990). This implies that for any viable extended inflation models, the probability of finding two same-sized bubbles separated by their size is at most $P \sim 10^{-\frac{200+8n}{5}}$. This extremely small probability indicates that, if observed multipoles of the CBR temperature are indeed of the SVBPs-origin, the fluctuation contribution is the most likely originated from a single bubble.

Now let us investigate the details. We note that the bubbles of radii greater than $H^{-1}(t_{eq})$ were, at least once, superhorizon-size ones. Therefore, it becomes necessary to distinguish the following two cases: (A) when the bubble perturbation, which is now subhorizon-sized, was once superhorizon-sized; and (B) currently, there is a superhorizon-sized bubble beyond our present horizon. Howver, as in the case of the SVPs and SAPs, observed quadrupole data of the CBR temperature disprove the case (A). Therefore, it is a (single mode) superhorizon-sized bubble of the case (B), which can "tilt" the Universe. For convenience, we denote H_o as the present Hubble parameter; L is scale of the bubble perturbation; and its horizon-crossing perturbation amplitude as δ_{H_L}. Then the Sachs-Wolfe formula yields the dipole, quadrupole and octopole moments of the CBR temperature as

$$\left(\frac{\delta T}{T}\right)_d \approx \delta_{H_L}\left(\frac{H_o^{-1}}{L}\right);\left(\frac{\delta T}{T}\right)_q \approx \delta_{H_L}\left(\frac{H_o^{-1}}{L}\right)^2;\left(\frac{\delta T}{T}\right)_{oct} \approx \delta_{H_L}\left(\frac{H_o^{-1}}{L}\right)^4 . \tag{5}$$

Now from the observed dipole data, the horizon crossing amplitude of the bubble perturbation as

$$\delta_{H_l} \;\approx\; 2\times\ 10^{-3}\left(\frac{L}{H_o^{-1}}\right) . \tag{6}$$

The the corresponding quadrupole component is $\left(\frac{\delta T}{T}\right)_q \sim 2\times 10^{-3}\left(\frac{H_o^{-1}}{L}\right)$. Since the observed limit is $(\Delta T/T)_q < 3\times 10^{-5}$ (Smoot 1991),

$$\left(\frac{L}{H_o^{-1}}\right) \geq \frac{2}{3} \times 100. \tag{7}$$

This implies that the nearest superhorizon-sized true vacuum bubble responsible for the observed dipole, and quadrupole moment should be located beyond $\sim 50H_o^{-1}$ of the present horizon-distance.

Also for the bubble-induced octopole limit, the observed quadrupole limit $\left(\frac{H_o^{-1}}{L}\right) < 1.5 \times 10^{-2}$, constrains the octopole moment as $\left(\frac{\Delta T}{T}\right)_{oct} < O(1) \times 10^{-9} < 10^{-8}$.

Having found the distance to the bubble, let us compute the probability of such occurrence. The occurrence-probability corresponds to a fraction of volume $(50 \times H_o^{-1})^3$ engulfed by bubbles greater than the volume: $f(> 50[H_e H_o^{-1}])$. Therefore, for $M_F \sim 10^{15} GeV$, the probability that observed multipoles of the CBR are of the superhorizon-sized bubble-origin is $[50H_e H_o^{-1}]^{-\frac{4}{b}} \approx 10^{-\frac{104}{b}}$. Since the range of the Brans-Dicke (-like) parameter for workable EI models is $1.5 < b < 25$ (Weinberg 1989; La and Steinhardt 1989), we find the occurrence probability

$$10^{-93} \sim 10^{-4}. \tag{8}$$

Therefore, the true vacuum bubble can "tilt" the Universe without any ad hoc assumptions, however, due to the small filling fraction, its occurrence probability in the FRW Universe is insignificantly low.

REFERENCES

Coleman, S. 1977, *Phys. Rev.*, **D15**, 2929.

Coleman, S., and De Luccia, F. 1987, *Phys. Rev.*, **D36**, 2919.

Collins, C. B., and Hawking, S. W. 1973, *M.N.R.A.S.*, **162**, 307.

Grishchuk, L. P., and Zel'dovich, Ya. B. 1978, *Sov. Astron*, **22**, 125.

La, D., and Steinhardt, P. 1989, *Phys. Rev. Letter*, **62**, 376.

La, D., and Steinhardt, P. 1989, *Phys. Letter*, **B220**, 375.

La, D. 1991, *Phys. Letter*, **B265**, 232.

Kolb, E., and Turner, M. 1990, *The Very Early Universe* (Addison-Wesley), p. 76 and p. 504.

Rebhan, A. 1991, CERN preprint CERN-TH6251/91.

Sachs, R. M., and Wolfe, A. M. 1967, *Ap. J*, **147**, 73.

Smoot, G. F., *et al.* 1991, *Ap. J. (Letters)*, in press.

Turner, M. 1991, Fermilab Preprint-pub-91/43A.

Weinberg, E. 1989, *Phys. Rev.*, **D40**, 3950.