# Real Time Scheduling for Computer-Aided Manufacturing (CAM) Systems with Instance-Based Rules

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CAM에서의 사례의존규칙을 이용한 실시간 일정계획

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## Abstract

An expert scheduling system on real time basis for computer-aided manufacturing systems has been developed. In developing expert scheduling system, the most time-consuming job is to obtain rules from expert schedulers. An efficient process of obtaining rules directly form the schedules produced by expert schedulers is proposed. By the process, a set of complete and minimal set of rules is obtained. During a real time scheduling, when given information on possible values of elements, the rules produce possible values of decision elements, where logical explanations of the result may be offered in terms of chaining rules. The learning and scheduling processes have been simulated with an automated manufacturing line engaged in the production of circuit boards.

## 1. Introduction

CAM systems are drawing increasing interest because of their capability in responding to various situation such as changes in product demand. The power of those systems, however, is very depending on the efficiency of the real time scheduling technique. In most cases, the problem of real time scheduling in a CAM system is very hard to solve because it cannot be well-formulated in a mathematical programming or other models where a good solution can be found in a reasonable time. Therefore, many systems use fixed rules which does not consider the overail situation of the systems so that the performance of them can be good or very bad in a dynamic situation. Recently, expert

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scheduling systems (Thesen and Lei, 1986; Yih, 1990) was introduced to overcome the difficulty. They have shown the superior performance of an expert scheduling system in a specific domain. With an expert scheduling system, a schedule is automatically selected by a rule chosen from a knowledge base for a specific situation. The performance of an expert system is determined by the quality and reliability of the rules in the knowledge base.

However, the acquisition of rules suffer from the problem of high cost and incompleteness (Nisbett and Wilson, 1977; Bainbridge, 1979; Ericsson and Simon, 1984; Collins, 1985). Conventionally, the acquisition of rules is carried out by interviewing human experts and analyzing collected verbal protocols. Most of the methods require the experts to describe or explain the process of decision making. In some domains, the experts cannot accurately express how they make decisions. The acquiring process may even change experts' view of the problem.

To avoid the difficulty, a different method of rule-acquisition which does not need direct involvement of human experts can be used(Quinlan, 1979; Dietterich and Michalski, 1985; Kass and Leake, 1987. Rhee and Yih, 1991). That is, the rules are automatically learned from schedules produced by expert schedulers. The set of rules must be complete and non-redundant to minimize cost. In this paper, an expert scheduling system with a process of learning a complete and minimal set of rules is proposed. With the learned rules, each decision making in a real time scheduling can be performed by an individueal rule, or by chaining rules. The process of rule acquisition and decision making has been conducted with an automated manufacturing line engaged in the production of circuit boards.

### 2. Instance Patterns and Rules

In Fig. 1. a simplified automated line engaged in producing circuit boards is represented. The line consists of chemical process tanks, an input buffer and an output buffer. There is a material handling robot in a track which transports a job among buffers and tanks. Each buffer and tank can contain a job at a time. Given a new job entry in Input buffer, it can be moved to the tanks by a prespecified sequence. To make the problem simple, it is assumed that each circuit board should

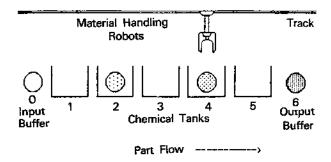


Fig. 1. A simplified circuit boards production line.

pass through the sequence of Tank 1, Tank 2, Tank 3, Tank 4, and Tank 5. Whenever a completed job is moved to Output buffer, it is automatically shipped out. The quality of a produced circuit board is determined by the time of length it spent in each tank. For each circuit board and each tank, a safe time length, [T\*, T\*+tolerance], is prespecified, (t\* is an optimal time length,) If a circuit board spends within the safe time length in each tank, it becomes as good product. On the other hand, if it stays shorter than the safe time length in a tank, it becomes a bad product. And if it stays longer than the safe time period in a tank, the product will be spoiled. In this research, it is assumed that the optimal time length can be different for each job but the tolerance is set to 24 minutes for all jobs and tanks. The objective of the system is to produce as many good circuit boards as possible in a period. At a time, the status of the system can be represented with the following elements:

E<sub>0</sub>=1, if Input buffer contains a new part, =0, otherwise.

 $E_i = T_i$ , if there is not a spoiled job in Tank i, where

 $T_i$  is the remained time until safe time length is up for the job in Tank i.

=0, if there is no job in Tank i,

=-1. if the job in Tank 1 is spoiled,

for  $i=1, 2, \dots 5$ .

 $E_6$ =the current position of robot(0, 1, 2, ...6).

Given a status of the system, the next move of the robot should be determined. The move of the robot can be represented with a couple of elements:

 $E_7$ = the position from which the contained job is to be moved (0, 1, 2,...5).

 $E_8$ = the position into which the job is to be moved (1. 2....6).

A real time schedule can be represented as a vector of the elements. That is,

 $(E_0, E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8)$ 

represent a schedule that given a status of the system is by the values of  $E_0$ ,  $E_1$ , ..., and  $E_6$ , the next move of a job is from  $E_7$ , to  $E_8$ . Such a vector will be called an "instance pattern." In this research, the value range of each  $E_i$ , for i=1, 2, ..., 5, is divided by several ranges and any value in a same range will be regarded as same. That is, for i=1, 2, 3, 4, and 5,

 $E_i = 1$ , if  $24 < T_i$ ,

=2, if  $16 < T_i < 24$ ,

=3, if  $8 < T_i < 16$ ,

=4, if  $0 < T_i < 8$ ,

and  $E_i$  can be 0 and -1, as described above. Note that we can interpret the meaning of the value  $E_{i*}$  i=1, 2, 3, 4, and 5,

if  $E_i=1$ , the job in Tank i is not ready,

if  $E_i=2$ , the job in Tank i is ready.

if E<sub>i</sub>=3, the job in Tank i is in hurry, and

if E<sub>i</sub>=4, the job in Tank i is urgent

for moving out to make a good product. Given a situation, that is, the values of  $E_0$ ,  $E_1$ ,  $E_2$ ,  $E_3$ ,  $E_4$ ,  $E_5$ , and  $E_6$ , a recommendable decision of the next move of the robot, that is, the values of  $E_7$ , and  $E_8$ , can be described by a rule. For example, we can imagine a rule,

if  $E_1=4$ ,  $E_2=0$ , and  $E_6=1$ ,  $E_7=1$  and  $E_8=2$ .

Such a rule can be obtained from the schedules of expert schedulers, and then, the rule is called an "instance-based rule." (For simplicity, it will be called just a rule).

From now on, the general concept of instance patterns and rules are explained. In general, let PATSET={PAT<sub>1</sub>, PAT<sub>2</sub>, ...} be a given set of instance patterns and  $E_i$ , for i=1, 2, ..., N, denote the i-th element in an instance pattern where N is the number of elements in each instance pattern. It is assumed that in each instance pattern, the value of each  $E_i$  is given. Let  $e_i[PAT]$  be the value of  $E_i$  in PAT. For example, assuming that N=4, if PAT=(1, 2, 1, 2), then  $e_i[PAT]=1$ ,  $e_2[PAT]=2$ , etc. From PATSET, we can find a relation of values of elements, or a rule, such as

"if  $E_1=1$  or 2 and  $E_2=2$ , then  $E_1=1$ ,  $E_2=2$ ,  $E_3=1$  or 2, and  $E_4=1$ "

Let's denote this rule by Rule. Rule, will be represented as

Rule<sub>a</sub>:  $(1 \lor 2, 2, *, *) \rightarrow (1, 2, 1 \lor 2, 1)$ , where " $\checkmark$ " means "or" and "\*" means "any value of E<sub>i</sub> existed in PATSET". For any rule rule<sub>x</sub>, let IF(Rule<sub>x</sub>) and THEN(Rule<sub>x</sub>) denote the left part and the right part, or called the "if" part and the "then" part, of Rule<sub>x</sub>, representing the condition and the conclusion of Rule<sub>x</sub>, For instance,

IF(Rule<sub>a</sub>) =  $(1 \lor 2, 2, *, *)$  and THEN(Rule<sub>a</sub>) =  $(1, 2, 1 \lor 2, 1)$ .

For convenience, "\*" will be used only in "if" parts of rules (not "then" parts). In this paper,  $\pi$  will denote an "if" part or a "then" part of a rule that can be formed from PATSET and

 $V_i(\pi)$ 

will denote the set of values of  $E_i$  assigned in  $\pi$ . For example,

 $V_1(IF(Rule_2)) = \{1, 2\},\$ 

 $V_2(IF(Rule_*)) = \{2\},$ 

V<sub>3</sub>(IF(Rule<sub>a</sub>))={all the values of E<sub>3</sub> existed in

PATSET<sub>1</sub>, and

 $V_4(IF(Rule_a)) = \{all \text{ the values of } E_4 \text{ existed in } PATSET\}.$ 

Note that for any rule rule<sub>x</sub>,  $V_i(THEN(Rule_x))$  $\subseteq V_i(IF(Rule_x)), \forall i$ , Assume that PATSET is given as follows:

#### Example 1: Instance patterns:

 $PAT_1 = (1, 1, 1, 1),$ 

 $PAT_2=(1, 1, 2, 2),$ 

 $PAT_3=(2, 1, 2, 2),$ 

 $PAT_4=(3, 2, 2, 2),$ 

 $PAT_5=(3, 3, 2, 3),$ 

 $PAT_6 = (3, 2, 2, 4).$ 

Form PATSET, we can form a rule,

Rule<sub>1</sub>:  $(1, *, *, *) \rightarrow$ 

 $(1, 1, 1 \lor 2, 1 \lor).$ 

which is interpreted as "if  $E_1=1$ , then  $E_1=1$ ,  $E_2=1$ ,  $E_3=1$  or 2 and  $E_4=1$  or 2" and is observed as such in PATSET because the condition " $E_1$  is 1" is satisfied only in PAT<sub>1</sub> and PAT<sub>2</sub>,  $E_1=1$ ,  $E_2=1$ ,  $E_3=1$  or 2,  $E_4=1$  or 2, Let

PATSET[π]

be the subset of PATSET such that for each instance pattern PAT in PATSET[ $\pi$ ].

 $e_i[PAT] V_i[\pi], \forall i$ .

As a matter of fact, a rule Rulex can be found from PATSET if and only if

PATSET[IF(Rule<sub>x</sub>)] $\neq \phi$ .

Let

i

 $V_i(PATSET[\pi])$ 

be the set of all the values of  $E_i$  existed in PAT-SET[ $\pi$ ], (For the purpose of convenience,  $V_i$ (PATSET) will denote the set of all the values of  $E_i$  given in PATSET.) Then,

 $V_i(THEN(Rule_x)) = V_i(PATSET[IF(Rule_x)]), \forall$ 

Consider the following rule,

Rule<sub>2</sub>:  $(1 \lor 2, *, *, 1 \lor 3 \lor 4) \rightarrow (1, 1, 1, 1)$ .

Note that only PAT<sub>1</sub> satisfies IF(Rule<sub>3</sub>) so that THEN(Rule<sub>3</sub>)=PAT<sub>1</sub>. Consider the following rule, Rule<sub>3</sub>: (\*, \*, \*, \*)  $\rightarrow$  (1  $\lor$  2  $\lor$  3, 1  $\lor$  2  $\lor$  3, 1  $\lor$  2  $\lor$  3, 1  $\lor$  2  $\lor$  3  $\lor$  4).

Because there are no specified values of elements in IF(Rule<sub>4</sub>), Rule<sub>4</sub> will be called an "unconditioned rule." Note that  $V_i(THEN(Rule_3)) = V_i(PA-TSET)$ ,  $\vee i$ .

**Definition**:  $\pi_1 \leq \pi_2$  if  $V_i(\pi_2) \subseteq V_i(\pi_1)$ ,  $\vee i$ . Here, if  $\pi_1 \neq \pi_2$ ,  $\pi_1 \leq \pi_2$ , which can be interpreted as " $\pi_2$  is more specific than  $\pi_1$ ."

Ex. (1,  $1 \lor 2$ ,  $1 \lor 2$ ,  $1 \lor 2 \lor 3$ )  $\angle$  (1, 1,  $1 \lor 2$ , 1).

Note that for any rule Rulex,  $IF(Rule_x) \leq THEN$  (Rulex).

## Complete and Minimal Set of Rules

From a set of instance patterns, PATSET, we can form many rules but they can be redundant. For example, consider the following rules,

Rule,:  $(1 \lor 2, *, *, *) \rightarrow (1 \lor 2, 1, 1 \lor 2, 1, \lor 2)$ ,

Rule<sub>5</sub>: (\*,  $1 \lor 2$ , \*,  $2 \lor 3$ ) $\rightarrow$ (1  $\lor$  2  $\lor$  3,  $1 \lor 2$ , 2, 2) and

Rule<sub>6</sub>:  $(1 \lor 2, *, *, 2 \lor 3 \lor 4) \rightarrow (1 \lor 2, 1, 2, 2),$ 

formed from PATSET in Example 1. Rule<sub>6</sub> is redundant with Rule<sub>4</sub> and Rule<sub>5</sub>. Let an initial state of elements is given as represented in IF(Rule<sub>6</sub>).

That is.

 $E_1=1$  or 2, and  $E_4=2$  or 3 or 4.

Then, IF(Rule<sub>4</sub>) is satisfied by the state so that from THEN(Rule<sub>4</sub>),

 $E_1=1$  or 2,  $E_2=1$   $E_2=1$  or 2 and  $E_4=1$  or 2, and therefore, the state of elements is updated as follows:

 $E_1=1$  or 2,  $E_2=1$   $E_3=1$  or 2 and  $E_4=2$ .

Now, IF(Rule<sub>5</sub>) is satisfied by the state (and the state of elements will be updated again by THEN(Rule<sub>5</sub>)). Note that

 $V_i(THEN(Rule_6)) = V_i(THEN(Rule_4)) \cap V_i$ (THEN(Rule<sub>5</sub>)),  $\forall i$ .

That is, if the state of elements is given as IF (Rule<sub>6</sub>), then IF(Rule<sub>4</sub>) and IF(Rule<sub>5</sub>) are satisfied "in chain" and the conclusion given in THEN(Rule<sub>6</sub>) can be produced by combining THEN(Rule<sub>4</sub>) and THEN(Rule<sub>5</sub>).

**Definition**: Rule<sub>x</sub> "can be explained by chaining" Rule<sub>s</sub>, Rule<sub>b</sub>, ..., and Rule<sub>t</sub> if, an initial state of elements is given as represented in IF(Rule<sub>x</sub>).

(I) if the initial state satisfies IF(Rule<sub>a</sub>), that is,

 $V_i(IF(Rule_x)) \subseteq V_i(IF(Rule_x)), \forall i,$ 

the updated state of elements by THEN(Rule<sub>a</sub>) satisfies IF(Rule<sub>b</sub>), that is,

 $V_i(IF(Rule_x)) \cap V_i(THEN(Rule_a)) \subseteq V_i(IF(Rule_b)), \forall i,$ 

the updated state of elements by THEN(Rule<sub>b</sub>) satisfies IF(Rule<sub>c</sub>), that is,

 $V_i(IF(Rule_x)) \cap V_i(THEN(Rule_a)) \cap V_i(THEN(Rule_b))$ 

 $\subseteq V_i(IF(Rule_c)), \forall i,$ 

..., and the updated state of elements by THEN (Rule<sub>s</sub>) satisfies IF(Rule<sub>s</sub>), that is,

 $V_i(IF(Rule_x)) \cap V_i(THEN(Rule_s)) \cap V_i(THEN(Rule_s)) \cap V_i(THEN(Rule_s))$ 

 $\subseteq V_i(IF(Rule_i)), \forall i,$ 

and

(II) Vi(THEN(Rulex))

= $V_i(THEN(Rule_s)) \cap V_i(THEN(Rule_b))$  $\cap \cdots \cap V_i(THEN(Rule_i)), \forall i$ .

Note that a rule Rulex "can be explained by a single rule" Ruley, if

 $IF(Rule_y) \angle IF(Rule_x)$  and  $THEN(Rule_x) = THEN(Rule_y)$ .

Let ALL(PATSET) be the set of all the rules that can be obtained from PATSET.

**Definition**: A set of rules obtainable from PAT-SET is defined as a complete and minimal set of rules, represented as MIN(PATSET), if

- i) each rule in ALL(PATSET) MIN(PATSET) can be explained by chaining rules in MIN(PAT-SET) and
- ii) each rule in MIN(PATSET) cannot be explained by chaining other rules in MIN(PATSET).

Note that MIN(PATSET) is a non-redundant set of rules and any rule that can be formed from PAT-SET is a rule of MIN(PATSET) or can be explained by chaining rules of MIN(PATSET)

# Learning Algorithm of a complete and minimal set of rules

Let COM(PATSET) be the set of rules obtainable from PATSET such that

- i) each rule in ALL(PATSET) COM(PATSET)
   can be explained by a single rule in MIN(PATSET)
   and
- ii) each rule in COM(PATSET) cannot be explained by a single rule in MIN(PATSET).

Note that MIN(PATSET) is a subset of COM (PATSET). Let PAT<sub>i</sub>,  $i=1, 2, \cdots$  be the i<sup>th</sup> instance pattern presented and PATSET<sub>i</sub>={PAT<sub>1</sub>, PAT<sub>2</sub>, ..., PAT<sub>i</sub>.

## **ALGORITHM**

Phose 1. Initialize. Obtain COM(PATSET<sub>1</sub>).

For p=2, 3,..., up to the fnal instance pattern PATSET, do the following:

Phase p. Obtain  $COM(PATSET_p)$  from  $COM(PATSET_{p-1})$  and  $PAT_p$ .

p-1: Find and modify incorrect rules in COM(PATSET<sub>p-1</sub>).

p-2: Create new rules.

p-3: Obtain each rule that can be explained by another single rule in COM(PAT.  $SET_{p_2}$ ) which was found incorrect in p-1.

p-4: From the rules in hand, obtain COM (PATSET,).

Phase\*. From the rules obtained, obtain MIN (PATSET).

In Phose 1, with the first instance pattern, PAT<sub>1</sub>, COM(PATSET<sub>1</sub>) is easily obtained.

Actually, it has one and only one rule, Rule<sub>x</sub>, such that

 $IF(Rule_x = (*, *, \dots, *) \text{ and}$ 

THEN(Rulex) = PAT<sub>1</sub>.

For example, if  $PAT_1 = (1, 1, 1, 1)$ , the only rule in  $COM(PATSET_1)$  is

Rulex:  $(*, *, *, *) \rightarrow (1, 1, 1, 1)$ 

because for any other rule Ruley in  $\mbox{ALL}(\mbox{PAT}_1)$  ,

 $IF(Rule_x) \angle IF(Rule_y)$  and  $THEN(Rule_y) = THEN(Rule_y)$ .

That is, Ruley can be explained by Rulex.

Consider Phase p-1. Let Rulex° be a rule in ALL

(PATSET<sub>p-1</sub>) and Rule<sub>x</sub> be the corresponding rule in ALL(PATSET<sub>p</sub>) such that Rule<sub>x</sub>° is modified to Rule<sub>x</sub> according to PAT<sub>p</sub>. For any rule, whether it is in ALL(PATSET<sub>p-1</sub>) or in ALL(PATSET<sub>p</sub>) will be represented by whether "o" exists or not in the name of the rule. For convenience, let

**Definition**: Rule<sub>x</sub>° is incorrect with PAT<sub>p</sub> in E<sub>m</sub> if PAT<sub>p</sub> satisfies  $IF(Rule_{x}^{\circ})$  and  $e_{i}^{\alpha} \notin V_{m}$  (THEN

Rulex° is modified to Rulex as follows:

 $IF(Rule_x) = IF(Rule_x^{\circ})$  and

 $e_i^{\alpha} = e_i[PAT_{\alpha}], \forall i.$ 

 $(Rule_x^\circ)$ ).

 $V_i(THEN(Rule_x)) = V_i(THEN(Rule_x^\circ)) \cup \{e_i^a\},\$ 

For example, assume that we have

Rule<sub>X</sub>°: (\*, \*, \*, \*) $\rightarrow$ (1, 1, 1, 1) and PAT<sub>p</sub>=(1, 1, 2, 2).

Rulex° is incorrect with PAT, in E2 and E3 so that Rulex° must be modified to

Rule<sub>x</sub>:  $(*, *, *, *) \rightarrow (1, 1, 1 \lor 2, 1, \lor 2)$ .

Note that if  $Rule_{x}^{\circ}$  is correct with  $PAT_{p}$ ,  $Rule_{x}=Rule_{x}^{\circ}$ .

Note that in phase p-1, we do not find and modify incorrect rule in the set of ALL(PATSET<sub>p-1</sub>)-COM (PATSET<sub>p-1</sub>). The reason is as follows:

Assume the Ruley° is a rule in the above set and is incorrect with PAT<sub>p</sub> in E<sub>m</sub>. Then, Ruley° can be explained by another rule Rule<sub>x</sub>° in COM (PATSET<sub>p:1</sub>). Because

IF(Rule<sub>Y</sub>°)  $\angle$  IF(Rule<sub>Y</sub>°) and THEN(Rule<sub>Y</sub>°)= THEN(Rule<sub>Y</sub>°),

Rule<sub>x</sub>° must be incorrect with PAT<sub>p</sub> in  $E_m$ , too. Therefore,

THEN(Rule<sub>x</sub>) = THEN(Rule<sub>x</sub>).

And because

 $IF(Rule_x) = IF(Rule_x^{\circ}) \angle IF(Rule_y) = IF(Rule_y^{\circ}),$ 

Ruley can be explained by Ruley, Therefore, Ruley cannot be in COM(PATSET<sub>p</sub>) so that it need not be obtained.

In Phase p-2, consider new rule Rulex, that is, there does not exist a corresponding rule  $Rule_{x}^{\circ}$ . Then  $PATSET_{p-1}(IF(Rule_{x}))$  must be empty because if not,  $Rule_{x}^{\circ}$  exists. Therefore  $PATSET_{p}$   $(IF(Rule_{x})) = \{PAT_{o}\}$  so that

THEN(Rulex) = PAT...

Note that if there is another new rule Rule<sub>Y</sub> such that  $IF(Rule_Y) \angle IF(Rule_X)$ , and  $THEN(Rule_Y) = PAT_p$ , Rule<sub>X</sub> can be explained by Rule<sub>Y</sub> so that Rule<sub>X</sub> cannot be in MIN(PATSET<sub>p</sub>). Therefore, if Rule<sub>X</sub> is a rule of MIN(PATSET<sub>p</sub>), it must be that for each element  $E_m$  such that  $V_m(IF(Rule_X)) \subset V_m$  (PATSET<sub>p</sub>) and for any rule Rule<sub>Z</sub> such that

 $V_i(IF(Rule_x)) = V_i(IF(Rule_2))$  for  $i \neq m$  and  $V_m(IF(Rule_x)) \subset V_m(IF(Rule_z)$ ,

Rule<sub>z</sub> is not a new rule, that is, Rule<sub>z</sub>° exists. Note that PAT<sub>p</sub> satisfies IF(Rule<sub>z</sub>°). Also note that Rule<sub>z</sub>° must be incorrect with PAT<sub>p</sub> in E<sub>m</sub>, that is,  $e_m^{\ \alpha} \not\in V_m(THEN(Rule_z°))$ . It is because if  $e_m^{\ \alpha} \in V_m(THEN(Rule_z°))$ , PATSET<sub>p-1</sub>(IF(Rule<sub>z</sub>°)) must have an instance pattern PAT where  $e_m$  [PAT]= $e_m^{\ \alpha}$  and because

 $IF(Rule_z) = IF(Rule_z^{\circ})$  and  $e_m^{\alpha} \in V_m(IF(Rule_z))$ ,

PAT will satisfy IF(Rule<sub>x</sub>) and therefore, PAT. SET<sub>p-1</sub> (IF(Rule<sub>x</sub>)) will not be empty. And for each value  $e_m^*$  of  $E_m$  such that  $e_m^* \in V_m(THEN(Rule_2^\circ))$ , it must be that

 $e_m^* \notin V_m(IF(Rule_x))$ 

because PATSET<sub>p-1</sub> (IF(Rule<sub>z</sub>°)) has an instance pattern PAT where  $e_mPAT]=e_m^*$ , so that if  $e_m^* \in V_m(IF(Rule_x))$ , PAT will satisfy IF(Rule<sub>x</sub>) and therefore,

 $PATSET_{p-1}(IF(Rule_x)) \neq \phi$ 

A new rule Rule<sub>x</sub> can be constructed from each rule Rule<sub>2</sub>° in ALL(PATSET<sub>p-1</sub>) and each element  $E_m$  such that Rule<sub>z</sub>° is incorrect with PAT<sub>p</sub>:

$$V_i(IF(Rule_x)) = V_i(IF(Rule_z^\circ))$$
 for  $i \neq m$ ,  
and .....(6)

Note that there could be multiple rules that satisfy (7). Among them, we only have to obtain Rule<sub>x</sub>\* such that

$$V_{m}(IF(Rule_{X}^{*})) = \{e_{i}^{\alpha}\} \cap (V_{m}(PATSET_{p-1}) - V_{m}$$

$$(THEN(Rule_{Z}^{\alpha})) \dots (8)$$

because all other rules will have more specific "if" part than  $IF(Rule_X^*)$  so that they can be explained by  $Rule_X^*$ . From now on  $Rule_X^*$  will be represented by  $(Rule_Z^\circ)_m^{\alpha}$ .

**Theorem**: In Phase p-2, we only have to generate  $(Rule_{w}^{\circ})_{m}^{\alpha}$  for each rule  $Rule_{w}^{\circ}$  and  $E_{m}$  such that  $Rule_{w}^{\circ}$  is incorrect with  $PAT_{p}$  in  $E_{m}$  and  $Rule_{w}^{\circ}$  is in  $COM(PATSET_{p-1})$ .

proof Consider a rule Rulez° which is in ALL (PATSET<sub>P-1</sub>)—COM(PATSET<sub>P-1</sub>). Then Rulez° can be explained by a rule Rulew° in COM(PATSET<sub>P-1</sub>). Assume that Rulez° is incorrect with PAT<sub>p</sub> in an element E<sub>m</sub>. Then, Rulew° is also incorrect with PAT<sub>p</sub> in E<sub>m</sub> so that we can consider (Rulew°)<sub>m</sub>° and (Rulez°)<sub>m</sub>°. Because the "if" part of (Rulew°)<sub>m</sub>° is less specific than the "if" part of (Rulez°<sub>m</sub>° and the "then" parts of (Rulew°)<sub>m</sub>° and (Rulez°)<sub>m</sub>° are the same, (Rulez°)<sub>m</sub>° can be explained by (Rulew°)<sub>m</sub>°. Therefore, (Rulez°)<sub>m</sub>° cannot be in COM(PATSET<sub>p</sub>) and need not be generated.

Let  $Rule_{U}^{\circ}$  be the unconditioned rule formed from  $PATSET_{p-1}$ . Note that  $Rule_{U}^{\circ}$  must be in COM  $(PATSET_{p-1})$ , and  $V_m(PATSET_{p-1})$  can be found in  $THEN(Rule_{U}^{\circ})$  because  $V_m(THEN(Rule_{U}^{\circ})) = V_m(PATSET_{p-1})$ . For example, assume that we have

Rule<sub>0</sub>°: (\*, \*, \*, \*) $\rightarrow$ (1  $\lor$  2  $\lor$  3, 1  $\lor$  2, 1  $\lor$  2  $\lor$  3, 1  $\lor$  2),

Rule<sub>W</sub>°:  $(1 \lor 2, *, 1 \lor 2, *) \rightarrow (1, 1, 1, 1, 1 \lor 2)$  and

 $PAT_{p}=(1, 2, 2, 2).$ 

Rulew° is incorrect with PAT<sub>p</sub> in  $E_2$  and  $E_3$  and new rules to be generated corresponding to  $E_2$  and  $E_3$  are

and

 $(\text{Rule}_{w^{\circ}})_{3}^{a}: (1 \vee 2, *, 2 \vee 3, *) \rightarrow (1, 2, 2, 2)$ 

Now, consider Phase p-3. Let Rule<sub>X</sub>° can be explained by a rule Rule<sub>Y</sub>° in COM(PATSET<sub>P-1</sub>). As defined above, let  $e_i \alpha = e_i [PAT_p]$ .  $\vee$  i.

**Lemma**: If Rulex° can be explained by Ruley° and if Rulex is a rule in  $COM(PATSET_p)$ , (A) there is an element  $E_m$  such that Ruley° is incorrect with  $PAT_p$  in  $E_m$  but Rulex° is not incorrect with  $PAT_p$  in  $E_m$ ,

(B) IF(Rule<sub>x</sub>°) = IF(Rule<sub>y</sub>°) except that  $e_a^{\alpha} \in V_i$ (IF(Rule<sub>y</sub>°)) but  $e_a^{\alpha} \notin V_i$ (IF(Rule<sub>x</sub>°)) for one and only one element  $E_a$  such that Rule<sub>y</sub>° is incorrect with PAT<sub>p</sub> in  $E_a$  but Rule<sub>x</sub>° is not incorrect with PAT<sub>p</sub> in  $E_a$ 

proof of A) Assume that there is no such element, that is, for each element  $E_i$  such that  $Rule_Y^\circ$  is incorrect with  $PAT_p$  in  $E_i$ ,  $Rule_X^\circ$  is also incorrect with  $PAT_p$  in  $E_i$ . Note that because  $IF(Rule_Y^\circ) < IF(Rule_X^\circ)$  and  $THEN(Rule_Y^\circ) = THEN(Rule_Y^\circ)$ 

°), if Rulex° is incorrect with PAT<sub>p</sub> in an element E<sub>i</sub>, Ruley° is also incorrect with PAT<sub>p</sub> in E<sub>i</sub>. Therefore, THEN(Rulex) = THEN(Ruley). And because IF(Ruley) = IF(Ruley°)  $\angle$  IF(Rulex°) = IF(Rulex), Rulex can be explained by Ruley, and therefore, Rulex should not be in COM(PATSET<sub>p</sub>).

proof of B) Note that the above conclusion (A) implies that  $IF(Rule_{x^o})$  is not satisfied by  $PAT_p$  because if satisfied,  $THEN(Rule_{x^o}) = THEN(Rule_{y^o})$  and  $Rule_{x^o}$  is incorrect with  $PAT_p$  in  $E_m$ . (therefore,  $Rule_{x^o}$  must be correct with  $PAT_p$ ). Then, there must exist at least one element  $E_a$  such that

$$e_s^a \in V_a(Rule_Y^o))$$
 but  $e_s^a \not\in (IF(Rule_X^o))$ .

consider Rulez° such that IF(Rulez°)=IF(Ruley°) except that  $e_a^a \in V_a(IF(Rule_Y^o))$  but  $e_a^a \not\in V_a$  (IF(IFRulez°)).

Assume that there is another element  $F_{\mathfrak{b}}$  such that

 $e_b{}^\alpha \in V_b(IF(Rule_Y{}^\circ))$  but  $e_b{}^\alpha \notin V_b(IF(Rule_X{}^\circ))$ . Then,  $IF(Rule_Y{}^\circ) \angle IF(Rule_Z{}^\circ) \angle IF(Rule_X{}^\circ)$ . Because  $THEN(Rule_X{}^\circ) = THEN(Rule_Y{}^\circ)$ , it must be that  $THEN(Rule_Z{}^\circ) = THEN(Rule_X{}^\circ)$ . Therefore,  $Rule_X{}^\circ$  can be explained by  $Rule_Z{}^\circ$ . Furthermore, because both rules are correct with  $PAT_p$ ,  $Rule_X$  can be explained by  $Rule_Z$  and  $Rule_X$  should not be in  $MIN(PATSET_p)$ . Therefore, in this case,  $Rule_X{}^\circ$  does not need to be restored. Therefore, there must be one and only one element  $E_A$  such that (1) holds. From (10),  $e_A{}^\alpha \notin V_\alpha(IF(Rule_X{}^\circ))$  and  $IF(Rule_X{}^\circ) \angle THEN(Rule_X{}^\circ)$ , so

 $e_a^{\alpha} \not\in V_a(THEN(Rule_{X^{\circ}})) = V_a(THEN(Rule_{Y^{\circ}}))$ .

And because  $e_a^{\alpha} \in V_a(IF(Rule_Y^{\alpha}))$ , Rule<sub>Y</sub> must

be incorrect with PAT<sub>p</sub> in E<sub>2</sub>.

The above lemma implies that in Phase p-3, for each rule Rule<sub>Y</sub>° that is a rule in COM(PATSET<sub>P</sub>) and for each element  $E_m$  such that Rule<sub>Y</sub>° is incorrect with PAT<sub>P</sub> in  $E_m$ , we need to restore the rule Rule<sub>X</sub>° as follows.

IF(Rule<sub>Y</sub>°) = IF(Rule<sub>Y</sub>°) except that  $e_m^{\alpha} \in V_m$ (IF(Rule<sub>Y</sub>°)) but  $e_m^{\alpha} \notin V_m$ (IF(Rule<sub>X</sub>°)),

(that is,  $V_m(IF(Rule_X^\circ)) = V_m(IF(Rule_Y^\circ)) - \{e_m\alpha\}$ ), and

THEN(Rule<sub>X°</sub>) = THEN(Rule<sub>Y°</sub>).

(Note that the rule Rule<sub>Y</sub>° is either in MIN(PAT. SET<sub>P1</sub>) or is restored in Phase 1-1). Here, it will be represented by

$$Rule_X^{\circ} = (Rule_Y^{\circ})_m^{\beta}$$
.

Note that in the case of  $V_m(Rule_{Y^o})) = V_m(PAT_-SET_{p-1})$ ,  $V_m(IF(Rule_{Y^o}))$  can be found from  $V_m$  (THEN(Rule<sub>U^o</sub>)). For example, assume that we have

Rule<sub>U</sub>°:  $(*, *, *, *) \rightarrow (1 \lor 2 \lor 3, 1 \lor 2, 1 \lor 2 \lor 3, 1 \lor 2),$ 

Ruley°:  $(1 \lor 2 *, 1 \lor 2, *) \rightarrow (1, 1, 1, 1 \lor 2)$  and

 $PAT_p = (1, 2, 2, 2).$ 

Ruley° is incorrect with PAT<sub>p</sub> in  $E_2$  and  $E_3$  and rules to be restored corresponding to  $E_2$  and  $E_3$  are

 $(\text{Rule}_{Y}^{\circ})_{2}^{\beta}$ :  $(1 \lor 2, 1, 1 \lor 2, *) \rightarrow (1, 1, 1, 1 \lor 2)$  and

 $(\text{Rule}_{Y}^{\circ})_{\xi}^{\beta}: (1 \lor 2, *, 1, *) \rightarrow (1, 1, 1, 1 \lor 2),$ 

Phose p-4 can be done as follows: Check each rule, Rulex, on hand in any sequence if Rulex can be explained by another rule on hand. If it is, delete Rulex.

Phose\* can be done as follows: Check each rule,

Rulex, on hand in any sequence if Rulex can be explained by chaining rules remained on hand. It can be done as follows:

Set an initial state as represented in IF(Rule<sub>x</sub>) and find another rule. Rule<sub>y</sub>, such that IF(Rule<sub>y</sub>) is satisfied by the state and update the state with THEN(Rule<sub>y</sub>) and similarly, find another rule whose "if" part is satisfied by the updated state and update the state again, etc. until there is no other rules whose "if" part is satisfied by the updated state. And then, we can check if Rule<sub>x</sub> can be explained by chaining those activated rules as described in the definition.

## Experiment

The real time scheduling of the manufacturing line engaged in the producing circuit boards, as described before, was performed by over 100 students. A student was selected as an expert scheduler based on performance. From the data of the schedules produced by the expert scheduler, a complete and minimal set of rules was obtained by the algorithm suggested in the previous section. The number of schedules made by the expert scheduler for obtaining rules was 2125 and the number of learned rules of a complete and minimal set was 176. In other words, the behavior of the expert scheduler can be explained by the 176 rules. Examples of learned rules are listed below:

- 1) If  $E_1=4$ ,  $E_2=0$ , and  $E_3$ ,  $E_4$ ,  $E_5=0$ , 1, 2 or 3, then  $E_7=1$ ,  $E_8=2$
- 2) If  $E_1$ ,  $E_2$ ,  $E_3$ ,  $E_4$ , =0, 1, 2, and  $E_5$ =2, 3 or 4, then  $E_7$ =5,  $E_8$ =6
- 3) If  $E_1=0$ , or 1.  $E_2=3$  or 4.  $E_3=0$ ,  $E_4=3$ .

- and  $E_5=0$ , 1, 2, then  $E_7=2$ ,  $E_8=3$
- 4) If  $E_1 = -1$ , and  $E_2$ ,  $E_3$ ,  $E_4$ ,  $E_5 = 0$ , 1, or 2, then  $E_7 = 1$ ,  $E_8 = 6$
- 5) If  $E_1=0$ , 1, 2, or 3,  $E_2$ ,  $E_3=1$ , 2, 3, or 4,  $E_4=1$ , 2, 3, or 4 and  $E_5=0$ , then  $E_7=4$ ,  $E_8=5$
- 6) If  $E_1=0$ , or 1,  $E_2=0$ , or 1,  $E_3=-1$ ,  $E_4=0$ , and  $E_5=0$ , or 1,  $E_7=3$ ,  $E_8=4$
- 7) If  $E_1$ ,  $E_2=1$  or 2,  $E_3=4$ ,  $E_4=1$  or 2, and  $E_5=0$ , then  $E_7=4$ ,  $E_8=5$
- 8) If  $E_1=0$  or 1,  $E_2=2$  or 3,  $E_3=0$   $E_4=-1$  and  $E_5=0$ , 1, or 2, then  $E_7=2$   $E_8=3$
- 9) If  $E_0=1$ ,  $E_2$ ,  $E_3$ ,  $E_4$ ,  $E_5=0$  or 1, then  $E_7=0$ ,  $E_8=1$
- 10) If  $E_1=1$  or 2,  $E_2=0$ ,  $E_3=0$  or 1n  $E_4=0$  or 1,  $E_5=0$ , and  $E_6=0$ , 1, or 2, then  $E_7=2$   $E_8=3$

The learned rules agree with the common sense even though some are more or less technical. For example, the rule of 1) represents the behavior of the expert scheduler that if the job in Tank 1 is urgent to be moved out, there is no job in Tank 2 (so that a job can be moved into Tank 2), and the other jobs in the other tanks are not urgent to be moved out, the next move of the robot is to move the job in Tank 1 to tank 2. The rules represents that if the job in Tank 1 is spoiled and the other jobs in the other tanks are not in hurry or urgent to be moved out, the next move of the robot is to take the job in Tank 1 away to Output buffer. The followings are some cases of schedules generated by the learned rules with random situations:

1) Situation:  $E_0=0$ ,  $E_1=1$ ,  $E_2=2$ ,  $E_3=0$ ,  $E_4=0$ ,  $E_5=1$ ,  $E_6=5$ 

Generated schedule:  $E_7=2$ ,  $E_8=3$ 

2) Situation:  $E_0=1$ ,  $E_1=0$ ,  $E_2=1$ ,  $E_3=1$ ,  $E_4=0$ ,  $E_5=1$ ,  $E_6=2$ 

Generated schedule:  $E_7=0$ ,  $E_8=1$ 

3) Situation:  $E_0=0$ ,  $E_1=0$ ,  $E_2=0$ ,  $E_3=0$ ,  $E_4=-1$ ,  $E_5=2$ ,  $E_6=4$ 

Generated schedule:  $E_7 = 5$ ,  $E_8 = 6$ 

4) Situation:  $E_0=0$ ,  $E_1=1$ ,  $E_2=1$ ,  $E_3=1$ ,  $E_4=2$ ,  $E_5=0$ ,  $E_6=1$ 

Generated schedule:  $E_7=4$ ,  $E_8=5$ 

5) Situation:  $E_0=1$ ,  $E_1=0$ ,  $E_2=1$ ,  $E_3=1$ ,  $E_4=-1$ ,  $E_5=0$ ,  $E_6=2$ 

Generated schedule:  $E_7 = 4$ ,  $E_8 = 6$ 

#### conclusion

In this paper, an expert scheduling system where rules for the knowledge base are obtained from schedules generated by expert schedulers was suggested. A case of computer-aided manufacturing line producing circuit boards was adopted for simulation. By the rule-generation method, the behavior of an expert scheduler in decision making was saved in a complete and minimal set of rules to be used in later real time scheduling.

Obviously, the learning algorithm offers a great advantage of bypassing the troublesome and costly procedure of obtaining rules from a domain expert. The quality of the obtained rules is closely related to the quality of the schedules used for obtaining those rules and how well the elements are set up. In some problem domains, a probabilistic conclusion is more realistic. The suggested system can be extended to the case of frequency-based probabilistic rules. This remains for future research.

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