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The Effect of Surface Tension on the Transient Free-Surface Flow near the Intersection Point

by

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교차점 부근의 과도자유표면유동에 미치는 표면장력의 영향

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Abstract

When a body starts to move, the flow near the intersection point between a body and a free surface changes violently and rapidly in a very short initial time interval. This flow phenomena must be investigated whenever one treats the interaction between a body and a fluid, such as the motion of a floating body, sloshing in a tank, wave maker problem, entry of a body into a fluid etc.. Until Roberts(1987), it was widely accepted that a singularity exists at the intersection point. However, he showed that the singularity does not exist if a body moves non-impulsively.

In this paper, an analytical solution cosistent for the case of impulsive motion of a body is obtained by including the effect of surface tension. From the characteristics of the newly obtained solution, a critical value associated with an oscillating phenomenon is found, and further more, it is shown that the oscillating phenomenon does not appear in the region where the distance form the intersection point is less than this critical value.

요 약

자유표면을 가지는 유체에서 물체가 운동을 하게되면 물체와 자유표면의 교차점 주위에서 유체유동이 매우 급격해진다. 이 유동현상은 부유체의 운동에 따르는 유체유동, 탱크내에서의 슬로싱, 조파기 문제, 물체의 입수등 여러문제에서 발생된다. 이 유동은 교차점 주위에서 특이성을 가진다고 알려져 있었으나 Roberts(1987)는 물체의 운동이 충격운동이 아닐 경우에는 특이성이 존재하지 않는다는 것을 밝혔다. 본 논문에서는 표면장력을 고려하여 물체의 운동이 충격운동인 경우에도 특이성을 가지지 않는 해석해를 구하였고, 해의 특성을 조사하여 진동현상에 대한 임계치를 구하여 교차점에서 떨어진 거리가 이 임계치보다 작은 곳에서는 진동현상이 존재하지 않는다는 것을 밝혔다.

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1. Introduction

The flow characteristics near the intersection point between a body and a free surface are of great interest in the field of water waves associated with the motion of a surface-piercing body. When a body starts to move, the flow near the intersection point changes violently and rapidly in a short initial time interval. The strong change of the flow produces significant difficulties in both analytic and numerical studies. In the numerical study, there exist many difficulties such as numerical error and numerical in stability in calculating the flow near the intersection point with high accuracy. In the analytic study, the possibility of the eistence of singularity at the intersection point makes the problem difficult to solve, so the problem should be analyzed with great care.

The problem of the intersection point occurs when one treats such problems in a time domain as the motion of a floating body, sloshing in a tank, wave maker problem, entry of a body into a fluid etc.

There is a series of studies about the flow characteristics near the intersection point. Kravtchenko[1] pointed out that the behavior of the velocity potential near this point is $z^2 \log z$ in the case of time harmonic motion, where z is a coordinate value in complex plane. The problem of a moving vertical plate was solved by Peregrine[2] by the method of small-time expansion and his results showed that the elevation of a free surface has a logarithmic singularity. Chwang[3] solved the problem to the third order by small-time expansion for the case of constant acceleration of a vertical plate and his results also had the singularity. Lin[4] analyzed the wave maker problem by both the small-time expansion and the numerical method. Also he obtained the singular solution and confirmed the logarithmic singularity. Until the work of Roberts[5], the logarithmic singularity was widely accepted. Roberts showed that the neglect of gravity makes the problem singular, and by including gravity he could obtain the solution non-singular at this point. His solution is valid in the case of non-impulsive motion of a vertical plate, but not so in the case of impulsive motion. In the case of impusive motion, he pointed out that the height of the intersection point is bounded in the form of $-t\log t$, which caused his solution to lose self-consistency. Thus he concluded that the acceleration of a plate must be bounded for his solution valid. Cointe[6] also showed that the velocity potential near the intersection point is weakly regular ($z^2 \log z$) for a weakly non-linear case.

The analytical solution near this point is important, because the numerical scheme that can simulate the flow cannot be developed unless the flow characteristics near this point are analyzed correctly. Since the analytical solution of non-linear problem cannot be obtained directly, the solution used to be obtained by applying a vaild asymptotic expansion. The linear solution, the solution of the leading order problem derived in an expansion scheme, has important meaning for the following reason. If the linear solution is singular or inconsistent, the higher order solutions become more singular. Hence the magnitude of the higher order solutions is no longer smaller than that of linear solution. As a result, the expansion scheme loses the uniform validness, and the linear solution ceases to be vaild. The flow near the intersection point changes very rapidly and the possibility of having a singularity is very high. Thus the problem of the flow near the intersection point should be analyzed in details considering existence, boundedness and validness.

In this paper, the initial asymptotic behavior of the flow near the intersection point is examined. By including surface tension, a consistent analytical solution is obtained for the case of impulsive motion. As a result, it is shown that both the height of the intersection point and the slope of free surface are bounded. And the critical value associated with oscillating phenomena is obtained. The oscillating phenomena does not appear where the distance from the intersection point is less than this critical value.

2. Formulation

There are two ways in describing the fluid motion[7]. One, the Lagrangian description, in which the motion of an individual fluid element based on its position is given. The other, the Eulerian description, in which the velocity field of the fluid is given. In most numerical methods dealing with the non-linear free surface problem, the flow is calculated in the Eulerian sense and the position of the free surface is treated by the Lagrangian concept[8].

Lagrangian formulation is treated in the work of Lee[9], in which he showed that the leading order poblems with Lagrangian description and with Eulerian description are identical mathematically, and the only difference is a body boundary condition and the way interpreting the results. Here we treat the Eulerian formulation only.

The fluid is assumed to be inviscid and incompressible, and the fluid motion is considered to start at initial instant $t = 0$. When the fluid is at rest the flow is irrotational, thus by the Kelvin's theorem the flow is irrotational at any time.

Two basic laws are used to describe the fluid motion : the mass and the momentum conservation. In the framework of the irrotational flow, the mass and the momentum conservation laws become Laplace equation and the Bernoulli equation, respectively.

$$\nabla^2\phi = 0 \qquad (2.1)$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + \rho g y = -\frac{1}{\rho}p, \qquad (2.2)$$

where ϕ is the velocity potential, p is the pressure, ρ is the dendsity of the fluid, g is the gravitational acceleration, y is the vertical coordinate taken positive upward, and ∇ is the gradient operator. Boundary conditions and the linearization procedure are well known, so we omit the details and just write down the results [10]. If the velocity of a body is small and can be represented by αU(where α is small number), the motion of the fluid is also small. Then the velocity potential and the elevation of the free surface can be expanded as follows,

$$\phi = \alpha\phi_1 + \alpha^2\phi_2 + \cdots\cdots, \qquad (2.3)$$

$$\eta = \alpha\eta_1 + \alpha^2\eta_2 + \cdots\cdots,$$

The resulting leading order problem is as follows.

$$\nabla^2\phi_1 = 0 \quad \text{in the fluid domain,} \quad (2.4)$$

$$\nabla\phi_1 \to 0 \quad \text{as } y \to -\infty, \quad (2.5)$$

$$\nabla\phi_1 \to 0 \quad \text{as } x \to \infty, \quad (2.6)$$

$$\phi_{1n} = U_n \quad \text{on the body surface,} \quad (2.7)$$

$$\phi_{1t} + g\eta_1 - \frac{T}{\rho}\eta_{1xx} = 0 \text{ on } y=0, \quad (2.8)$$

$$\eta_{1t} = \phi_{1y} \quad \text{on } y=0, \quad (2.9)$$

$$\phi_1(x,0,0) = \phi_{1t}(x,0,0) = 0, \quad (2.10)$$

where T is the coefficient of surface tension (see Ref.[11], [12] for more details of surface tension and the free surface boundary condition.) For the fresh water at 20℃, T = 0.07275N/m, ρ = 998.3kg/m^3. Eq.(2.8) and Eq.(2.9) may be combined to yield a single free surface condition for the velocity potential.

$$\phi_{1tt} + g\phi_{1y} - \frac{T}{\rho}\phi_{1xxx} = 0 \quad \text{on } y = 0 \qquad (2.11)$$

3. Initial Asymptotic Behavior of the Flow near the Intersection Point

The initial asymptotic behavior of the flow near the intersection point will be treated for a simple model (see Fig.1). We take the position of the vertical plate in x-direction to be

$$x = \alpha X_p(t) = \frac{\alpha t^{p+1}}{\Gamma(p+2)} \qquad t \geq 0, \qquad (3.1)$$

where Γ is the Gamma function, α is the amplitude of the motion and $p \geq 0$. It is not necessary that $X_p(t)$ is a power of t, but to avoid the mathematical complexity the power of t is chosen. When p=0, Eg.(3.1) represents an impulsive motion of the plate with constant velocity, and when p=1 a motion with a constant acceleration. If $p < 0$, there is no possible motion. The velocity and the acceleration of the vertical plate can be written as follows,

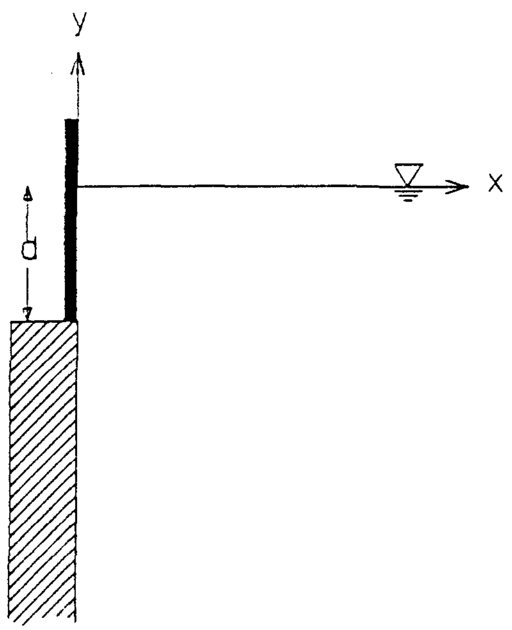

Fig. 1 Model and Coordinate System

$$u = \alpha U p(t) = \frac{\alpha t^{p}}{\Gamma(p+1)}, \qquad (3.2)$$

$$a = \alpha A p(t) = \frac{\alpha t^{p-1}}{\Gamma(p)}. \qquad (3.3)$$

The flow characteristics are studied first by Peregrine[2], who used the method of small-time expansion. In small-time expansion, the velocity and the height of the free surface are assumed to be represented by the following regular perturbation expansion.

$$\phi = t^{p}\phi_0 + t^{p-1}\phi_1 + t^{p+2}\phi_2 + \cdots\cdots,$$
$$\eta = t^{p+1}\eta_0 + t^{p+2}\eta_1 + t^{p+3}\eta_2 + \cdots\cdots. \qquad (3.4)$$

When the leading order problem is drawn by using the above expansion, the free surface boundary condition becomes

$$\phi_0 = 0 \text{ on } y = 0. \qquad (3.5)$$

For the case of a finite water depth, the solution of the impulsive motion problem was obtained by Peregrine[2], and the solution of the constant acceleration problem by Chwang[3]. For the case of the infinitely deep water, the solution was given by Roberts[5]. All their solutions have a logarithmic singularity in the free surface elevation even if the motion of the vertical plate is smooth. This singularity comes from the confluence of the boundary conditions. Thus, because of the free surface condition ϕ = 0, the small-time expansion loses the validness near the intersection point even if it is valid far from this point.

In the small-time expansion, the free surface boundary condition becomes ϕ = 0, and gravity is ignored. Roberts[5] pointed out that the flow near the intersection point can be analyzed properly provided that gravity is not ignored. He used the following expansion,

$$\phi = a\phi_1 + a^2\phi_2 + \cdots\cdots,$$
$$\eta = a\eta_1 + a^2\eta_2 + \cdots\cdots, \qquad (3.6)$$

where a is the amplitude of the motion and taken as small. Using the above expansion, the free surface boundary condition of the leading order problem is

$$\phi_{1tt} + g\phi_{1y} = 0 \quad \text{on} \quad y = 0 \qquad (3.7)$$

Roberts obtained the solution of the leading order problem and examined the properties of the solution from its asymptotic expansion. For the case of impulsive motion, his solution has a square root singularity in the slope of the free surface. Thus Roberts concluded that the acceleration must be bounded for his solution self-consistent. Until now a self-consistent solution for the case of impulsive motion has not been reported.

3.1 The Analytical Solution with Surface Tension

When the amplitude of the plate motion is small, the velocity potential and the height of the free surface are expanded as in Eq.(3.6). Resulting body boundary condition on the vertical plate is as follows,

$$\phi_{1x} = U_p(t) \quad \text{on} \quad x = 0. \qquad (3.8)$$

At the wall below the plate, the velocity normal to the wall is zero. The solution satisfying Eq.(3.8) and Eq.(2.4)-(2.11) can be obtained by using the Green function derived in Appendix. (For convenience, the subscript '1' will be omitted henceforth.) At $x = 0$, ϕ_x is independent of y, so the strength of source potential becomes constant on the vertical plate. It is easy to see that this strength is twice the velocity normal to the plate and equals to zero on the wall. The complex velocity potential can be obtained by integrating the Green function from $-d$ to 0, multiplying by 2, and carrying out the convolution integral with $U_p(t)$.

$$\phi = U_p(t)\frac{i}{\pi}[2z(\log z-1)-(z+id)(\log(z+id)-1)-(z-id)(\log(z-id)-1)]$$
$$-\frac{2}{\pi}H(t)\int_0^\infty (e^{-ikz}-e^{-ik(z-id)})f_p dk, \qquad (3.9)$$

where $z = x + iy$, $\beta = T/\rho$, $H(t)$ is the unit Heaviside function, and f_p is the convolution part as follows,

$$f_p = \int_0^t U_p(\tau)(g+\beta k^2)\frac{\sin(\sqrt{gk+\beta k^3}(t-\tau))}{k\sqrt{gk+\beta k^3}}d\tau \qquad (3.10)$$

Once $U_p(t)$ is given, f_p can be calculated and by substituting f_p into Eq.(3.9) the complete velocity potential can be obtained. For convenience, the function in Eq.(3.2) is used.

$$f_0 = \frac{1}{k^2}(1-\cos(\sqrt{gk+\beta k^3}t)$$
$$f_1 = \frac{1}{k^2\sqrt{gk+\beta k^3}}(\sqrt{gk+\beta k^3}t - \sin(\sqrt{gk+\beta k^3}t))$$
$$f_2 = \frac{1}{k^2(gk+\beta k^3)}\left(\frac{t^2}{2}(gk+\beta k^3) - 1+\cos(\sqrt{gk+\beta k^3}t)\right)$$

Differentiating the Eq.(3.9) with respect to z, the complex velocity becomes

$$q = u - iv$$
$$= \frac{t^p}{\Gamma(p+1)}\frac{i}{\pi}(\log(z-id)-\log(z+id))$$
$$-\frac{2}{\pi}H(t)i\int_0^\infty (e^{-ikz}-e^{-ik(z-id)})h_p dk, \qquad (3.11)$$

where

$$h_0 = \frac{1}{k}\cos(\sqrt{gk+\beta k^3}t),$$
$$h_1 = \frac{1}{k\sqrt{gk+\beta k^3}}\sin(\sqrt{gk+\beta k^3}t),$$
$$h_2 = \frac{1}{k(gk+\beta k^3)}(1-\cos(\sqrt{gk+\beta k^3}t)).$$

The following integral formula is used to obtain Eq.(3.11),

$$\int_0^\infty \frac{1}{k}(e^{-ikz} - e^{-ik(z-id)})dk$$
$$= -\log z + \log(z-id).$$

Examining closely Eq.(3.11), we see that the height of the free surface can be represented as follows,

$$\eta_p = \text{Re}(iq_{p+1}). \qquad (3.12)$$

Thus the height of the free surface is

$$\eta = \frac{2}{\pi}\text{H}(t)\,\text{Re}\int_0^\infty (e^{-ikx} - e^{-ik(x-id)})\,h_p dk, \qquad (3.13)$$

where h_p is

$$h_0 = \frac{1}{k\sqrt{gk+\beta k^3}}\sin(\sqrt{gk+\beta k^3}\,t),$$
$$h_1 = \frac{1}{k(gk+\beta k^3)}(1-\cos(\sqrt{gk+\beta k^3}\,t)).$$

3.2 Flow Characteristics

For the case of impulsive motion, it was shown that the analytical solution without surface tension has no self−consistency because of the highly−oscillating term from the stationary phase integral by Roberts[5]. We now investigate the characteristics of the analytical solution, and the effect of surface tension on the highly−oscillating term.

For the case of impulsive motion, the height of the free surface is represented as follows as in the preceding section.

$$\eta = \frac{2}{\pi}\text{H}(t)\text{Re}\int_0^\infty (e^{-ikx} - e^{-ik(x-id)})\frac{\sin(\sqrt{gk+\beta k^3}\,t}{k\sqrt{gk+\beta k^3}}dk. \qquad (3.14)$$

Let $\sqrt{gK} = u$, then the integral part becomes

$$\text{E} = \frac{1}{i}\left[\int_0^\infty + \int_0^{-\infty}\right]\frac{(1-e^{-u^2 d/g})}{u^2\sqrt{1+\sigma u^4}}\exp(i[-u^2 x/g + ut\sqrt{1+\sigma^4}])du, \qquad (3.15)$$

where $\sigma = \text{T}/(\rho g^3)$. In the above representation, the main contribution of the integral is given by two integrals : the integral near $u = 0$ and the integral near the stationary point. Because the highly−oscillating term results from the stationary phase integral, let's examine this integral more closely.

We choose two parameters as follows

$$X^2 = \frac{x}{gt^2}\ , \quad \gamma = \sigma\left(\frac{gt}{x}\right)^4 \qquad (3.16)$$

□ Case of $x \ll gt^2$ ($i.e. X^2 \ll 1$)

Let's introduce a variable $u = (gt/x)s$, then Eq.(3.15) becomes

$$\text{E} = \frac{x}{igt}\left[\int_0^\infty + \int_0^{-\infty}\right]\frac{1-\exp(-\frac{d}{x}\frac{s^2}{X^2})}{s^2\sqrt{1+\gamma s^4}}\exp\left\{i\frac{1}{X^2}f(s)\right\}ds. \qquad (3.17)$$

where the phase function function $f(s)$ is as follows,

$$f(s) = -s^2 + s\sqrt{1+\gamma s^4}. \qquad (3.18)$$

The stationary phase point is the root of $f' = 0$, *i.e.*,

$$f'(s) = -2s + \frac{1+3\gamma s^4}{\sqrt{1+\gamma s^4}} = 0. \qquad (3.19)$$

The real roots of the above equation are positive because the second term of f' is positive and an even function. Also the second term is a monotonically increasing funcition and so is its derivative, hence two roots can exist at most. As shown in Fig.2, if γ is small enough, two positive real roots exist. As γ gets larger, the values of two roots approaches to each other, and for a certain value of γ two roots coincide, *i.e.*, we have a double root. If γ is greater than this value, no real root exists.

The critical value at which the stationary phase point is a double root can be obtained from Eq.(3.19),

$$2s\sqrt{1+\gamma s^4} = 1+3\gamma s^4. \qquad (3.20)$$

For the case of doubel root, the derivatives of

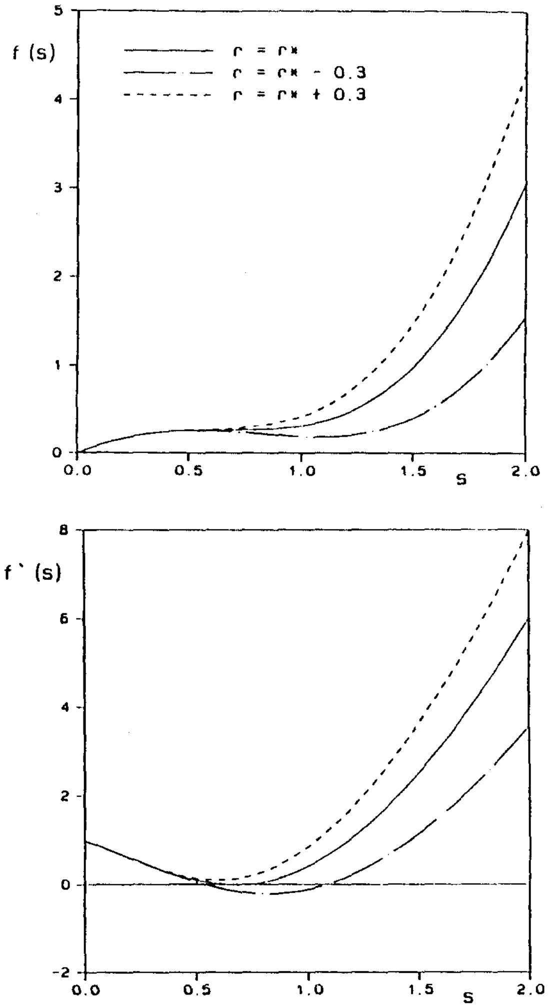

Fig. 2 Phase Function And Its Derivative for x << gt* *2

the left hand side and the right hand side of the above equation must coincide with each other also, thus the following equation must hold.

$$2\frac{1+3\gamma s^4}{\sqrt{1+\gamma s^4}}=12\gamma s^3 \text{ or } 6\gamma s^3\sqrt{1+\gamma s^4}=1+3\gamma s^4. \tag{3.21}$$

The critical value of γ and s can be obtained from the two simultaneous equations Eq.(3.20) and Eq.(3.21). Since γ is positive, the critical value γ * can be obtained as below.

$$\gamma^* = \frac{1}{6\sqrt{3}-9} = \frac{3+2\sqrt{3}}{9}. \tag{3.22}$$

The corresponding value of s is

$$s^* = \sqrt{2\sqrt{3}-3}.$$

The critical value of x can be obtained as follows by using the difinition of γ.

$$x^* = (\gamma^*)^{-1/4}\sigma^{1/4}gt \tag{3.23}$$

Let the two stationary phase points be represented by s_1 and s_2 and s_2 ($s_1 < s_2$). As in Fig. 3, if x is greater than the critical value x^*, s_1 nearly equals to 1/2 and s_2 has a great value. As x becomes smaller, s_1 becomes larger and s_2

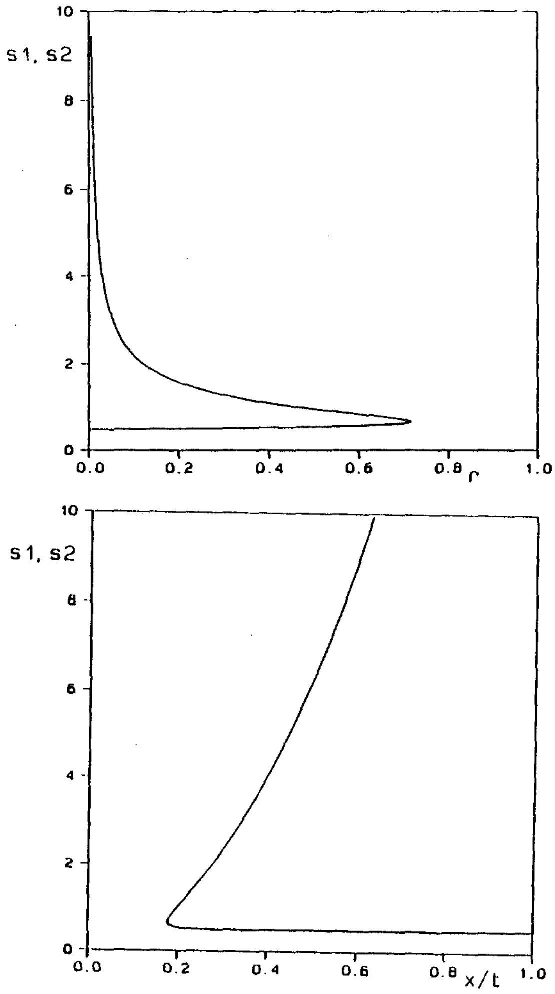

Fig. 3 Stationary Phase Point vs. r and x/t for x << gt* *2

becomes smaller. If x coincides with x^*, s_1 equals to s_2 and their values become s^*. If x is less than x^*, no real root exists.

Let the stationary phase integral of Eq.(3.17) be represented by E_{st}, when $x > x^*$, E_{st} can be obtained by simply adding the stationary phase integrals associated with s_1, s_2 together.

$$E_{st} \sim \frac{x}{igt} \frac{1}{s_1^2\sqrt{1+\gamma s_1^4}} \sqrt{\frac{2\pi X^2}{-f''(s_1)}} \exp \left(\frac{i}{X^2} f(s_1) - \frac{\pi}{4} i\right),$$
$$+ \frac{x}{igt} \frac{1}{s_2^2\sqrt{1+\gamma s_2^4}} \sqrt{\frac{2\pi X^2}{f''(s_2)}} \exp \left(\frac{i}{X^2} f(s_2) + \frac{\pi}{4} i\right), \quad (3.24)$$

as $1/X^2 \to \infty$.

When x approaches the critical value x^*, s_1 and s_2 become closer to each other, and f'' nearly equals to zero, so that the above expansion is not valid any more (Chester *et al.*[13]). Thus the asymptotic expansion must be obtained from representing the phase function by cubic representation. The phase function Eq.(3.18) is expanded around $s_c (s_1 < s_c < s_2)$, which is corresponding to a root of $f'' = 0$, up to the third order.

$$f(u) = \frac{f'''(s_c)}{6} u^3 + f'(s_c)u + f(s_c), \quad (3.25)$$

where $u = s - s_c$ We may rewrite Eq.(3.25) as

$$\frac{1}{X^2} f(u) = N(u^3/3 + \zeta u) + \mathrm{NA}, \quad (3.26)$$

where N, A, ζ are as follows.

$$\mathrm{N} = \frac{1}{X^2} \frac{f'''(s_c)}{2}, \quad \mathrm{A} = \frac{2f(s_c)}{f'''(s_c)},$$
$$\zeta = \frac{2f'(s)}{f'''(s_c)}.$$

Substituting Eq.(3.26) into Eq.(3.17) we get the integral around the stationary phase point. As x approaches x^*, ζ becomes very small since so is f'. Thus the main portion is the integral around zero, and E_{st} can be approximated as follows,

$$E_{st} \approx \frac{x}{igt} \frac{1 - \exp\left(-\frac{d}{x} \frac{s^2}{X^2}\right)}{s_c^2 \sqrt{1+\gamma s_c^4}} e^{i\mathrm{NA}} \int_{-\delta}^{\delta} e^{i\mathrm{N}(u^3/3 + \zeta u)} du. \quad (3.27)$$

where δ is chosen to include the stationary phase point. As $\mathrm{N} \to \infty$, the above integral can be represented by the Airy function. The definition of the Airy function is

$$\mathrm{A}i(x) = \frac{1}{2\pi} \int_{c} e^{i(xk + k^3/3)} dk.$$

where the contour C is the Sommerfeld contour from $\infty e^{i(\pi - \pi/6)}$ to $\infty e^{i\pi/6}$. Using the definition of the Airy function, Eq.(3.27) can be expanded asymptotically as follows,

$$E_{st} \sim \frac{x}{igt} \frac{1}{s_c^2\sqrt{1+\gamma s_c^4}} \frac{1}{\mathrm{N}^{1/3}} e^{i\mathrm{NA}} \mathrm{A}i(\mathrm{N}^{2/3}\zeta), \quad (3.28)$$

where the sign of ζ is negative when $x > x^*$, positive when $x < x^*$, and ζ equals to zero when $x = x^*$.

When $x > x^*$, as in Eq.(3.24) the stationary phase integral consists of two oscillating terms, one is a highly oscillating term compared with the other. When x becomes smaller and approaches x^*, the frequencies of the two oscillating terms become closer, and when $x \approx x^*$ it is reduced to one oscillating term which is multiplied by the Airy function as in Eq.(3.28). When $x < x^*$, ζ is positive and the argument of the Airy function has very large value, while tha Airy function decays exponentially. Thus as x becomes smaller, the stationary phase integral decays exponentially.

□ Case of $x \gg gt^2$. ($i.e.$ $X^2 \gg 1$)

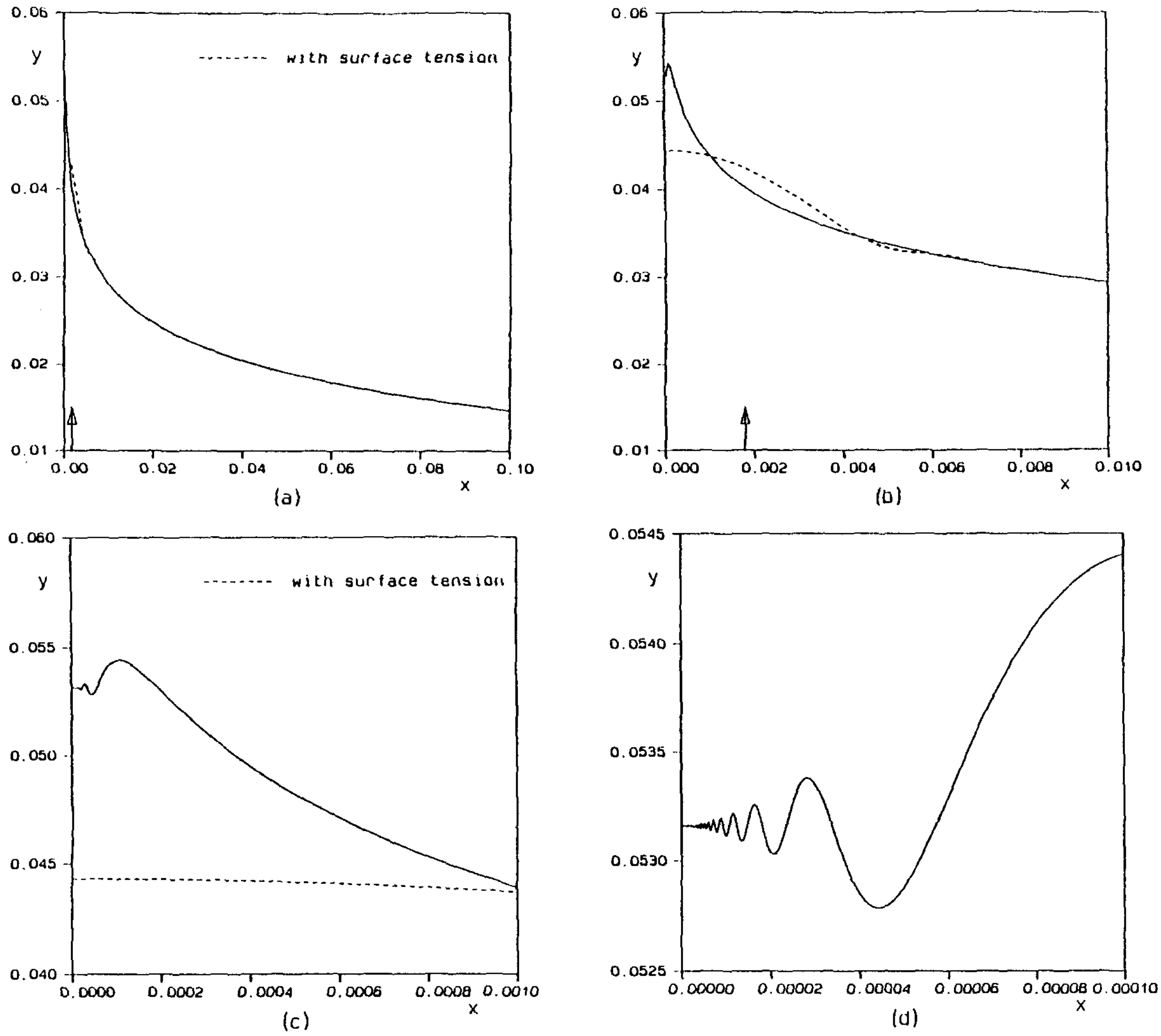

Fig. 4 Free Surface Elevation at t=0.01 Impulsive Motion

Making change of variable as $u=x/(gt\sqrt{\sigma})$. s in Eq.(3.15) gives

$$E=\frac{gt\sqrt{\sigma}}{ix}\left[\int_0^{\infty}+\int_0^{-\infty}\right]\frac{1-\exp\left(-X^2\frac{xd}{\sigma g^2}s^2\right)}{s^2\sqrt{1+s^4/\gamma}}\exp\left(iX^2\frac{x^2}{\sigma g^2}f(s)\right)ds, \tag{3.29}$$

where $x^2/(\sigma g^2)$ is not small. The phase function is

$$\begin{aligned} f(s) &= -s^2+s\sqrt{\gamma}\sqrt{1+s^4/\gamma} \\ &= -s^2+s\sqrt{\gamma+s^4} \end{aligned} \tag{3.30}$$

Similarly as in the previous case, the critical values γ^* and s^* can be obtained as follows,

$$\gamma^*=\frac{3+2\sqrt{3}}{9},$$

$$s^*=\frac{\sqrt{3}}{3}. \tag{3.31}$$

The critical value γ^* is the same as that of Eq.(3.22) which is obtained when $x \ll gt^2$. Thus the behavior of the analytical solution is the same as the case $x \ll gt^2$.

Let us examine the characteristics of the highly oscillating term from the stationary phase integral. When x is greater than the criti-

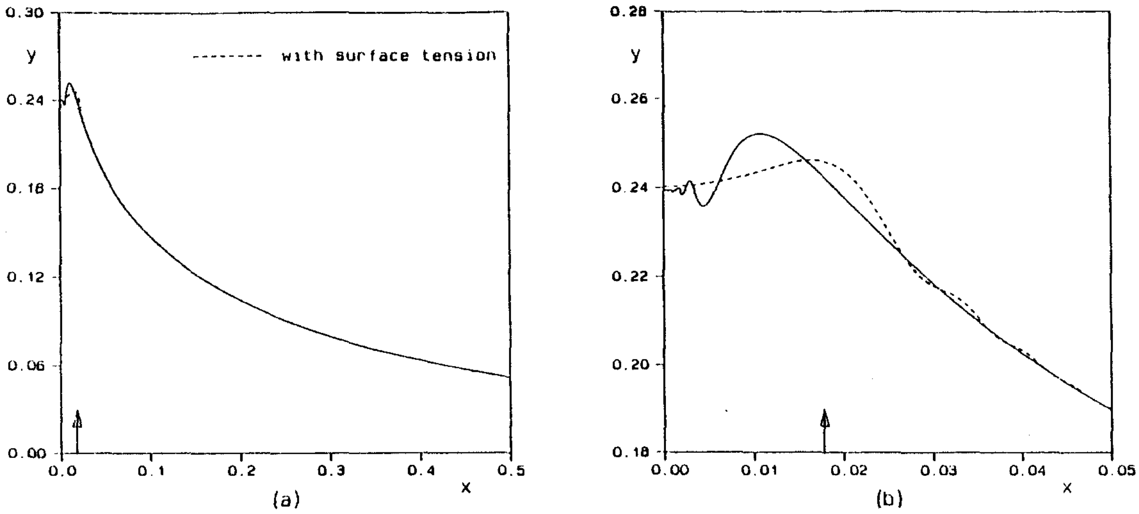

Fig. 5 Free Surface Elevation at t=0.1 Impulsive Motion

cal value x^*, there exists a highly oscillating term, and when x is greater than the critical value x, there exists a highly oscillating term, and when x is less than the critical value, the amplitude of oscillating term decays exponentially, so the oscillating term disappears eventually. Therefore, the slope of the free surface becomes smoother, an as a result, the problem of the self-consistency will be resolved if surface tension is included. If surface tension is ignored there exist gravity waves only, which propagate slowly as the wave length become smaller. Thus short waves do not propagate far from the intersection point. Therefore, the slope of the free surface cannot be bounded. But if surface tension is included, short waves are affected by surface tension more than long ones, and propagate faster, so the very short waves do not exist any longer near the intersection point.

The existence of the oscillating term depends on a scale proportional to t, while it depends on the scale gt^2 when surface tension is ignored. Eq.(3.23) shows that the critical point $x^*$is proportional to t and the velocity of critical point is x^*/t = 0.1776m/sec. If x is less than this critical value, the oscillating term vanishes.

In Fig.4, the height of the free surface is shown for the case of impulsive motion at t = 0.01. For large x , the height of the free surface bahaves like a logarithmic function as shown in Fig.4(a). Fig.4(b) shows that the slope of the free surface becomes smoother near the intersection point when surface tension is included, and the oscillating term appears when x is greater than the critical value. If surface tension is ignored, there exists a highly oscillating term as shown in Fig.4(d).

In Fig.5, the height of the free surface is shown for the case of impulsive motion at t = 0.1 Also there is no highly oscillating term if the distance x is less than the critical value 0.01776.

Fig.6, at time t = 1, shows that the effect of gravity wave is large in the region where x is large, and there is no highly oscillating term in the region where x is less than the critical value 0.1776.

In Fig.7−Fig.9, the height of the free surface is shown for the case of constant acceleration. Fig.7, at time t = 0.01, show that the effect of

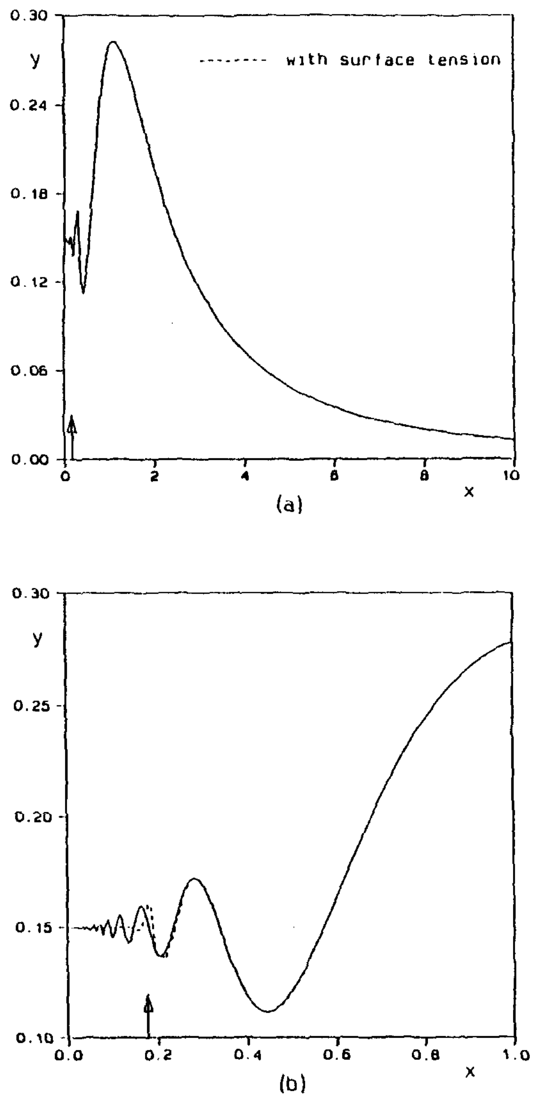

Fig. 6 Free Surface Elevation at t=0.1
Impulsive Motion

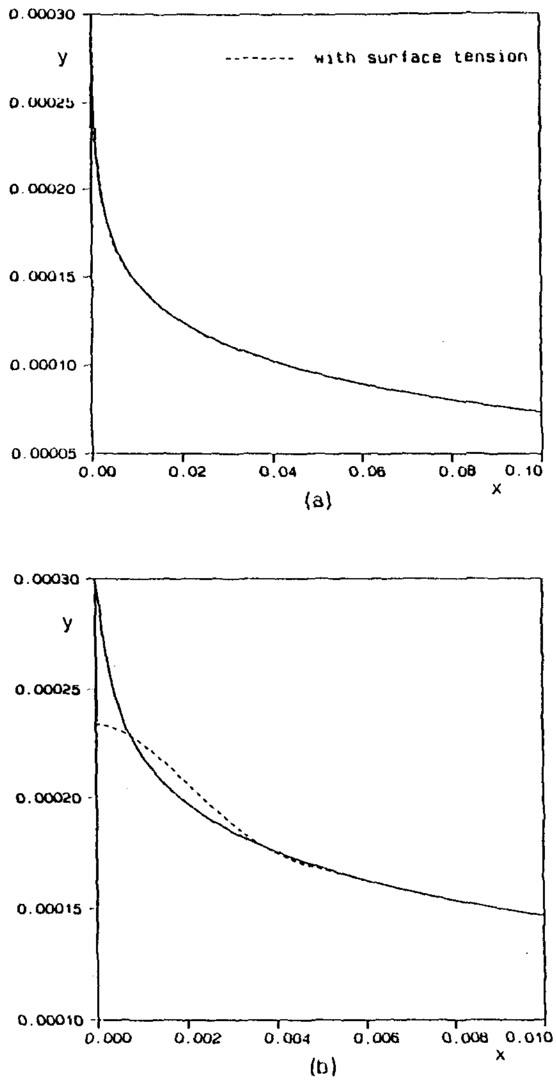

Fig. 7 Free Surface Elevation at t=0.01
Constant Acceleration

surface tension is large near the intersection point, the slope of the free surface is bounded even if surface tension is ignored. In Fig.8 and Fig.9 when $t = 0.1$ and $t = 1$ respectively, the difference between the analytical solutions with and without surface tension becomes small as time goes. In Fig.9, the difference is nearly zero.

In Fig.10, the height of the intersection point is shown. We see that the height of the intersection point becomes smaller for the time interval of interest if surface tension is included, and that the behavior in time for the case of impulsive motion differs from the one for the case of constant acceleration.

We may conclude that the effect of surface tension becomes larger for the case of impulsive motion than for the case of constant accelera-

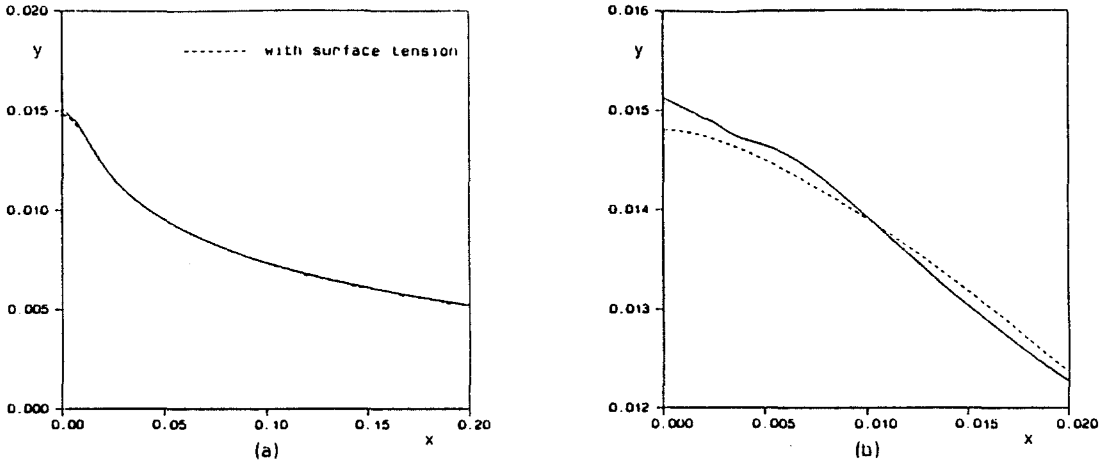

Fig. 8 Free Surface Elevation at t=0.1 Constant Acceleration

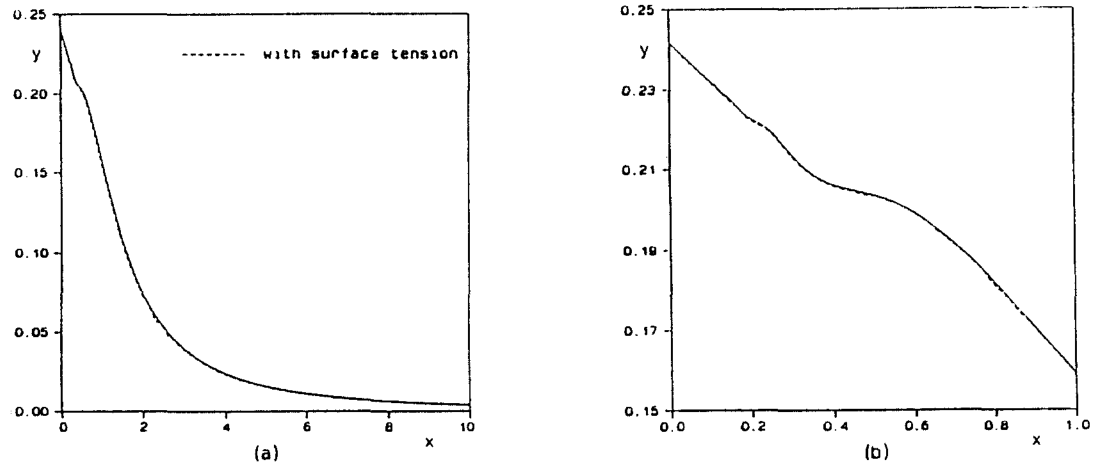

Fig. 9 Free Surface Elevation at t=1.0 Constant Acceleration

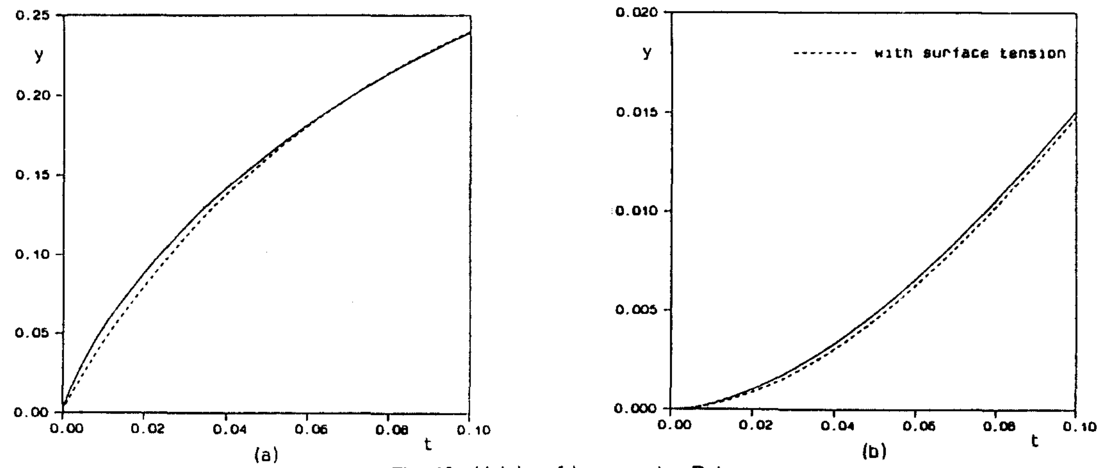

Fig. 10 Height of Intersection Point

(a) Impulsive Motion (b) Constant Acceleration

tion. If surface tension is taken into account, the slope of the free surface becomes smooth. The critical value obtained in Eq.(3.23) agrees well with the results shown in figures.

4. Conclusions

In this paper, the fluid flow near the intersection point which is one of the difficult problems in fluid mechanics is studied. The effect of surface tension is investigated, and an analytical solution which has a self-consistency even for the impulsive motion of a vertical plate is obtained. Following conclusions can be drawn.

(1) In the case of impulsive motion of a vertical plate, the analytical solution without surface tension has no self-consistency. But if surface tension is included, the solution with self-consistency can be obtained.

(2) In the case of impulsive motion, if we neglect the effect of surface tension, an oscillating phenomenon exists in a small region near the intersection point(its length scale is gt^2). If surface tension is included, there exists a critical value associated with oscillating phenomenon which is proportional to time. In the region where the distance from the intersection point is less than this critical value $0.1776t$, no oscillating phenomenon exists. Only if the distance is greater than the critical value, an oscillating phenomenon occurs.

(3) In the case of constant acceleration, the effect of surface tension is important for a short time duration. The height of the intersection point becomes smaller and so does the slope of a free surface near the intersection point. As time passes, the effect decreases.

It is hoped that a further study for the case when a body meets a free surface at a certain angle can be made.

Acknowledgment

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Appendix

Green Function with Surface Tension

The initial−boundary value problem to be solved is as follows.

$$\nabla^2\phi = 0$$

$$\phi_{tt} + g\phi_y - \frac{T}{\rho}\phi_{yxx} = 0 \quad \text{on} \quad y = 0$$

$$\nabla\phi \to 0 \quad \text{as} \quad y \to -\infty$$

$$\nabla\phi \to 0 \quad \text{as} \quad x \to \pm\infty$$

$$\phi(x,0,0) = \phi_t(x,0,0) = 0$$

The Green function in which the strength of a source is a Dirac delta function $\delta(t)$ is used. First, the Green function is assumed as the sum of a source velocity potential and a harmonic function K.

$$G = \frac{\delta(t)}{2\pi}\left[\frac{1}{2}\log((x-a)^2 + (y-b)^2) - \frac{1}{2}\log((x-a)^2 + (y+b)^2)\right] + K(x,y,t), \quad \text{(A.1)}$$

where (a,b) is the position of a source. Substituting above representation into the free surface boundary condition, then we have

$$K_{tt} + gK_y - \frac{T}{\rho}K_{yxx} = -\frac{g}{\pi}\delta(t)\,\text{Re}\int_0^\infty \left(1 + \frac{T}{\rho g}k^2\right)e^{kb}e^{-ik(x-a)}dk, \quad \text{(A.2)}$$

where an integral representation of a logarithmic function has been used. Because the harmonic function K must satisfy Laplace equation and the bottom boundary condition, it is convenient to represent it as follows,

$$K = \text{Re}\int_0^\infty B(k,t)e^{k(y+b)}e^{-ik(x-a)}dk. \quad \text{(A.3)}$$

Substituting above representation into Eq.(A.2), then we have an equation for B,

$$B_{tt} + gkB + \frac{T}{\rho}k^3B = -\frac{\delta(t)}{\pi}\left(g + \frac{T}{\rho}k^2\right) \quad \text{(A.4)}$$

The initial condition for B is $B(k,0) = B_t(k,0) = 0$. Using the initial condition, Laplace transform of the above equation can be written as

$$\left(s^2 + gk + \frac{T}{\rho}k^3\right)B = -\frac{1}{\pi}\left(g + \frac{T}{\rho}k^2\right).$$

B can be obtained by Laplace inverse tranform.

$$B = -\frac{1}{\pi}H(t)\left(g + \frac{T}{\rho}k^2\right)\frac{\sin(\sqrt{gk + T/\rho k^3}\,t)}{\sqrt{gk + T/\rho k^3}}, \quad \text{(A.5)}$$

where $H(t)$ is the unit Heaviside function. The harmonic function K can be obtained by substituting B into Eq.(A.3), then the Green function which satisfies all boundary conditions except a body boundary condition can be obtained(represented using complex number).

$$G = \frac{\delta(t)}{2\pi}[\log(z-c) - \log(z-\bar{c})] - \frac{1}{\pi}H(t)\int_0^\infty e^{-ik(z-\bar{c})}\left(g + \frac{T}{\rho}k^2\right)\frac{\sin(\sqrt{gk + T/\rho k^3}\,t)}{\sqrt{gk + T/\rho k^3}}dk. \quad \text{(A.6)}$$

where $z = x + iy$, $c = a + ib$, and an overbar denotes the complex conjugate.