High Energy Photon Beam Modeling Using Transport Theory for Calculation of Absorbed Dose Distribution

Dong Rak Choi, Ph. D.* Ha Chung Chun, M.D. and Myung Za Lee, M.D.

*Department of Radiology, Catholic University Medical College, Seoul, Korea**
Department of Therapeutic Radiology, Hanyang University Hospital, Seoul, Korea

A mathematical model is presented for the calculation of the depth absorbed dose in water phantom irradiated by high energy photon beam (10MV X-ray), based on transport theory. The parameters of this model are obtained from the experimental values which were simulated by non-linear regression process method. The calculated absorbed dose distribution is extended to 3-D by using trial function from beam profile field sizes, SSD and depth in water phantom irradiated by high energy photon beam.

The calculated values using this model are in good agreement with the measured values.

Key Words: Dosimetry, Transport theory, PDD

INTRODUCTION

It has been demonstrated that the success or failure in radiation treatment depends on the absorbed dose delivered to the tumour and this should not vary by more than a few percent from the prescribed values[1]. In fact, it is seldom possible to measure absorbed dose distribution directly in patient treated with radiaion. Data on absorbed dose distribution are almost derived from measurements in phantoms. These basic data are used in a dose calculation system devised to predict absorbed dose distribution in an actual patient[2,3]. The absorbed dose distributions in tissues depend on output and energies of photon beam and on the characteristics of tissues[2].

Historically, two distinct theories have been developed in dealing with multiple scattering problems. One is called analytical theory[4,5] and the other transport theory[6~8]. In the analytical theory, we start with basic differential equations such as the Maxwell equations or the wave equation. This is mathematically rigorous in the sense that all the multiple scattering, diffraction, and interference effects can be included. However, in practice, it is impossible to obtain a formulation which completely includes all these effects. The transport theory, on the other hand, does not start with the wave equation. It deals directly with the transport of energy through a medium containing particles. The development of the theory is heuristic and it may lack the mathematical rigor of the analytical theory. Transport theory, also called radiative transfer theory, was initiated by Schuster in 1903[7]. The basic differential equation is called the equation of transfer and is equivalent to Boltzmann's equation[9,10] used in the kinetic theory of gases and in neutron transport theory. The formulation is flexible and capable of treating many physical phenomena[11,12]. Even though transport theory gives an adequate description of many physical phenomena, a general solution of the resulting transport equation is not known. Numerical solutions of the general transport equation require extensive computer time and memory. In the majority of practical cases, it is desirable to obtain approximated solutions. An approximated method requires no more than simple algebraic operations, and has been found to give reasonably good agreement with experimental data. Recently, the methods for obtaining approximated solutions of transport equation have been extensively studied in applied optics[7,8,13].

In the present work, the absorbed dose distributions in water phantom irradiated by 10MV X-ray are calcuated by using 3-flux model based on transport theory. As a result, we can predict absobed dose rate at any point in water phantom. The obtained results are compared with the relevant experiments.

MATERIALS AND METHODS

1. Transport Theory

Let us consider a specific intensity I (**r**, **s**) (**r**: displacement vector at a given point, **s**: directional unit vector) incident upon a cylindrical elementary volume with unit cross section and length ds. The volume ds contains ρds particles where ρ is the number of particles in a unit volume. Each particle absorbed the power $\sigma_a I$ and scatters the power σ_s I, and therefore, the decrease of the specitic intensity dI (**r**, **s**) for the volume ds is expressed as

$$dI(\mathbf{r},\mathbf{s}) = -\rho ds(\sigma_a+\sigma_s)I = --\rho ds\sigma_t I. \quad (1)$$

At the same time, the specific intensity increases because a portion of the specific intensity I (**r**, **s**) incident on this volume from other directions **s**′ is scattered into the direction **s** and is added to the intensity I (**r**, **s**). In order to deterimine this contribution, let us consider a wave incident in the direction **s**′ on a particle. The incident flux density through a small solid angle dω' is given by $S_i = I(\mathbf{r},\mathbf{s}')\,d\omega'$. This flux is incident on particles in the volume ds. The power flux density S_r of the wave scattered by a single particle in the direction **s** at a distance R from the particle is then given by $S_r = [|f(\mathbf{s},\mathbf{s}')|^2/R^2]S_i$, where f (**s**, **s**′) is the scattering amplitude. The scattered specific intencity in the direction **s** due to S_i is therefore

$$S_r R^2 = |f(\mathbf{s},\mathbf{s}')|^2 S_i = |f(\mathbf{s},\mathbf{s}')|^2 I(\mathbf{r},\mathbf{s}')d\omega'. \quad (2)$$

Adding the incident flux from all directions **s**′, the specific intensity scattered into the direction **s** by ρds particles in the volume ds is given by

$$\int_{4\pi} \rho ds|f(\mathbf{s},\mathbf{s}')|^2 I(\mathbf{r},\mathbf{s}')d\omega' \quad (3)$$

where the integration over all ω' is taken to include the contributions from all directions **s**′. We can express equation (3) using phase function p (**s**, **s**′):

$$p(\mathbf{s},\mathbf{s}') = \frac{4\pi}{\sigma_t}\left|f(\mathbf{s},\mathbf{s}')\right|^2, \quad \frac{1}{4\pi}\int_{4\pi} p(\mathbf{s},\mathbf{s}')d\omega = \frac{\sigma_s}{\sigma_t}. \quad (4)$$

Adding the contributions equation (1) and (3), we get the equation of transfer:

$$\frac{dI(\mathbf{r},\mathbf{s})}{ds} = -\rho\sigma_t I(\mathbf{r},\mathbf{s}) + \frac{\rho\sigma_t}{4\pi}\int_{4\pi} p(\mathbf{s},\mathbf{s}')\,I(\mathbf{r},\mathbf{s}')d\omega'. \quad (5)$$

In order to simplify the above equations, we assume that high energy photon beam experiences highly forward scatteing in homogeneous medium. As a result, the transport equations can be solved analytically by using 3-flux model proposed in this work. Scattering of high energy photon beam in random media is approximated by creating a simple pattern of weighted directional fluxes. As shown in Fig. 1, the scattering pattern is represented by 2 fluxes and a third flux is introduced to represent the collimated photon beam. Then, the differential equations for 3-flux model based on transport theory can be written as follows.

$$\frac{dI_c(z)}{dz} = -\gamma I_c(z), \quad (6)$$

$$\frac{dI_f(z)}{dz} = -\gamma I_f(z) + \gamma\,(P_f I_f(z) + P_b I_b(z)) + \gamma P_f I_c(z), \quad (7)$$

and

$$\frac{dI_b(z)}{d(-z)} = -\gamma I_b(z) + \gamma(P_f I_b(z) + P_b I_f(z)) + \gamma P_b I_c(z) \quad (8)$$

where, $\gamma \equiv \rho\sigma_t$, $\sigma \equiv \sigma_s$, $P_f + P_b = \frac{\sigma}{\gamma}$ hold and, for highly forward scattering ($P_b = gP_f$), $P_f = \frac{\sigma}{\gamma(1+g)}$ and $P_b = \frac{g\sigma}{\gamma(1+g)}$ hold.

I_f and I_b represent radiances to +z and -z

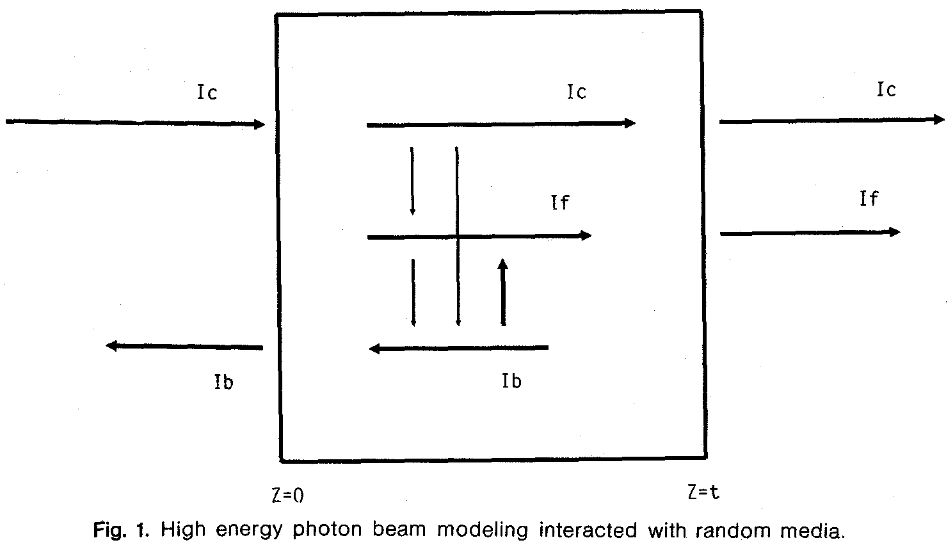

Fig. 1. High energy photon beam modeling interacted with random media.

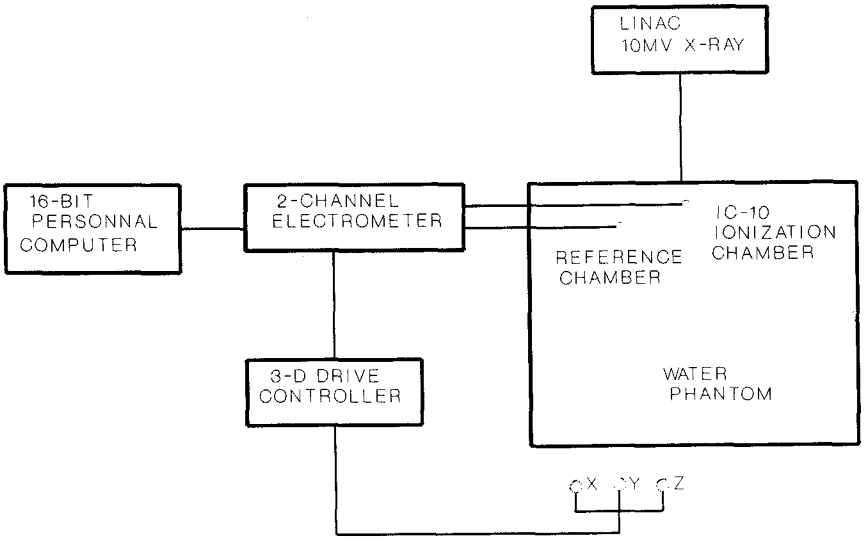

Fig. 2. WP-600 dosimetry system.

directions respectively, and I_c represents collimated radiance to +z direction. Also, P_f and P_b represent scattering contributions to the forward and the backward directions. By solving Eq. (6), (7), and (8), we can obtain the following results.

$$I_c(z) = C_0 e^{-rz}, \tag{9}$$

$$I_f(z) = C_1 e^{az} + C_2 e^{-az} - C_0 e^{-rz}, \tag{10}$$

and

$$I_b(z) = C_1 b e^{az} + \frac{C_2}{b} e^{-az} \tag{11}$$

where, $a = \sqrt{(n(n+2g\sigma/(1+g))}$ and

$$b = \frac{(1+g)(n+a)}{g\sigma} + 1 \text{ hold.}$$

In the case when the medium has finite thickness, the boundary conditions can be written as $I_c(z=0) = I_o$. $I_f(z=0) = 0$, and $I_b(z=t) = 0$. From these boundary conditions, we can solve the Eq. (9), (10), and (11) completely. The scattered radiance I_f and I_b must be added to the collimated radiance I_c. Therefore, the total radiance distributions in random media irradiated by photon beam are as follows.

$$I(z) = I_c(z) + I_f(z) + I_b(z)$$

$$\cong I_c(z) + I_f(z) = \frac{I_0}{1 - b^2 e^{2at}} (e^{az} - b^2 e^{2at-az}). \tag{12}$$

2. Determination of Parameters

The absorbed dose distributions in water phantom irradiated by 10MV X-ray are measured by using WP-600 dosimetry system. WP-600 dosimetry system consists of IC-10 ionization chamber, water phantom, WP-5006 electromenter, dual processor based 3 dimensional drive control, and Duet-16 personal computer (Fig. 2). To compare experimental values with calculated values derived from 3-flux model, the absorbed doses are measured by moving ionization chamber 3-dimensionally in water phantom irradiated by 10MV X-ray at various field sizes and at 100 cm SSD (Source-Surface Distance). Dmax (Maximum Dose) is defined at depth 2.5 cm.

3. Dose Distribution Model

Basic dose distribution data are usually measured in water phantom which closely approximates the radiation abosrption and scattering properties of muscle and other soft tissues[2]. As the beam is incident on a phantom, the absorbed dose in the phantom varies with depth. This variation depends on many conditions such as beam energy, depth, field size, distance from source, and beam collimation system[2,3,14].

In order to compare the experimental values with the calculations, we should modify the absorbed dose distributions as follows.

$$\text{Dose rate}(W, L, x, y, z) = [-p_1 W_0 + p_2 I_n (p_3 W_0 + p_4)]$$

$$\times \frac{e^{a(Z-Dmax)} - (b + c\sqrt{W_0})^2 e^{2at - a(Z-Dmax)}}{1 - (b + c\sqrt{W_0})^2 e^{2at}}$$

$$\times \frac{1}{1 + ue^{(|x| - W/2 - Wz/(2*SSD))/V}}$$

$$\times \frac{1}{1 + ue^{(|y| = W/2 - Wz/(2*SSD))/V}}$$

$$\times \frac{(SSD+Dmax)^2}{(SSD+z)^2+x^2+y^2} \qquad (13)$$

where, W and L represent field width and field length respectively. And Wo=2WL/(W+L). In Eq. (13), paramenter p_1, p_2, p_3, p_4, a, b, c, u, and v must be determined from the measured values. The first term of the right-hand side represents output dependence on field size, and the second term Percent Depth Dose on field size and on depth. The third and fourth terms of the right-hand side represent beam profile in XY-plane perpendicular to incident beam and the last term of the right-hand side represents inverse square law depended on source-to-detector distance.

For various field sizes (10 cm×10 cm and 20 cm×20 cm) and depths (Dmax, 5 cm, and 15 cm), we should obtain the absorbed doses rates on the central axis and on beam edge (off-axis) in order to determine the values of parameters. We now obtain the parameters of Eq. (13) by using non-linear regression process method. For 10MV X-ray, obtained parameters are shown in table 1.

Table 1. The Parameters of Absorbed Dose Distributions for 10MV X-ray Derived from 3–Flux Model

Parameter	Value
p_1	−0.008827
p_2	0.3560
p_3	1.039
p_4	10.67
a	0.0026
b	0.9944
c	0.01451
u	0.8942
v	0.2619

RESULTS

One way of characterizing the central axis dose distribution is to normalize dose at depth with respect to dose at reference depth[2,3]. The quantity percentage depth dose may be defined as the quotient of the absorbed dose at any depth to the absorbed dose at a fixed reference depth along the central axis of the beam. In the present work, we will define a fixed reference depth to 2.5 cm in the case of 10MV X-ray. The PDD curves of 10MV X-ray for various field sizes (10 cm×10 cm, 20 cm×20 cm, and 30 cm×30 cm) are shown in Fig. 3.

Compared with the theoretical values and experimental ones of PDD, the differences are below the error of 2%. Fig. 4 shows the beam profile of 10MV X-ray at various depths (2.5 cm, 5 cm, 15 cm, 25 cm) for field size 10 cm×10 cm. Because of rapid decrease and inaccurate measurement of absorbed dose at penumbra region, comparison with the calculated values and the measured values of absorbed dose rate at any

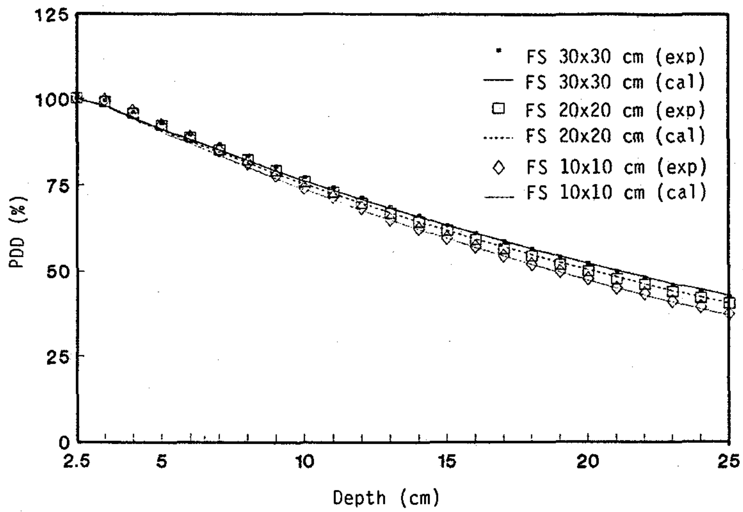

Fig. 3. The PDD curves of 10MV X-ray for various field sizes.

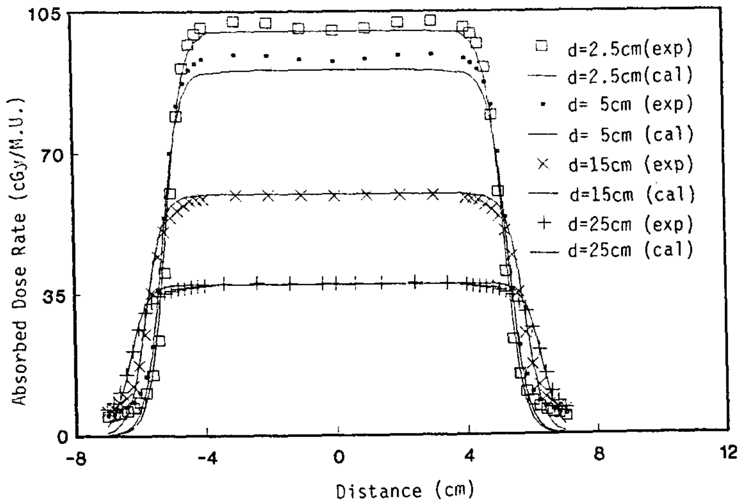

Fig. 4. The Beam Profile of 10MV X-ray for various depths (field size = 10 cm × 10 cm).

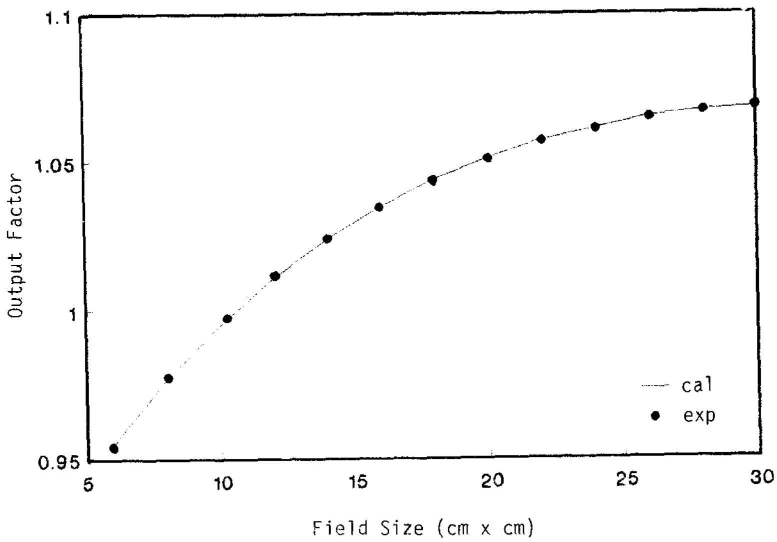

Fig. 5. The comparison of output factors derived from trial functions and experiments.

point shows that the differences are above the error of 10%. But, in practice, this comparison is useless in terms of isodose chart. It is desirable to compare with the calculated distances and the measured ones for a given absorbed dose rate. From this point of view, Fig. 4 shows that the differences are below the error of 2 mm. Fig. 5 compares the prediction for output factor with the relevant experimental data. As shown in this figure, the trial function for output factor is reasonable for calculating output factor for various field sizes.

DISCUSSION

The absorbed dose distributions in water phantom irradiated by 10MV X-ray are calculated by using 3-flux model based on transport theory[7,8]. This simple model predicts the absorbed dose distribution accurately.

Therefore, the proposed model is reasonable for calculating absorbed dose rates in water phantom irradiated by the high energy photon beam. However, the absorbed dose distributions at build-up region[2,3] cannot be calculated because Fresnel reflection is not considered.

In conclusion, we have proposed a simple and practical beam model which can be used in predicting the absorbed dose distributions accurately in tissues irradiated by high energy photon beam. Gap calculation, off-axis ratio, and absorbed dose distribution in various SSDs can be calculated with simple method, when this model is applied.

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= 국문초록 =

흡수 선량 분포의 수송방정식을 이용한 10 MV X-선의 모델

가톨릭대학교 의과대학 방사선과학교실

최 동 락

한양대학교 의과대학 치료방사선과학교실

전 하 정 · 이 명 자

물팬톰내에 조사된 10 MV X-선의 심부율을 입자의 수송이론을 근거로 한 1차원적인 모델을 이용하여 계산하였다. 계산된 이론식의 매개상수는 9개로 줄일 수 있었으며 실측치를 이용하여 비선형 회귀분석 방법으로 얻을 수 있다. 조사면과 선원간의 거리 및 깊이에따른 3차원적인 흡수선량 분포의 계산식은 고에너지 광자선이 조사된 물팬톰내에서의 Beam Profile에 대한 시도함수를 이용하여 수송이론에의한 심부율계산을 3차원적으로 확장하였으며 흡수 선량 분포는 3차원적 위치의 함수로 널리 계산할 수 있다.

이 모델을 사용하여 계산된 이론값은 실험값과 ±2% 이내의 만족할만큼 잘 일치하였다.