Technical Paper

Transactions of the Society of Naval Architects of Korea
Vol. 28, No. 1, April 1991
大韓造船學會論文集
第28卷 第1號 1991年 4月

On the Transverse Strength of SWATH Ship

−Reliability Analysis against Ultimate and Fatigue Strength−

by

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SWATH선의 횡강도에 관한 연구

−최종강도와 피로강도에 대한 신뢰성 해석−

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Abstract

This paper is an illustration of the application of the reliability analysis to the transverse structure of a SWATH ship. The ultimate strength of plate members on the cross structure and upper part of strut are considered in the reliability analysis. The fatigue reliability analysis has been also carried out at the junction of cross structure, sponson and strut. Included is also an example of the allowable fatigue damage level. Demonstrated is the reliability study of series system of which elements are the ultimate and fatigue failure as well. Doing this would be desirable to get a truer solution of the structural safety level.

The paper ends with a brief summary of the present reliability study and some important points which may be useful at the design stage.

요 약

본 논문에서는 SWATH선의 횡 구조물에 신뢰성 해석법을 적용하였다. cross−structure와 strut의 윗부분에 있는 판부재를 선정하여 그 압축 최종강도에 대한 신뢰성해석을 수행하였고, 또한 cross−structure와 strut, sponson과 strut의 연결부에 대한 피로 신뢰성해석을 수행하였다.

피로손상의 허용수준의 설정에 대한 일례를 보여주었으며, 최종강도와 피로강도를 동시에 고려한 일종의 series system에 대한 신뢰성해석을 포함하였다. 이는 구조의 안전성 평가시 보다 정확한 결과를 얻을 수 있다는 점에서 바람직하다고 하겠다.

발표 : 1990년도 대한조선학회 추계연구발표회(1990. 11. 10)

Manuscript received : January 12, 1991, revised manuscript received : March 14, 1991.

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1. Introduction

SWATH ship has been known to have advantages over the mono−hull ships, in especial with regard to stability, motion and resistance characteristics. More than 70% of the researchs on SWATH ship have been concerned with the hydrodynamic side. This paper is aimed at a better strutural design of SWATH ships. Apart from the mono hull ships, SWATH structural design much depends on the transverse strength rather than longitudinal stregth due to its own structural configuration characterististics quite different from that of mono hull ship (SWATH ship in general shows a good longitudinal strength any way).

This is not the place to go into detail structural analysis nor design. This study is in principle concerned with the evaluation of the reliability(or safety level) against both ultimate and fatigue strength under primary loading condition. Advanced level−II reliability method (AFORM) is employed for the reliability analysis as in Reference 1.

2. Swath ship model

The SWATH ship originally designed by KRISO is chosen for the present study[2]. Its principal dimensions are listed in Table 1. It has the transverse frame system to effectively resist the transverse loading. The frame space is 500 mm and there is bulkhead at every 6 frames. The structural module of cross−struture and strut between bulkheads are taken as the present structural model see(Fig. 2) and the transverse side load and bending moment are applied to the structural module. The structural material is the aluminum alloy with the elastic modulus of 69,000MPa and yield stress of 163MPa. Fatigue reliability is evaluated at the junction of cross−structure and sponson (at "A" in Fig.1) and at the junction of strut and sponson (at "B" in Fig.1).

Aluminum alloy is well appreciated to be prone to fatigue than steel and it also suffers from a condition known as "stress corrosion" which produces cracks of similar apperance.

Transverse side load is obtained by using the Sikora's formula[3]. That is given by :

$$F=9.81(k+LBT)D\Delta \quad (1)$$

where Δ is loaded displacement in tons, $k=-0.5$, and L, B, T and D are given as :

$$L=0.511+0.041\ L_S^2/\Delta\frac{2}{3}$$

$$B=0.9582+0.027\ B_S^2/\Delta\frac{2}{3}$$

$$T=0.3234+2.051\ d^2/\Delta\frac{2}{3}$$

$$D=2.12-0.995\ ln\ \Delta$$

and with(see Fig. 3)

L_S=strut length(m) d=draft(m)

B_S=distance between lower hull centre lines (m)

Bending moment at the upper part of strut M_{St} and at the centre line of the cross − structure M_w due the side load are given by :

$$M_{St}=F\ L_{St} \quad (2)$$

$$M_W=F\ L_W+0.1F\ B_S/2 \quad (3)$$

Still water bending moment at the centre of the cross−structure M_{sw} is given by :

$$M_{SW}=9.81\Delta'\ B_S/8 \quad (4)$$

Table 1 Principal Dimensions of the Present SWATH Ship Model

Length B.P.(scantling)	17.3m
Length of strut	17.7m
Breadth (Max)	9.2m
Distance between lower hull centrelines	7.4m
Depth to main deck	4.5m
Draft (scantling)	2.4m
Lower hull diameter	1.6m
Displacement(full load)	70.0ton

and is added to M_W and then the total bending moment at the centre line of the cross-structure is :

$$M_T = M_{SW} + M_W \tag{5}$$

In Eqs(1) thru (5) units are m and KN. In Eq. (4) Δ' is mass of cross structure and is given as $\Delta' = 0.8\Delta$ if there are no available data. With data in Table 1, bending moments acting on the cross-structure centre line are obtained as :

$$M_W = 3342.70 \text{ KN-m} \tag{6.a}$$

$$M_{SW} = 508.16 \text{ KN-m} \tag{6.b}$$

and on the upper part of strut is :

$$M_W = 1636.64 \text{ KN-m} \tag{7}$$

3. Reliability Analysis Against Ultimate Strength

In this section the ultimate strength is defined as that of plate under compression and then it can be seen that the safety margin is simply expressed as Eq. (8) in terms of axial stress :

$$M^U = X_{MR}\sigma_U - (\sigma_{SW} + \sigma_W) \tag{8}$$

where σ_{SW} and σ_W are the applied on plate due to still water bending moment wave bending moment and M^U is referred to as the safety margin for the ultimate strength and X_{MR} is the strength modelling parameter introduced to account for the uncertainty in strength model. The uncertainties of σ_{SW} and σ_W are explicity considered by imposing their COV vallues. σ_U is the ultimate compressive strength of plate given as :

$$\sigma_U = \sigma_Y\left(\frac{2}{\beta} - \frac{1}{\beta^2}\right) \tag{9}$$

in which σ_Y is yield stress of plate and β is the plate slenderness parameter defined as :

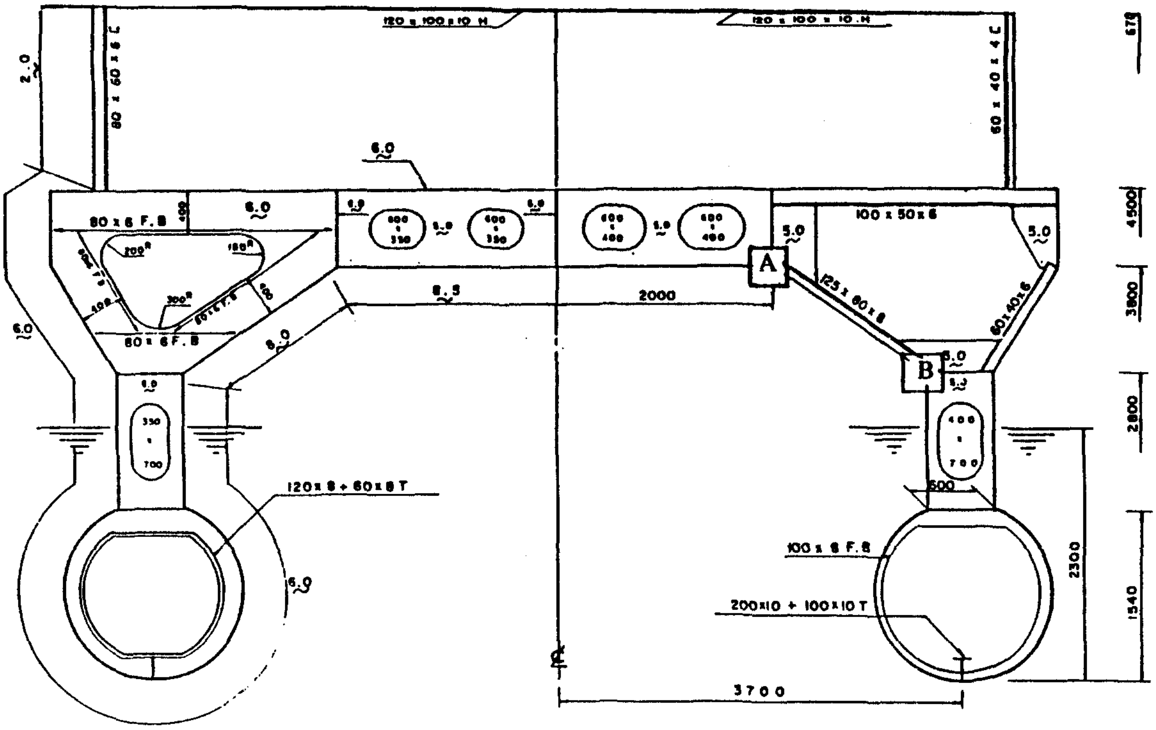
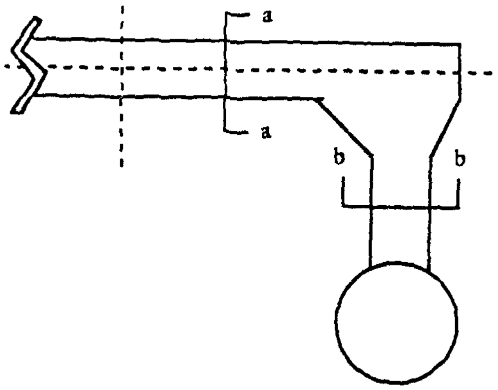
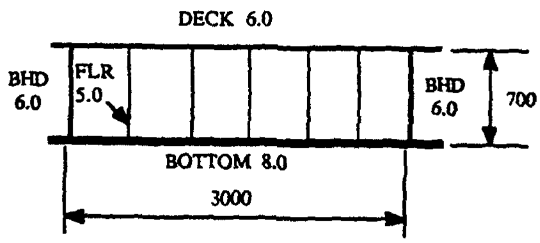


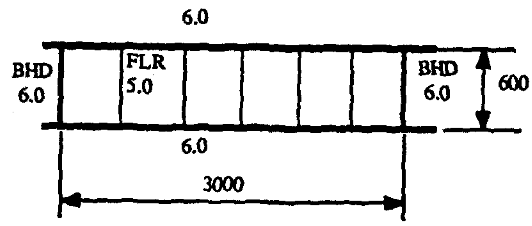
Fig. 1 Midship of the Present SWATH Ship, Passenger Room Section (after Reference 1)

$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}}$$





(a) Cross-Structure : Section a-a



(b) Strut : Section b-b

Fig. 2 Structural Module for the Present Study

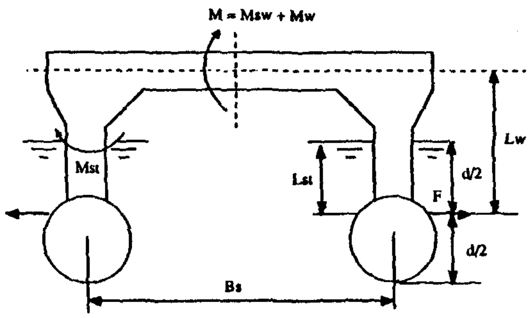


Fig. 3

Dara for the reliability analysis are listed in Table 2. The COV of wave bending stress, i. e. dynamic load effect, is assumed to be 20 to 30 % since there must be much inherent uncertainty in predicting the transverse side lode with the Sikora's formula(This is not, of course, clear). Reliability indices have been derived for the upper part of strut, and the results are presented in Table 3.

Table 2 Data for Reliability Analysis against Ultimate Strength(unit : mm, N)

Variable	mean	COV	dist. type
width of plate	500	0.04	N
thickness of plate	8 at bottom of cross-structure 6 at upper part of strut	0.04	N
yield stress	165	0.08	L−N
strength modelling parameter	1.0	0.10	L−N
σ_{SW}	4.61 at bottom of cross−structure 0 at upper part of strut	0.10	L−N
σ_W	30.32 at bottom of cross−structure 21.31 at upper part of strut	0.2−0.3	L−N

dist, type : N=normal L−N=log−normal

Table 3 Reliability Indices against Ultimate Strength of Plate

COV of wave bending stress	at bottom of cross −structure	at upper part of strut
0.2	4.46 (0.41×10^{-5})	5.25 (0.78×10^{-7})
0.25	3.83 (0.64×10^{-4})	4.47(0.38×10^{-5})
0.3	3.36 (0.39×10^{-3})	3.91 (0.46×10^{-4})

note : figures in ( ) is the probability of failure

4. Fatigue reliability Analysis

Fatigue failure is one the most important failure mode (or element) in a structure mainly subjected to variable loads. Above all, as previously mentioned, since aluminum alloy structure is less immune to fatigue failure than steel structure, it should likely be more seriously considered than the ultimate failure. As well known, there is much more uncertainties associated with fatigue strength than the ultimate strength and then, reliability analysis must be a suitable tool to rationally assess the structural safety. In this paper failure due to fatigue damage is concerned. The characteristic S−N curve is given by :

$$NS^m = A \qquad (10)$$

where m is the fatigue strength exponent, A is the fatigue strength coefficient, S is stress range and N is cycles to failure at the stress range S. Assuming that fatigue strength is defined by Eq. (10) and that Miner's rule works, fatigue damage ratio D can be written as :

$$D = N_T B^m E\,[S^m]/A \qquad (11)$$

where N^T is the total number of cycles applied and E[·] is expected value. B is introduced as a bias factor of stress range and reflect the uncertainty in estimating the stress range, S. The Weibull distribution is commonly employed for the long−term distribution of S. Its cumulative distribution function is given as :

$$F_S(s) = 1 - \exp[-(s/\delta)^{\xi}], \text{ for } s>0 \qquad (12)$$

in which δ and ξ are Weibull scale and shape parameters. Let S_C be the design stress range, say the characteristic stress range, and then the probability that S_C is exceeded by s on an average of once every N_T times is :

$$P(s>S_C) = 1/N_T \qquad (13)$$

Let N_T be the total number of cycles in the service life. It can be easily shown that the scale parameter δ is expresses in terms of S_C, ξ and N_T as :

$$\delta = S_C[ln\ N_T]^{-1/\xi} \qquad (14)$$

With Eq. (14) for δ, Eq. (12) is rewritten as :

$$F_S(s) = 1 - \exp[-(\mathrm{In}\ N_T)\ (s/S_C)^{\xi}], \quad \text{for } s>0 \qquad (15)$$

Then it is evident that the shape parameter ξ plays a key role which describes implicitly both the environmental and the structural system.

Using the cumulative distribution function for S given as Eq. (15) , the expected value $E[S^m]$ in Eq.(11) is written as :

$$E[S^m] = \int_0^{\infty} s^m f_S(s)\,ds = S_C^m (ln\ N_T)^{-m/\xi}\ (1+m/\xi) \qquad (16)$$

where FS(s) is the probability density f unction for S and $\Gamma(\cdot)$ is the Gamma function defined as :

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \qquad (17)$$

Substituting Eq. (16) for $E[S^m]$ in Eq. (11), we can have the fatigue damage ratio as :

$$D = \frac{N_T}{A} B^m S_c^m [(lnN_T)^{-m/\xi}\ \Gamma(1+m/\xi] \qquad (18)$$

Let Δ be the fatigue strength(damage index at failure) and then failure occurs when $\Delta < D$. Hence the safety margin is written as :

$$M^F = \Delta - D = \Delta - \frac{N_T}{A} B^m S_c^m [(lnN_T)^{-m/\xi}\ \Gamma(1+m/\xi)] \qquad (19)$$

The superscipt "F" means that the safety margin is associated with the fatigue failure. The random variable are Δ, B and A. Their uncertainties usually vary within a wide range. The S−N curve of which m and A are 3.0 and $10^{12.044}$ is used in this study[5]. Table 4 illustrates the uncertainty modelling for the present reliability study. Distribution of all variables are assumed to be log−normal as in Reference[6]. COV of Δ (C_Δ) represents the inherent uncertainty of the Miner's damage model and COV of A (C_A) represents the inherent variability of the fatigue strength. C_B is

COV of the bias factor B. Several case studies have been carried out by varying the values of COVs, say C_A, C_Δ and C_B, within the ranges in Table 4. Their values for case studies are :

C_Δ=0.3, 0.4, 0.5, 0.6

C_B=0.2, 0.35, 0.5

C_A=0.4, 0.5, 0.6

Fatigue reliability indices are evaluated at "A" and "B" in Fig.1. The characteristic stress range at "A" is taken as the value due to wave bending stress, i. e.,

$(S_C)A=60.64\ N/mm^2$

With regard to the stress range at "B", from the finite element analysis in Reference[1] the stress concentration occured mainly at "B" and hence the characteristic stress range at "B", $(S_C)_B$ is taken from the result in the reference. The value is :

$(S_C)_B=57.70 N/mm^2$

Table 5 shows the fatigue reliability indices of the case studies. We can find that effects of C_A and C_Δ on the fatigue reliability index are nearly same and less than the effect of C_B. That is, fatigue reliability is more sensitive to the variation of C_B than those of C_A and C_Δ. This means that the advanced loading model is needed to raise the fatigue reliability level.

Design criteria documents frequently specify the allowable fatigue damage ratio, Δ_0(often called the "target damage ratio, "). With the assumed log-normal distribution for Δ, B and A, the allowable fatigue damage ratio is given in the simple form as [6] :

$$\Delta_0=\lambda\Delta/B_m \exp(\beta_0\, \sigma\, ln\ T) \qquad (20)$$

where β_0 is the allowable reliability index(or "target reliability index") and

$$\lambda=\exp(2\sigma_{ln\,A}) \text{ with } \sigma_{ln\,A}=\sqrt{\ln(1+C^2_A)} \qquad (21)$$

$$\sigma^2 ln_T=ln[1+C^2{}_\Delta)(1+C^2_A)(1+C^2_B)^{m'} \qquad (22)$$

Upon seletion of an apporite target reliability index β_0 the value of Δ_0 can be easily established from Eqs(20)-(22). Selection of β_0 is influenced by consideration of importance and inspectability of the structural member.

Table 4 Data for Fatigue Reliability Analysis

Variable	mean	COV	distribution type
Δ	1.0	0.3-0.6	log-normal
B	1.0	0.2-0.5	log-normal
A	$10^{12.044}$	0.4-0.6	log-normal

Table 5 Results of Fatigue Reliability Analysis

(a) C_Δ=0.3

position	at "A"			at "B"		
C_A \ C_B	0.2	0.35	0.5	0.2	0.35	0.5
0.4	3.93	2.77	2.20	4.20	2.95	2.33
0.5	3.66	2.66	2.14	3.91	2.84	2.27
0.6	3.40	2.55	2.07	3.68	2.72	2.02

(b) C_Δ=0.4

position	at "A"			at "B"		
C_A \ C_B	0.2	0.35	0.5	0.2	0.35	0.5
0.4	3.70	2.68	2.15	3.95	2.85	2.28
0.5	3.46	2.58	2.09	3.70	2.75	2.22
0.6	3.23	2.47	2.03	3.46	2.63	2.15

(c) C_Δ=0.5

position	at "A"			at "B"		
C_A \ C_B	0.2	0.35	0.5	0.2	0.35	0.5
0.4	3.46	2.58	2.09	3.70	2.75	2.22
0.5	3.25	2.48	2.03	3.48	2.65	2.16
0.6	3.05	2.38	1.97	3.26	2.54	2.10

(d) C_Δ=0.6

position	at "A"			at "B"		
C_A \ C_B	0.2	0.35	0.5	0.2	0.35	0.5
0.4	3.23	2.47	2.03	3.45	2.63	2.15
0.5	3.05	2.38	1.97	3.26	2.54	2.10
0.6	2.87	2.28	1.91	3.08	2.44	2.04

Choosing the values of C_Δ, C_B and C_A, an example of Δ_0 is given in Table 6.

5. Safety Level by Consideration of Ultinate and Fatigue Strength

In this section the safety level by considering both the ultimate and fatigue strengths is briefly described. Let think the failure dus to lack of ultimate strength and due to fatigue damage as failure elements[7]. Then a truer safety level of a structure may be obtained from the series system of which elements are the ulltimate and fatigue failure. That is the probability of failure is evaluated from :

$$P_{fs}=[(M^U<0)\ U\ (M^F<0)] \tag{23}$$

where M^U and M^F are the safety margin associated with ultimate and fatigue failure, respectively given as Eqs(8) and (19). The probability of failure to the safety margins are obtained from the separate analysis :

$$P_f^U=P[(M^U<0)] \text{ and } P_f^F=P[(M^F<0)]$$

Hereafter the superscript "U" and "F" denotes that they are associated with the ultimate and fatigue failure. The corresponding reliability indices are :

$$\beta^U=\Phi^{-1}(P_f^U) \qquad \beta^F=\Phi^{-1}(P_f^F)$$

where $\Phi(\cdot)$ is the cumulative distribution function of standard normal distribution. The probability of failure P_{fs} in Eq. (23) is then

Table 6 Example of the Allowable Fatigue Damage Ratio(Δ_0)

(a) reference data

variable	mean	COV
Δ	1.0	0.3
B	1.0	0.3
A	$10^{12.044}$	0.5
m	3.0	—

(b) allowable damage ratio

β_0	Δ_0
2.0	0.32
2.5	0.19
3.0	0.11

given by :

$$P_{fs}=1-[\Phi(\beta^U)\ \Phi(\beta^F)+\int_0^{\rho}\Phi_2(\beta^U,\ \beta^F : z)dz] \tag{24}$$

where ρ is the correlation coefficient between two safety margins, M^U and M^F and $\phi_2(\cdot)$ is the binormal density function (e. g, see Reference 7) :

$$\phi_2(x, y : \rho)=\frac{1}{2\pi\sqrt{1-\rho^2}}\exp[-\frac{1}{2\sqrt{1-\rho^2}}(x^2+y^2-2\rho xy)] \tag{25}$$

Let the safety margins M^U and M^F be rewritten in the linear form as :

$$M^U=\sum_{i=1}^{n}\alpha^U_iX^U_i+\beta^U$$
$$M^F=\sum_{j=1}^{n}\alpha^F_jX^F_j+\beta^F \tag{26}$$

in which n is the number of random variables. X's are random variables and α's are corresponding sensitivity factors. The correlation coefficient is defines as :

$$\rho=\{\alpha^U\}^T\{\alpha^F\} \tag{27}$$

The sensitivity factors corresponding to the common variables between the safety margins M^U and M^F are included in calculation of ρ from Eq. (27). Only the wave bending stress is common between safety margins M^U and M^F in this study.

The data in Tables 1 and 6 are used except that COV of σ_w for M^U is assumed to be 0.3 as for M^F. The reliability indices for M^U and M^F are :

$\beta^U=3.36$ $\quad\beta^F=2.93$ $\quad$ at "A" in Fig.1

$\beta^U=3.91$ $\quad\beta^F=3.07$ $\quad$ at "B" in Fig.1

and the correlation coefficient is 0.778 and 0.785 at "A" and "B", respectively. That is two failure elements have close correlation. From Eqs(24) and (25) the probability of failure P_{fs} and the corresponding reliability index β_S are :

$P_{fs}=0.193 \times 10^{-2}$ $\beta^F=2.89$ $\quad$ at "A" in Fig.1

$P_{fs}=0.109 \times 10^{-2}$ $\beta^F=3.06$ $\quad$ at "B" in Fig.1

The reliability index β_S is smaller than those of each failure element and is closer to the fatigue reliability index than the ultimate reliability index. This also confirms that fatigue consideration is important in the structural design of SWATH ship.

6. Conclusions

The present paper is a preliminary work concerning with the reliability study of SWATH ships as designed considering the ultimate and fatigue failure. Bending moment due to transverse side load is applied on the centre line of cross−structure and the upper part of strut. Ultimate strength and fatigue damage are considered. It can be drawn that the present SWATH ship model possesses enough transverse strength and safety and more strength could be added from the box−girder effect. From the fatigue reliability point's of view the fatigue reliability indices of most marine structures lies between 2.0 to 2.5 and the present SWATH model seems to have good fatigue reliability level.

This paper also includes an example of the allowable fatigue damage level as use for design criteria against fatigue failure but more study would be required in this area before applying to the practical design. The reliability study of a simple series system of which elements are the ultimate and fatigue failure as presented in the last section may be recommended to get a truer solution of the structural safety level.

It should be pointed out that the fatigue consideration is more important than the ultimate strength especialy when aluminum alloy is used as the structural material as frequently found in high speed light craft.

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