# Determining the Optimum Target Value for Filling Operations with Nondestructive Sampling Plans

## —비파괴 샘플링 계획을 갖는 Filling 작업에 대한 최적 목표치 결정—

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### Abstract

We consider a filling process problem on a production line. Up to present this problem have examined by 100% inspection. Thus a target value is determined which takes into account the regular selling prices, the reprocess cost, the excess quality cost and the process variability and so on. However, in this paper we propose a solution under specified sampling plan when the inspection is nondestructive.

### 1. Introduction

There are often specification limits for the quality characteristics of items produced in industrial processes. Items which fulfill specification are accepted and sold at regular price.

It is very important that the problem of finding the process level of a production process, such that is maximized with regard to production costs, selling prices, specification levels and so on.

This problem has been studied earlier underassuming 100% inspection of items after filling. Moreover, it is assumed that the quality characteristic is normally distributed with known variance.

Bettes[1] treated the problem of simultaneously choosing the optimal upper specification limit and process level.

Springer[7] studied the problem under the assumption of constant net income functions with both upper and lower specification limits.

Hunter and Kartha[5] solved the problem with one specification limit, assuming the net income function for accepted items to be linearly decreasing, and constant for rejected items.

Carlsson[3] extended the work of Hunter and Kartha to include both fixed and variable costs and revenue function whereby the customer paid extra for good quality and must be compensated for poor quality.

Golhar[4] treated the case in which rejected product was recycled so that it would be sold in the primary market.

Schmidt and Pfeifer[6] used Golhar's model to evaluate the economic effects of process variance reduction. Boucher and Jafari[2] extended the problem under a sampling plan as opposed to 100% inspection.

In this paper sampling plans are specified, for example, in the production of food products for military assumption and in situations where producers choose not to use automatic inspection equipment. Moreover, the inspection is nondestructive.

### 2. Formulation and Solution

In this study we consider a filling process. The random variable X in this process represents the quantity

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of material in an individual container(or can, case and so on). If we define L as a lower specification limit, a target value of filling process is greter than L, because of a variability of filling process.

In this way, produced lot of size N is inspected only drawn n items by specified sampling plan. If filled quantity of container is less than lower specification limit ($X<L$) it is justified as defective item, otherwise ($X \geq L$) it is justified as nondefective item. We assume that D is the number of defective items found and do is the allowable number of defective items.

If $D \leq do$, the lot is accepted and if $D>do$, D item is reprocessed with cost R per item and return to lot. Then the lot is sold at regular price A.

We construct the following the expected gross income function

$$E(G \mid D)=\begin{cases} AN & \text{if } D \leq do \quad (1) \\ AN-R \cdot D & \text{if } D>do \quad (2) \end{cases}$$

where $E(G \mid D)$ is the expected gross income for a lot of size N, given D. For a target value of t, the expected value of the material cost per lot is $(2N-D)ct$, where c is the cost of excess quality per accepted item and t is the expected value of X.

Therefore, the expected marginal profit function is

$$\begin{aligned} E[\pi(t)] &= ANPr(D \leq do)+(AN-DR)Pr(D>do)-(2Nct-Dct) \\ &= AN-DRPr(D>do)-(2N-D)ct \end{aligned} \tag{3}$$

whwew $E[\pi(t)]$ is the expected maginal profit for a lot of size N and target value of t.

We assume that random variable X follows a normal distribution with mean t and variance $\sigma^2$. For industrial process, the sample size is typically small in comparison to the lot size. Therefore,

$$\begin{aligned} Pr(D>do) &= 1-Pr(D \leq do) \\ &= 1-\sum_{d=0}^{do} \frac{n!}{d!(n-d)!} q^d(1-q)^{n-d}, \quad 0 \leq q \leq 1 \end{aligned} \tag{4}$$

where $q=Pr(X<L)=\Phi(-z)$, $z=\frac{t-L}{\sigma}$

$\Phi(\cdot)$ is the c.d.f of standard normal distribution

Equation (3) can be written as

$$\frac{E[\pi(t)]}{N}=A-\frac{DR}{N}Pr(D>do)-(2N-D)\frac{ct}{N} \tag{5}$$

let

$$\begin{aligned} M &= Pr(D \leq do) \\ &= \sum_{d=0}^{do} \frac{n!}{d!(n-d)!} q^d(1-q)^{n-d} \end{aligned} \tag{6}$$

Defferentiating equation (5) with respect to t,

$$\frac{\partial}{\partial t}\left[\frac{E[\pi(t)]}{N}\right]=-(2N-D)\frac{c}{N}-\frac{DR}{N} \cdot \frac{\partial}{\partial t}Pr(D>do) \tag{7}$$

Therefore, equation (7) is zero at maximum profit

where

$$\frac{\partial}{\partial t}\Pr(D>do)=\frac{\partial(1-M)}{\partial t}=-\frac{\partial q}{\partial t}\cdot\frac{\partial M}{\partial q} \tag{8}$$

because

$$\frac{\partial q}{\partial t}=\frac{\partial}{\partial t}\Phi(-z)=-\frac{1}{\sigma}\phi(-z) \tag{9}$$

where $\phi(\cdot)$ is the p.d.f for the standard normal distribution.
and

$$\frac{\partial M}{\partial q}=\sum_{d=0}^{d_0}\frac{n!}{d!(n-d)!}\cdot dq^{d-1}(1-q)^{n-d}-(n-d)q^{d}(1-q)^{n-d-1}$$

$$=\sum_{d=0}^{d_0}\frac{n!}{d!(n-d)!}dq^{d-1}(1-q)^{n-d}-\sum_{d=0}^{d_0}\frac{n!}{d!(n-d)!}(n-d)q^{d}(1-q)^{n-d-1}$$

$$=\sum_{d=1}^{d_0}\frac{n!}{(d-1)!(n-d)!}q^{d-1}(1-q)^{n-d}-\sum_{d=0}^{d_0}\frac{n!}{d!(n-d-1)!}q^{d}(1-q)^{n-d-1} \tag{10}$$

Let d'=d−1, equation (10) is

$$\sum_{d'=0}^{d_0-1}\frac{n!}{d'!(n-d'-1)!}q^{d'}(1-q)^{n-d'-1}-\sum_{d=0}^{d_0}\frac{n!}{d!(n-d-1)!}q^{d}(1-q)^{n-d-1}$$

$$=-\frac{n!}{do!(n-do-1)!}q^{d_0}(1-q)^{n-d_0-1}$$

$$=-\frac{n!}{do!(n-do)!}(n-d_0)q^{d_0}(1-q)^{n-d_0-1}$$

$$=-\frac{n!}{do!(n-do-1)!}q^{d_0}(1-q)^{n-d_0-1} \tag{11}$$

By substitution (9), (11) into (7) and (8)

$$\phi\left(\frac{t^*-L}{\sigma}\right)\frac{n!}{do!(n-d_0-1)!}\cdot q^{d_0}(1-q)^{n-d_0-1}$$

$$=\frac{(2N-D)c\cdot\sigma}{DR} \tag{12}$$

where $t^*$ is the optimal target value.
If n=1 and d=0, equation (12) reduces to

$$\phi\left(\frac{t^*-L}{\sigma}\right)=\frac{(2N-D)c\cdot\sigma}{DR} \tag{13}$$

equation (13) represents the solution for lot sampling when the sample size is 1. The lot size is large in relation to the sample size, and the allowable number of defective items is zero.

## 3. Example

We shall now illustrate the method by an example.
For this we use data from Hunter and kartha[5] and assume nondestructive inspection.

A=67.5₵
c=55₵ per lb

L=1.00 $^{l}$b
$\sigma$ =0.00563

Further suppose the N=100, q= 0.0064, R=200¢ Per item
For n=1 and do=0, using (12), then $t^{*}$=1.00406
For another cases are shown in Table 1.

Table 1 was created using FORTRAN program(see Appendix). In this program, we conducted conclusion through changing variable value for another casese.

**TABLE 1. Optimal Target Value and Unit Profit**

| n | do | D | t | E[$\pi$(t)]/N |
|---|---|---|---|---|
| 1 | 0 | 1 | 1.00406 | 0.67487 |
| 10 | 0 | 1 | 1.00132 | 0.84377 |
| | | 5 | 1.03901 | 0.78675 |
| | 1 | 1 | 1.00547 | 0.67877 |
| | | 5 | 1.16203 | 0.54263 |
| | 2 | 1 | 1.02411 | 0.61923 |
| | | 5 | 1.71379 | 0.57395 |

## 4. Summary

It is important that the problem of setting the process level of a production line. Generally this problem have supposed 100% inspection of procuct after filling. However, in special cases, this assumption is not appropriate. For example, in the production of food products for military consumption. In this study we have determined the optimum target value of a filling process under specified sampling plan. We hope that the determing of target value problem will be developed further for the destructive inspection.

### Appendix

```
MAIN PROGRAM
REAL D, DO, NL, Q, ET, TS, SM, EPT, NF, DF, NDF
A=67.5
C=55
L=1
SIG=0.00563
R=200
NL=100
Q=0.0064
ET=0
D=1
DO=0
N=10
CALL FACT(D, DO, N, Q, ET)
  GG=SQRT(6.28)*(2*NL-D)*C*SIG/(D*R)
  SQ=2*ALOG(GG)/ET
  TS=SIG*SQRT(SQ)+L
  SM=0
```

```
    DO 100 D=0, DO
    CALL FA(D, DO, N, Q, ET)
      SM=SM+ET
100 CONTINUE
    EPT=A-(2*N-D)*(C*T)/NL-(D*R/NL)*(1-SM)
    EPT=EPT/NL
    WRITE(7, 900)TS, EPT
900 FORMAT(10X, F8.5, 10X, F8.5)
    STOP
    END
C
    SUBROUTINE FACT(D, DO, N, Q, ET)
    ET=0
    NF=1
    DF=1
    NDF=1
    ND=N-DO-1
    IF(N.EQ.0) THEN
      N=1
    ELSE
    ENDIF
    IF(ND.EQ.0) THEN
      ND=1
    ELSE
    ENDIF
    IF(D.EQ.0) THEN
      DF=1
    ELSE
    ENDIF
    DO 10 NN=N, 1, -1
      NF=NN*NF
 10 CONTINUE
    DO 20 KK=D, 1, -1
      DF=KK*DF
 20 CONTINUE
    DO 30 DD=ND, 1, -1
      NDF=DD*NDF
 30 CONTINUE
    ET=(NF/(DF*NDF))*(Q**DO)*((1-Q)**ND)
    RETURN
    END
```

## References


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