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A Two-Stage Batch Production System with Integer Ratio Lot Requirements Lot－Sizing Policy

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정수비율 조달정책하에서의 이단계 배치 생산체계

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Abstract

This paper deals with a production-inventory model for two-stage batch production system. We propose a lot-sizing scheme which combines the integer multiple lot requirements (IMLR) policy and the integer split lot requirements (ISLR) policy. An iterative search procedure for optimal decision variables is presented and numerical examples are solved to illustrate the validity of the model. The results show that the proposed scheme outperforms the existing policies.

1. Introduction

Two-stage batch production system has been treated extensively under various conditions since Goyal[1] proposed with the name of integrated inventory model. In this system, the economic batch size for the final product and economic order quantities for raw materials are determined simultaneously in such a way that the total variable cost of the production inventory system is minimized. Korgaonker[5] developed an iterative search algorithm which can be applied to the multi-stage multi-product batch production systems.

Park[7], Kim and Hwang[3] and Raafat[8] developed algorithms which can be used for decaying stocks. Kim and Chandra[4] suggested a heuristic procedure which classifies the raw materials into groups so that the total inventory cost per unit time is minimized under the assumption that all the raw materials within a group have the same time interval between successive orders.

However, all these studies assumed that the lot size of each raw material is such that it satisfies the demand requirements generated by an integer

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multiple of production lots of its final product. Moily[6] referred to this type of policy as the "integer multiple lot requirements(IMLR)" lot-sizing.

Under another lot-sizing policy, namely lot-splitting policy, the lot size of a raw material is allowed to cover only a fraction of final product. Jensen and Khan[2] and Szendrovits[9, 10] provided examples in which lot splitting policies result in better solutions to the multi-stage lot-sizing problem. Moily[6] proposed a lot-splitting policy in which an integer number of production lots of each component item are required to satisfy the demand requirement caused by a single production lot of its parent item. This policy is referred to as the "integer split lot requirements(ISLR)" lot sizing.

Since in real world problems, it may be more economical to employ IMLR as lot-sizing policy for some raw materials and ISLR for the rest, we define this type of policy the "integer ratio lot requirements(IRLR)" lot-sizing. The term "integer ratio" means either the ratio of the order size to the demand requirement of the raw material caused by each production run or its reciprocal is an integer.

The objective of this paper is to formulate the two-stage batch production system under the IRLR lot-sizing policy and to provide a solution procedure to this model.

2. The Mathematical Model

The following assumptions are employed in this paper regarding the two-stage batch production system.

1. the demand for the final product is continuous and time invariant;
2. the production rate for final product is constant;
3. time horizon is infinite;
4. lead times are zero;
5. no stock-outs are permitted.

Under the IRLR lot-sizing policy, the m raw materials are partitioned into two mutually exclusive and exhaustive groups I_1 and I_2 such that $I_1 \cup I_2 = \{1, 2, \cdots, m\}$ and $I_1 \cap I_2 = \phi$. The IMLR lot-sizing policy is employed for the raw materials contained in I_1, and the ISLR policy for the raw materials in I_2. The problem we want to solve is how to determine the partition with the objective of minimizing the total inventory cost of the production-inventory system.

The following symbols are used in the cost expressions.

(1) For the final product;

d : demand per year,

p : production rate per year($p > d$),

$\rho = d/p$,

s_p : manufacturing set-up cost,

h : stock holding cost per unit per year,

m : number of raw materials required for the product,

q : economic production lot-size.

(2) For the j^{th} raw material($j = 1, 2, \cdots, m$);

r_j : amount of raw material required to make one unit of the product,

s_j : cost of placing a purchase order,

h_j : stock holding cost per unit per year,

q_j : economic order quantity,

k_j : relative order quantity in positive integer and

$$k_j = \begin{cases} q_j / r_j q & \text{if } j \in I_1 \\ r_j q / q_j & \text{if } j \in I_2 \end{cases}$$

with $r_j q$ = amout required for each production run.

The production-inventory system characterized by the above ordering policy is represented in the Fig. 1.

Since the demand of the final product is assumed to be continuous and time-invariant, the inventory level of the product follows the saw-toothed shape. With the production batch size q, the annual variable cost from production setup and inventory holding is

$$ds_p/q+(1-\rho)hq/2 \quad \cdots\cdots (1)$$

The inventory levels of the raw materials behavior differently in accordance with to which group the raw material belongs. The annual variable cost of the j^{th} raw material can be expressed as the followings.

$$V_{1j}(q,k_j)=ds_j/k_jq+(\rho+k_j-1)h_jr_jq/2, \text{ for } j\in I_1 \quad \cdots\cdots (2)$$

$$V_{2j}(q,k_j)=ds_jk_j/q+\rho h_jr_jq/2k_j, \text{ for } j\in I_2 \quad \cdots (3)$$

From a necessary condition for a local minimum of the discrete functiion, with q fixed, the optimal relative order quantities, k_j, which minimize $V_{ij}(q, I_i)$ are the smallest positive integers which satisfy eq. (4) and (5), respectively.

$$k_j^*(k_j^*+1)\geq 2ds_j/h_jr_jq^2, \quad j\in I_1 \quad \cdots\cdots (4)$$

$$k_j^*(k_j^*+1)\geq h_jr_jq^2/2ps_j, \quad j\in I_2 \quad \cdots\cdots (5)$$

The annual variable cost for the raw materials from purchase order, and inventory holding is

$$\sum_{j\in I_1} V_{1j}(q,k_j)+\sum_{j\in I_2} V_{2j}(q,k_j)$$

$$=\sum_{j\in I_1}(ds_j/k_jq+(\rho+k_j-1)h_jr_jq/2)+\sum_{j\in I_2}(ds_jk_j/q+\rho h_jr_jq/2k_j) \quad \cdots\cdots (6)$$

Hence the total annual variable cost for the system is given by

$$TC(q,k_j,I_i)=ds_p/q+(1-\rho)hq/2$$

$$+\sum_{j\in I_1}(ds_j/k_jq+(\rho+k_j-1)h_jr_jq/2)+\sum_{j\in I_2}(ds_jk_j/q+\rho h_jr_jq/2k_j) \quad \cdots\cdots (7)$$

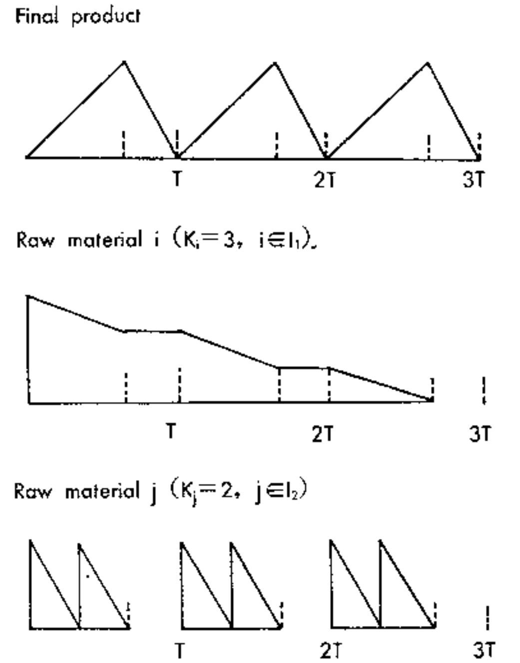

Fig. 1. Behavior of the Production/Inventory System.

By differentiation and after some calculus, we find that the optimal production batch size which minimizes eq. (7) becomes

$$q^*=\{2d(s_p+\sum_{j\in I_1}s_j/k_j+\sum_{j\in I_2}s_jk_j)/((1-\rho)h+\sum_{j\in I_1}(\rho+k_j-1)h_jr_j+\sum_{j\in I_2}\rho h_jr_j/k_j)\}^{0.5} \quad \cdots\cdots (8)$$

The corresponding minimum total cost is

$$TC(q^*,k_j,I_i)=\{2d(s_p+\sum_{j\in I_1}s_j/k_j+\sum_{j\in I_2}s_jk_j)((1-\rho)h+\sum_{j\in I_1}(\rho+k_j-1)h_jr_j+\sum_{j\in I_2}\rho h_jr_j/k_j)\}^{0.5} \quad \cdots\cdots (9)$$

Note that in order to find q^*, we have to partition the raw materials into two groups, I_1 and I_2, optimally and to find the optimum relative order quantities, k_i^*. The following property can be used to find the optimum partition (I_1, I_2). It is observed that with q and partition (I_1, I_2) prescribed, $TC(q, k_j, I_i)$ is separable for each raw material, and thus k_j^* can be derived independently from eq. (4) and (5).

Property. For a given production batch size q, the following statements can be made.

(a) When $\{2ds_j/h_jr_j\}^{0.5} \leq q\rho^{0.5}$, the ISLR policy is preferable to the IMLR policy for raw material j.

(b) When $\{2ds_j/h_jr_j\}^{0.5} > q\rho^{0.5}$, the IMLR policy is preferable to the ISLR policy for raw material j.

(pf) Proof of statement(a) : Since the annual variable cost of raw materials ar separable at a given value of q, it is sufficient to show that if $\{2ds_j/h_jr_j\}^{0.5} < q\rho^{0.5}$ holds, then $\min_{k_j=1,2,\cdots} V_{1j}(q, k_j) \geq \min_{k_j=1,2,\cdots} V_{2j}(q, k_j)$. The given condition can be converted to $h_jr_jq^2 > 2ps_j$. Let $f(x) = V_{1j}(q, x) - V_{2j}(q, x)$ for real $x \geq 1$. It can be shown that f(x) is continuous and differentiable for all $x \geq 1$. Differentiation of f(x) with respect to x and substitution of the condition yields

$$f'(x) = (h_jr_jq^2(1+d/px^2) - 2ds_j(1+1/x^2))/2q$$

$$> (2ps_j(1+d/px^2) - 2ds_j(1+1/x^2))/2q$$

$$= (p-d)s_j/q$$

$$> 0,$$

i.e., f(x) is strictly increasing function and $f(1)=0$. Therefore, $f(x) \geq 0$ for all $x \geq 1$ and so does for positive integers ≥ 1. Noting that $V_{1j}(q, x) \geq V_{2j}(q, x)$ for all positive integers x implies $\min_{x=1,2,\cdots} V_{1j}(q, x) \geq \min_{x=1,2,\cdots} V_{2j}(q, x)$, we conclude that $\min_{k_j=1,2,\cdots} V_{1j}(q, k_j) \geq \min_{k_j=1,2,\cdots} V_{2j}(q, k_j)$ i.e., the ISLR policy is preferable.

Proof of statement(b) : The second statement can be proved with the similar argument. (Q.E.D.)

3. Heuristic Procedure for Minimization

As seen above, q^* of eq. (8) cannot be obtained without the prior knowledge of (I_1, I_2) and k_j which in turn cannot be determined without q. Therefore, the following search procedure is proposed.

Step 1. Calculate $X_j = \{2ds_j/h_jr_j\}^{0.5}$, $j = 1, 2, \cdots, m$, and renumber all the raw materials in the sense of nonincreasing sequence.

Step 2. To obtain the initial estimate of q^*, let $I_1 = \{1, 2, \cdots, m\}$, $I_2 = \phi$, $k_j = 1$ for all $j \in I_1$ and compute q^* from eq. (8).

Step 3. Find the largest value of j satisfying $X_j \geq q^*\rho^{0.5}$ and denote by M. Update the partition such as $I_1 = \{1, 2, \cdots, M\}$ and $I_2 = \{M+1, M+2, \cdots, m\}$.

Step 4. Obtain $k_j^*(q^*, I_1)$, $j \in I_1$ and $k_j^*(q^*, I_2)$, $j \in I_2$ by applying eq. (4) and (5), respectively. Revise q^* from eq. (8) and go to step 3.

Step 5. Repeat Step 3 and Step 4 till convergences of decision variables are obtained.

Step 6. Compute the optimal production batch size, q^*, and the economic order quantities for the raw materials, q_j^*, as follows.

$$q^* = \{2d(s_p + \sum_{j \in I_1} s_j/k_j^* + \sum_{j \in I_2} s_jk_j^*)/((1-\rho)h + \sum_{j \in I_1} (\rho + k_j^* - 1)h_jr_j + \sum_{j \in I_2} \rho h_jr_j/k_j^*)\}^{0.5} \quad \cdots\cdots (10)$$

$$q_j^* = k_j^* r_j q^*, \ j \in I_1, \quad \cdots\cdots (11)$$

$$q_j^* = r_j q^*/k_j, \ j \in I_2 \quad \cdots\cdots (12)$$

4. Numerical Example

To illustrate the validity of the model and the algorithm devgeloped, a real world problem from newsprint manufacturing company is considered. Waste paper and ground pulp are two major raw materials in newsprint manufacturing. To produce one unit of newsprint, 0.35 units of waste paper and 0.715 units of ground pulp are needed. These and other relevant data are given in Table 1. The problem is to determine simultaneously an optimal production batch size and an optimal lot size of each raw material that minimize an annual variable cost of the inventory system. Applying the iterative algorithm to the case problem, we obtain $k_1^*=2$(i.e., waste paper is ordered once every two manufacturing setups) and $k_2^*=1$(i.e., ground pulp is ordered every manufacturing setup). The optimal production batch size q^* of newprint and the corresponding minimum total variable cost are 5296 units and $149550.5/year, respectively.

Now, the proposed policy is compared with two other policies, (1) Goyal's[1] IMLR policy and (2) The ISLR policy of Moily[6]. To facilitate the comparisions, it is assumed that the ordering cost, s_j, for each raw material is the same, and the setup cost, s_p, for the final product is expressed as a multiple M of s_j. With $s_1=s_2=\$1500$/order, $s_p=\$1500\times M$/setup, and other parameter values being equal to the case problem, various values of M are examined.

For each policy, we determine the minimum total costs and the percentages of cost savings achieved by the proposed method over the competing methods. The results are shown in Table 2. For instance, when the setup cost of the final product is ten times as large as the ordering cost of the raw material(i.e., M=10), the proposed method generates the minimum annual cost of $266300 and achieves 0.8 percent cost savings over the Goyal's method. If M=0.1, the amount of the savings can reach 25.38 percent compared to the Moily's policy. Fig. 2 shows the effect of the ratio M of the manufacturing setup cost to the ordering cost on the cost savings. It can be observed that with the raw material ordering cost being fixed, the cost savings over IMLR increase as the manufacturing setup cost increases, while those over ISLR policy increase as the manufacturing setup cost decreases. These phenomena can be explained by

Table 1. Parameter Values for the Case Problem

Parameter	final product	raw materials	
	newsprint	waste paper (j=1)	ground pulp (j=2)
d : demand rate (tons/year)	80000		
p : production rate (tons/year)	130000		
s_j : setup/order cost ($/setup)	3000	1500	1200
h_j : holding cost ($/unit/year)	48	9.2	10.4
r_j : raw materials required per unit of newsprint (units/unit)		0.35	0.715

Table 2. Existing Policies vs. Proposed Policy

M*1	Minimum Annual Cost (unit : $1000)			Cost Savings (%)*2	
	Goyal	Moily	proposed	over Goyal	over Moily
0.001	72.1	109.6	72.1	0.00	52.06
0.01	76.4	109.9	76.4	0.00	43.77
0.1	89.6	112.3	89.6	0.00	25.38
1	130.2	134.2	130.2	0.00	3.11
10	268.4	266.3	266.3	0.80	0.00
100	782.6	720.7	720.7	8.59	0.00
1000	2453.	2160.	2160.	13.56	0.00
10000	7750.	6711.	6711.	15.47	0.00

*1 $M = \dfrac{\text{the setup cost of the final product}}{\text{the ordering cost of the raw material}}$

*2 2 cost savings(%)

$$= \frac{[\text{cost by competing policy} - \text{cost by proposed policy}]}{\text{cost by proposed policy}} \times 100$$

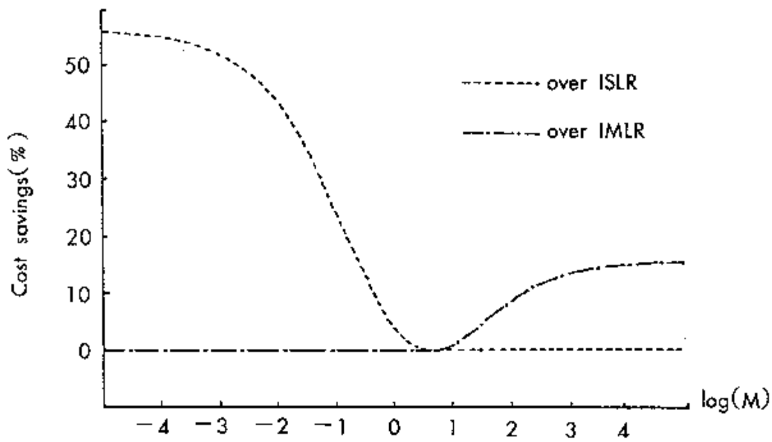

Fig. 2. Cost Savings of IRLR Policy.

the trade-off among setup, ordering and holding costs of the inventory system.

That is, with large manufacturing setup cost, i.e., M is large, the production lot size tends to be large which, in turn, makes the quantity of raw materials required for the production run large. Therefore, it is more economical to split the required raw material into several small batches. This explains why IMLR is not effective when M is large. Contrary to the first case, suppose the manufacturing setup cost is small(i.e., M is small), the production batch size tends to be sma...

In this case, the quantity of the raw material required for one batch of the final product becomes small and so it may be more economical to increase the ordering quantity of the raw materials such that it can be sufficient to cover more than one production cycle. This explains why the proposed model outperforms ISLR policy. Similar arguments can be made with respect to the holding costs of the product and raw materials.

5. Concluding Remarks

In this paper, we propose a new lot sizing policy for the lot-sizing problem in a two-stage batch production system. The proposed policy is developed based on the IMLR policy of Goyal[1] and the ISLR policy of Moily[6]. From the numerical example, we ascertain that the IRLR policy always outperforms both the IMLR and ISLR policies. The reasons why the proposed policy can generate a better solution are explained in terms of the ratio M of the setup cost to the raw material ordering cost. Additional computational experiences indicate that the procedure converges in short steps.

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