

COSMOLOGY, EPISTEMOLOGY AND CHAOS

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ABSTRACT

We may consider the following three fundamental epistemological questions concerning cosmology. Can cosmology at last understand the origin of the universe? Can computers at last create? Can life be formed at last synthetically?

These questions are in some sense related to the liar paradox containing the self-reference and, therefore, may not be answered by recursive processes in finite time. There are, however, various implications such that the chaos may break the trap of the self-reference paradox. In other words, Goedel's incompleteness theorem would not apply to chaos, even if the chaos can be generated by recursive processes. Internal relations among cosmology, epistemology and chaos must be investigated in greater detail.

I. INTRODUCTION

Electrons are believed to have the same property irrespectively of whether they are in a plasma, in a molecule, in a metal, in the sun, in the Large Magellanic Cloud, in the past, or in the future. It was J. Wheeler who proposed an idea that the reason of the identity of the properties of electrons is that there is only a single electron which exhibits itself everywhere. The legitimacy of the idea can be neither proved nor disproved, unless different kinds of information are added. It is true that the universe is turning around the earth in the rotating coordinate adopted, the alt-azimuth coordinate which is the most convenient for the meridian observations. The coordinate becomes inconvenient, if the motions of planets, stars and galaxies should be described at the same time.

Since our knowledge on the universe is finite in any case, it is the matter of epistemology to decide which is the most convenient description of the universe the cosmology of a single electron in a very sophisticated space-time or the cosmology of innumerable electrons in 4-dimensional space-time. The optimum answer may be in the intermediate, because there are not only electrons but families of elementary particles that interact in various complicated forms in the universe. Perhaps, the universal properties of elementary particles should better be attributed to the nature of the multi-dimensional space, while the origin of the hierarchical macroscopic structures should better be attributed

to the nonlinear dynamical interactions, i.e., chaos. There are, however, serious doubts about the legitimacy of such dual character in the epistemology. If chaos is important in the macroscopic universe, it must be important also in the microscopic universe. Our question then turns out to be: what are the intrinsic meanings of chaos in cosmology?

Before entering into the discussion on the three questions stated in the abstract, we will summarize the basic properties of chaos in the following section.

II. THREE THEOREMS ON CHAOS

The origin of chaos is the nonlinear interactions among three or more modes of variations in which at least one mode is a growing mode. Thus, we may take the first theorem of chaos to be "3 gives chaos". The coupling of two nonresonant modes results at most in a limit cycle, (Spiegel 1985). A good example is the binary motion which is periodic, while the general problem of three bodies possesses the chaotic solution.

In order to describe complex phenomena of n degrees of freedom unambiguously, we need to provide an n -dimensional space to embed their variations, if the variations are monotonic. But, if the variations are chaotic, we need a $(2n + 1)$ -dimensional space (Takens 1981). This is one of the basic theorem of the dynamical system theory of chaos. The theorem is apparent in the case of $n=1$ so that the position on an entangled string ($n=1$) is not possible to identify on the 2-dimensional projection because of the occurrence of crossings and tangential contacts.

There are, therefore, at least two reasons to consider cosmology in multi-dimensional space. They are that anything universal can be described most simply as the property of the space and that the introduction of higher dimensional considerations is the best resolution of confusions.

The third theorem of chaos is that a small difference in the initial condition gives rise to a large difference in time. This is the very reason why chaos evolves unrepeatedly, and this property of chaos is important for the evolution. Combining this theorem with the first theorem, we see that the coupling of three elements causes chaos, and the chaos causes unrepeated evolution. Since religions should include a kind of cosmology in their philosophy, many world famous religions have interpretation of this property of chaos by having three principal elements or gods. In natural history, on the other hand, the classification is the important step to build sciences, and in this case the hierarchy of binary classifications is normally considered to be the best stable classification. However, for the evolutionary considerations, the binary classification becomes inadequate. We have to consider viruses and bacteria as the third entity to be added to fauna and flora for the evolution of the latter.

The structure of chaos is fractal having self-similar structures in finer and finer scales. This is compulsory for linear unstable motions restricted by nonlinear dampings within a finite domain. The self-similarity of the chaos is the mapping function in which the structure itself is involved. Therefore, the self-similarity in chaos shows the same structure as the self-reference in the liar paradox. It is because of this same structure that the

machine which includes the chaos as its sub-routine job may be able to break the trap of the liar paradox. Our epistemological questions on cosmology are the following: 1) Can the cosmology at last understand the origin of the universe? 2) Can computer finally create? 3) Can life be formed synthetically? All of these questions are considered to have inherent relations with the liar paradox through Goedel's incompleteness theorem to which we shall now turn to discuss.

III. THE LIAR PARADOX AND GOEDEL'S INCOMPLETENESS THEOREM

"This sentence is false." This is the sentence that we cannot decide whether it is true or false. Because, if the sentence is assumed to be true, the assumption contradicts the sentence, and if the sentence is assumed to be false, the assumption means that the sentence should be true and contradicts the sentence. We can even show the paradox of the divine liar (see, Grim 1991).

There are two theorems in mathematical logic due to Goedel in the present context (see J. N. Crossley et al. 1972). The Goedel-Henkin completeness theorem tells that the universally valid formulae in the predicate calculus are provable and vice versa and that their models can always be constructed. But, at the same time, Goedel's incompleteness theorem tells that there are formulae in the predicate calculus that are neither provable nor disprovable. The proof of the incompleteness theorem goes like the logic of the liar paradox asking the provability of the formula which states that the formula is not provable.

Since the predicate calculus and the logic are essentially the same, we may understand that all the truth we know is provable in logic but there are always problems that cannot be logically judged with our present knowledge. In other words, the ideal computer, the universal Turing machine cannot create new ideas. In order to study essentially unknown problems, therefore, we must work in a higher dimensional world by introducing new experience or ideas either from the outside world or by intuition that is not entirely predicative, for instance, by yoga or by music, etc..

Cosmology is the mapping of the universe onto the human knowledge which is a part of the universe. It is, therefore, an action of self-reference. The fundamental nature of life is the reproducibility of self-similar descendants. If the life can be formed synthetically, the mechanism must be self-similarity generating. Also, a bio-computer which can create must involve the procedure to generate self-similarity hierarchy. In the following section, we will discuss if chaos can be such mechanism.

IV. SELF-CONSISTENCY OF ABSOLUTE CONTRADICTION

Paradox is different from contradiction and has resolution or model. The famous resolution by Russell for the liar paradox, "I am a liar", is the theory of type which distinguishes I in the sentence from I who is talking the sentence. It is a reasonable

resolution, since a person is not completely predicative and cannot always tell a lie. But, the paradox is strictly unavoidable only during the time when he was so speaking. Within that one second, whether he was a liar or not is uncertain. Therefore, that one second plays the role of the Planck constant in the uncertainty principle. A question then arises whether this uncertainty is empty or has structures. Goedel's incompleteness theorem implies that there can be infinite formulae. Perhaps, in the liar paradox, he was exclaiming something spiritual beyond his statement. The paradoxical uncertainty may not be empty. Actually, Shankara, an Indian philosopher in the eighth century claimed that "sunya", the zero or the nothingness, has structures.

In his epistemology, Kitaro Nishida (1870-1945) considered the paradoxical relation between the individuality and the universality such that there can be no individuality without universality and no universality without individuality, but, if they are restricted by each other, their own property, freedom and generality, respectively, will be lost. He called such paradoxical relation as "self-consistency of absolute contradiction". By means of this concept, he described the function of mind in both the pure experience before consciousness and the insight cooperating with action that are closely related to the philosophy of Zen-Buddhism (Nishida 1953).

A question arises whether it is a logically sound concept or not. Is it possible for anything to exist in absolute contradiction? If it does exist, it gives a new category of paradox. The liar paradox has proved the existence of uncertainty, and the new paradox would give the type theory of the structure of that uncertainty in the form of the self-consistency of absolute contradiction. It now remains to construct a model having literally the latter property. The discussion in the last section suggests that the model would contain a fractal structure.

A particular example of the self-identity of absolute contradiction has been modeled in the previous paper (Unno 1991) in a different context. It is the straight line segment of infinite length. The construction is quite simple as described below. A sine curve of one wave length is drawn between two points A and B, as expressed by

$$y = A \sin(2\pi x/L), \quad (0 < x < L), \quad (1)$$

L being the distance between A and B, and A the amplitude. Then, a sine curve having the wave length just half of that of the previous curve and with the amplitude (Aa) larger than half of that of the previous curve ($1 > a > 0.5$) is drawn between A and B,

$$y = Aa \sin(2\pi 2x/L). \quad (2)$$

The new curve (two wavelengths between A and B) is certainly longer than the previous curve. If the process is repeated infinite times, the limiting line will have zero amplitude (straight line) and infinite length. The n -th curve will be represented by

$$y = Aa^n \sin(2\pi 2^n x/L), \quad (3)$$

having approximately a length of $4A(2a)^n$, tending to infinity for large n . The secret of the paradox lies in the fractal structure with the fractal dimension between 1 and

2. The self similarity is there (sine curve!) as in usual fractal figures, but here length ratios between successive sine curves are different in the direction parallel to AB and perpendicular to it. The involvement of these two directions gives rise to the fractal structure in the limit. The self similarity corresponds structurally to the self-reference of the liar paradox. The latter paradox is so to say a one dimensional paradox. Therefore, in the liar paradox, the addition of other dimension, the type, dissolves the paradox. Only within one second during which a Cretan was exclaiming, "I am a liar!", he was in the self-consistency of absolute contradiction. In that, he was in a higher fractal dimension, exclaiming something spiritual which is beyond his statement. The new paradox may be called the fractal structure paradox.

The construction of the line segment of infinite length tells that there are innumerable models for the self-consistency of absolute contradiction, since there is no limit of dimensions of figures to start with. The model not only proves the logical validity of the concept but also implies that the concept is the result of the limiting procedure with amplification from a higher dimension. We can construct even a point of infinite length by diminishing the distance between A and B, since the length of the n -th curve tends to

$$l = 4A(2a)^n, \quad (2a > 1), \quad (4)$$

for the limit of large n , which is independent on L . So, one can choose L as small as possible from the beginning, which proves the existence of a point of infinite length. Quite similarly, we can start with a surface:

$$z = A \sin(2\pi x/L) \sin(2\pi y/L), \quad (0 < x, y < L), \quad (5)$$

and then, we can proceed to the n -th surface given by

$$z = Aa^n \sin(2\pi 2^n x/L) \sin(2\pi 2^n y/L), \quad (0 < x, y < L), \quad (6)$$

having the area:

$$s = 4A^2(2a)^{2n}, \quad \text{for large } n. \quad (7)$$

Since this limiting value is independent on L , we can reduce L to zero after taking n to infinity, obtaining a point of infinite surface area.

A point of infinite length and a point of infinite area can be identified literally as models of the self-consistency of absolute contradiction. It is actually possible, therefore, that "sunya", the zero, can have not only structure but also infinite structure, a statement beyond Shankara of the 8th century. The self-consistency of absolute contradiction is the basic concept of Nishida's epistemology which is in a sense the philosophical description of Zen's spirit. Since the point of infinite length or infinite area is the simplest (dimension $1+$ or $2+$) model of the self-consistency of absolute contradiction which itself is a partial interpretation (model) of the mind, the simple mathematical model of the mind should be suitably constructed in part with the structure having the property of the point of

infinite length or of infinite area (Unno 1992). The origin of the fractal structure of mind, the self-consistency of absolute contradiction, is the evolution in which the chaos is always involved. We will consider how the chaotic evolution has retained its structure in the epistemology.

V. CHAOS IN EPISTEMOLOGY AND EPISTEMOLOGY IN COSMOLOGY

It is said that the cognition consists mainly of historical understanding, beauty understanding, and logical understanding, corresponding to goodness, beauty, and truth in ethics. These three types of understandings (the epistemological dimension) are rooted mainly in the brain governing reflection, emotion, and reason, respectively, but, they are coupled and supported with each other, forming a chaotic change of mind. Here, we see a typical manifestation of the theorems of chaos, "three causes chaos and chaos drives evolution". The trace of the chaotic evolution should be the fractal structure in the self-consistency of absolute contradiction. Three hundred million years ago, amphibia developed curiosity and climbed up on land in spite of the danger of death by dryness. Thirty million years ago, mammals developed empathy with handicapped babies for living, forming family-society hierarchy. Three million years ago, human beings developed logic by inventing language. Forming the psychological dimension, curiosity, empathy, and logic cooperate to generate a new ability, the creation, with which human beings can construct the simulation model of the universe. This could be the reason why the human being can be the micro-cosmos.

Apart from the epistemological and psychological dimensions, there must be the biological dimension also, as required from the necessity of conserving individual life and species. We will not enter to discuss this dimension at all. But, the fact that life originated in the average size between the universe and the Planck length seems to be fundamental for the understanding of the simultaneous generation of the universe and the elementary particles. In addition to these internal dimensions of life, there are external space, time and material dimensions. All those internal and external dimensions combined together form a life-centered cosmology which should be equivalent to the usual scientific cosmology. As a natural consequence of the equivalence, there can be an ample chance for the epistemology to be enrolled in the scientific cosmology, as the embedding theory (Takens 1983) would imply.

The human principle in cosmology may better be understood in the same context. The presence of human beings, beginning to understand the universe after ten billion years of the universe evolution, could be taken as a principle which has significance beyond a mere boundary condition. As an example, let us consider ten or twenty-six dimensions invoked in the super-string theory that could be the basic theory for the inflation and/or dark-matter universe. These multi-dimensions are introduced for the sake of renormalization avoiding unphysical infinity. However, it is our suspicion that these multi-dimensions might be the consequence of the human frame of reference. in epistemology. Ten and

twenty-six are equal to $2(2n + 1)$ for $n = 2$ and 6 . The factor 2 would come from the principle of dualism in the judgment, i.e., positive/negative, yes/no, past/future, etc.. Wheeler's hypothesis of a single universal electron reminds us of an idea that there were neither matter nor anti-matter in the absence of the principle of judgment and that the principle sets the coordinate origin in symmetry in the middle of the matter and the anti-matter. The other factor $(2n + 1)$ would come from the embedding theory for the requirement of unambiguous description of an n -dimensional entangled space. If the chaos were in the origin of the universe, the intrinsic Lagrangian coordinate following the motion of the universe would naturally be entangled. The reason why two or two times three is preferred for n should be given from the study of the cosmological chaos.

After all, cosmology is a modeling of the universe which the human reason constructs in the human frame of reference. Being a product of chaos, Life and especially the human being have installed the chaos operating processes in the brain and in the genome. Thus, we conclude that the human being could at last understand the origin of the universe, that a computer should be able to create, once the appropriate chaos operating process is installed, and that life would be able to be formed synthetically, if the chaos generating mechanism could be regenerated by itself. However, the algorithms to find those models in a systematic way are never available in principle. The internal relations among cosmology, epistemology, and chaos should further be studied with curiosity, empathy, and logic in greater detail (cf. Hofstadter 1979).

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