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A Study on the Dynamic Stability of the Long Vertical Beam Subjected to the Parametric Excitation

by

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파라메터 기진에 의한 긴수직보의 동적안정성에 관한 연구

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Abstract

The dynamic stability of the long vertical beam subjected to the periodic axial load is investigated. As a solution method, the Galerkin's method is used to obtain a set of coupled Mathieu type equations. To obtain the stability chart, both the perturbation method and numerical method are used, and the results of the both methods are compared with each other.

The stability regions for the various boundary conditions are obtained. Also the effects of the viscous damping, the mean tension and the multi−frequency parametric excitation are studied in detail.

요 약

축방향의 주기적인 하중으로 가진되는 긴 수직보의 동적안정성에 관하여 연구하였다. 해석방법으로서 Galerkin방법을 이용하여 무한원 연립 Mathieu형 미분 방정식을 얻었으며, 안정성영역을 나타내는 도표를 얻기 위하여, 섭동법과 수치적인 방법을 사용하였다. 또한 이두가지 방법으로 구한 결과를 서로 비교 검토하였다.

여러가지 경계조건에 대한 안정영역을 구했으며, 감쇠의 영향, 평균인장력 및 다중 주파수 파라메터 기진의 영향에 관해서 집중적으로 연구하였다.

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Nomenclature

ϕ_m : the m^{th} Mode Shape
$[\phi]$, $[\varphi]$: Fundamental Matrix
β : Dimensionless Euler Buckling Load
λ_j : Characteristic Multiplier
ω_j : the j^{th} Natural Frequency
P_{cr} : Euler Buckling Load
$P(x, t)$: Periodic Axial Load
$P_0(x)$: Spacially varying tension due to gravity
$P_1(x)$: Dynamic Component of the Axial Load
$\sigma(x)$: Nondimensionalized Static Tension
$\mu_m(t)$: Generalized Coordinate
ω : Lateral Deflection of the Beam
x : Coordinate along the Beam
ξ : Excitation Amplitude Parameter

1. Introduction

In recent decades, the dynamic stability of the elastic system subjected to the parametric excitation has been studied by many researchers.

The term, parametric excitation can be characterized by the harmonic variation of coefficients of the governing differential equations.

In the case of the simply supported beam under the periodic axial loads, the governing equation contains the periodic coefficient and it can be transformed to a set of uncoupled Mathieu type equation. Therefore, only the usual parametric resonanses occur, and then the instability region corresponding to the exciting frequency which is close to twice the natural frequency is possible.

When the beam does not have simply supported ends, the governing equation gives a set of coupled Mathieu type equations. In this case, the combination resonances as well as the parametric resonances may occur.

Historically, Bolotin[2, 3] was the first to study the problem of the parametric instability of the elastic system under periodic loads. He obtained only instability regions corresponding to parametric resonances by means of the method of Hill's infinite determinant.

Iwatsubo *et al.*[4] investigated the stability of cantilevers under parametric excitation by digital simulation technique.

Iwatsubo and Saigo[5] demonstrated experimentally the exsistence of combination resonances in the responses of clamped−clamped and clamped−simply supported columns under periodic axial load. Using analog simulation technique, Sugiyama *et al.*[6] found the combination resonaces in the case of the clamped−free columns subjected to the periodic tangential loads.

Nayfeh and Mook[7] obtained approximate transition curve for many resonance situations by perturbation technique. Hsu[8] obtained approximate solutions for various resonant situations.

Hsu[9, 10, 11] also developed the numerical method to approximate transition matrix to overcome the deficiency of numerical works. Friedmann *et. al* [12] developed the efficient numerical scheme for computing the transition matrix at the end of one period.

The main purpose of the present paper is to make comparisions among the existing methods in the literature and to study the dynamic stability of the long vertical beam subjected to the periodic axial loads for various constraint conditions. The supporting legs of Tension Leg Platforms can be modeled as such kind of system.

Finally, the instability regions for various boundary conditions are obtained. Also the effects of viscous damping, mean tension and the multi−frequency parametric excitation on the instability regions are studied in detail.

2. Equation of Motion

In order to derive the equation of motion, let's assume the followings :

The beam is long and slender, such that the shear deformation and the rotary inertia effect can be neglected. Furthermore, the beam is excited by the axial harmonic forces at the upper end. Then, referring to Fig. 1, the equation of motion of the beam with varying tension due to gravity force is given by

$$EI\frac{\partial^4 \omega}{\partial x^4} + \frac{\partial}{\partial x}[P(x, t)\frac{\partial \omega}{\partial x}] + m\frac{\partial^2 \omega}{\partial t^2} + c\frac{\partial \omega}{\partial t} = 0, \qquad (1)$$

Where E is Young's modulus, I is the moment of inertia of the cross-section, m is the mass of beam per unit length and c is the damping coefficient. The space and time-varying tension $P(x, t)$ can be divided by two parts, i. e., static and dynamic, $P(x, t) = P_0(x) + P_1\cos\Omega t$.

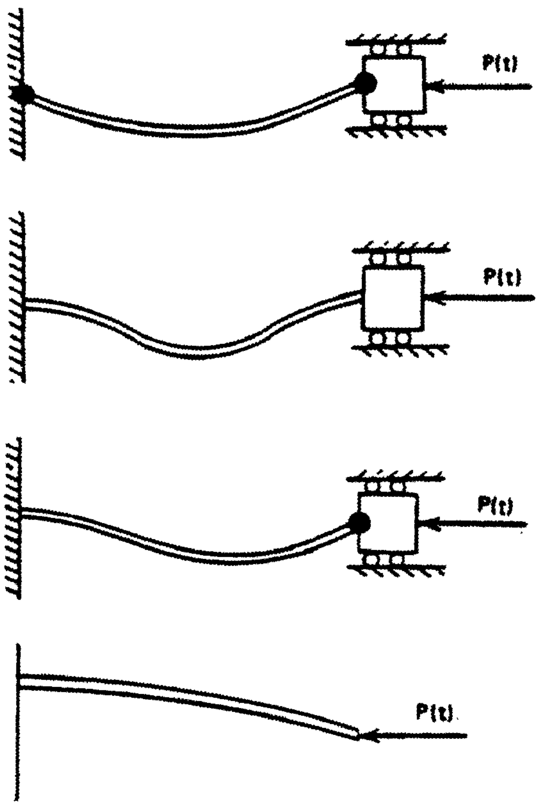


Fig. 1 Beams subjected to the periodic axial force

The typical boundary conditions can be described as follows :

$\omega(0) = 0,\ \omega(L) = 0,$

$\omega''(0) = 0,\ \omega''(L) = 0$ for simply supported ends, (2a)

$\omega(0) = 0,\ \omega''(L) = 0,$

$\omega'(0) = 0,\ \omega'''(L) = 0$ for clamped-free ends, (2b)

$\omega'(0) = 0,\ \omega(L) = 0,$

$\omega(0) = 0,\ \omega''(L) = 0$ for clamped-simply supported ends, (2c)

and

$\omega(0) = 0,\ \omega(L) = 0,$

$\omega'(0) = 0,\ \omega'(L) = 0$ for clamped-clamped ends, (2b)

With introduction of the following dimensionless parameters

$$x^* = x/L,\ \omega^* = \omega/L,\ t^* = t\sqrt{EI/mL^4},$$
$$\mu = cL^2/\sqrt{mEI},$$
$$\beta = P_{cr}L^2/EI,$$
$$\sigma = P_0/P_{cr}$$
$$\xi = P_1/P_{cr}$$
$$\Omega^* = \Omega L^2\sqrt{m/EI} \qquad (3)$$

Eq. (1) becomes

$$\frac{\partial^4 \omega^*}{\partial x^{*4}} + \frac{\partial^2 \omega^*}{\partial t^{*2}} + \mu\frac{\partial \omega^*}{\partial t^*} + \beta\frac{\partial}{\partial x^*}[\{\sigma(x^*) + \xi\cos\Omega^* t^*\}\frac{\partial \omega^*}{\partial x^*}] = 0, \qquad (4)$$

For convenience we shall drop the asterisk in what follows.

Since we can not have a closed form solution of Eq.(4), we express the approximate solution as an expansion of the free-oscillation modes.

$$\omega(x, t) = \sum_{n=1}^{\infty} u_n(t)\phi_n(x) \qquad (5)$$

The mode shape functions $\phi_n(x)$ must satisfy the following equation.

$$\phi_n^{IV}(x) + \beta[\sigma(x)\phi_n'(x)]' - \omega_n^2\phi_n(x) = 0, \qquad (6)$$

With corresponding boundary conditions. In the case of spacially varying tension of the beam, the eigen functions $\phi_n(x)$ were found in closed form by KIM[13] by using WKB method and reported in [14]. In the case of linearly varying tension, the difference between the eigen values using the mean tension value ($T_{mean}=[T_{min}+T_{max}]/2$) and those using the variable tension is very samll. However, the mode shape has some difference. when we use the variable tension, the maximum amplitude in mode shape lies in the lower half of the beam.

Substituting Eq. (5) into Eq.(4), and multiplying $\phi_m(x)$ both sides, and integrating from $x=0$, to $x=1$, we can obtain the following equations.

$$\ddot{u}_m+\mu\dot{u}_m+\omega_m^2 u_m+\beta\xi\cos\Omega t\sum_{n=1}^{\infty}f_{mn}\,u_n=0 \qquad (7)$$

$$m=1,2,\cdots$$

where

$$f_{mn}=\frac{\int_0^1\phi_m\phi_n''dx}{\int_0^1\phi_m^2dx} \qquad (7a)$$

The equation (7) are an infinite set of coupled linear equtions having periodic coefficients. Thus, this is a problem of a parametrically excited system having infinite degrees of freedom.

3. Solution Methods

There are a number of techniques available for the determination of the stability boundaries. These techiques can be divided broadly into three classes :

First one is the Hill's Method of Infinite Determinants which had used extensively for single degree-of freedom system. But it has proved to be cumbersome and inefficient when applied to multidegree-of-freedom system. The second one is perturbation methods based on the assumption that the term containing periodic coefficients is small compared to other terms. This assumption makes the limit of the use of the perturbation method. The third method is the numerical method which is the most general one, but it requires enormous computational efforts for obtaining the transition matrix of large systems. Owing to the development of the high speed computer, this deficiency of the numerical method can be easily overcome.

In this paper, we will use both perturbation method and the numerical method to obtain the stability characteristics and to make comparisions between the two methods.

3.1 Perturbation Method

To analyze the dynamic response and the stability of parametrically excited systems by perturbation methods, we assume that the parametric excitation can be expressed in terms of small parameter ε. To accomplish this, we have to specify an order of parameters of Eq.(7) so as to be small in some sense. Letting

$$\xi=2\varepsilon\hat{\xi},\ \mu=2\varepsilon\hat{\mu} \qquad (8)$$

Eq.(7) becomes

$$\ddot{u}_n+2\varepsilon\hat{\mu}\dot{u}_n+\omega_n^2u_n+2\varepsilon\beta\hat{\xi}\cos\Omega t\sum_{m=1}^{\infty}f_{nm}\,u_m=0$$

$$n=1,2, \qquad (9)$$

The straight forward asymptotic expansion will produce the secular terms. Therefore, in order to eliminate the secular terms the method of multiple scales will be used. To get the uniformly valid expansion, let's assume that the solution is of the following form :

$$u_n(t;\varepsilon)=u_{n0}(T_0,\ T_1,\ T_2)+\varepsilon^1u_{n1}(T_0,\ T_1,\ T_2)+ \qquad (10)$$

where $T_n=\varepsilon^n t$. And the time derivatives become

$$d/dt = D_0 + \varepsilon^1 D_1 + \varepsilon^2 D_2 + \cdots,$$

$$d^2/dt^2 = D_0 + 2\varepsilon^1 D_0 D_1 + \varepsilon^2(2D_0D_2 + D_1^2) + \quad (11)$$

where $D_n = \partial/\partial T_n$. Substituting Eq.(10) and (11) into Eq.(9) and equating coefficients of like powers of ε yields

$$D_0^2 u_{n0} + \omega_n^2 u_{n0} = 0, \quad (12)$$

$$D_0^2 u_{n1} + \omega_n^2 u_{n1} = -2D_0D_1u_{n0} - 2\hat{\mu}D_0u_{n0} - \beta\hat{\xi}\sum_r f_{nr}u_{r0}[\exp(i\Omega T_0) + \text{c. c.}], \quad (13)$$

and

$$D_0^2 u_{n2} + \omega_n^2 u_{n2} = -2D_0D_2u_{n0} - D_1^2u_{n0} - 2D_0D_1u_{n1} - \beta\hat{\xi}\sum_r f_{nr}u_{r1}[\exp(i\Omega T_1) + \text{c. c.}], \quad (14)$$

where c. c. represents the compex conjugate.

The general solution of Eq. (12) can be expressed in the complex form

$$u_{n0} = A_n(T_1, T_2)\exp(i\omega_n T_0) + \text{c. c.} \quad (15)$$

Then, Eq. (13) become

$$D_0^2u_{n1} + \omega_n^2u_{n1} = -2i\omega_n(D_1A_n + \mu A_n)\exp(i\omega_nT_0) - \beta\hat{\xi}\sum_r f_{nr}A_r\{\exp[i(\omega_r+\Omega)T_0] + \exp[i(\omega_r-\Omega)T_0]\} + \text{c. c.}. \quad (16)$$

Form the above equation, A_n is to be chosen in such a way as to eliminate the secular terms.

(Case Ⅰ) Ω is away from $\omega_p \pm \omega_q$

The secular terms will be eliminated from u_{n1} if

$$D_1A_n + \hat{\mu}A_n = 0, \text{ or}$$

$$A_n = a_n\exp(-\hat{\mu}T_1), \text{ for all } n. \quad (17)$$

Therefore, a particular solution of Eq. (10) can be obtained as follew :

$$u_{n1} = \beta\hat{\xi}\sum_r f_{nr}A_r\left\{\frac{\exp[i(\omega_r+\Omega)T_0]}{(\omega_r+\Omega)^2-\omega_n^2} + \frac{\exp[i(\omega_r-\Omega)T_0]}{(\omega_r-\Omega)^2-\omega_n^2}\right\}$$

(Case Ⅱ) Ω is near $\omega_p + \omega_q$

By introducing the detuning paraameter σ,

$$\Omega = \omega_p + \omega_q + \varepsilon\sigma, \quad (18)$$

we can express $(\Omega-\omega_q)\ T_0$ as $\omega_pT_0 + \sigma T_1$ and $(\Omega-\omega_p)\ T_0$ as $\omega_qT_0+\sigma T_1$. From Eq. (16) the secular terms are eliminated from u_{p1} and u_{q1} if

$$2i\omega_p(D_1A_p + \hat{\mu}A_p) + \beta\xi f_{pq}\bar{A}_q\exp(i\sigma T_1) = 0 \quad (19)$$

and

$$2i\omega_q(D_1A_q + \hat{\mu}A_p) + \beta\xi f_{qp}\bar{A}_p\exp(i\sigma T_1) = 0 \quad (20)$$

Eq.(19) and (20) admit nontrivial solutions having the form

$$A_p = a_p\exp(-i\lambda T_1) \text{ and } A_q = a_q\exp[i(\bar{\lambda}+\sigma)T_1], \quad (21)$$

where

$$\lambda = -0.5[\sigma + 2i\hat{\mu} \pm (\sigma^2 - \Lambda_{pq})^{1/2}], \quad (22)$$

$$\Lambda_{pq} = \beta\hat{\xi}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} \quad (23)$$

From Eq. (21), to get the bounded value of A_p and A_q, λ must be real. Therefore, the transition curves can be defineded by

$$\Omega = \omega_p + \omega_q \pm 2\mu\left[\frac{(\varepsilon\beta\hat{\xi})^2}{4\mu^2}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} - 1\right]^{1/2} + O(\varepsilon^2) \quad (24)$$

such that

$$\frac{(\varepsilon\beta\hat{\xi})^2}{4\mu^2}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} - 1 \geq 0.$$

In this paper we shall construct stability regions in the $\Omega/2\omega_1$ vs ξ plane with increasing viscouse damping μ, so that we express Eq. (24) having the form :

$$\frac{\Omega}{2\omega_1} = \frac{\omega_p+\omega_q}{2\omega_1} \pm \frac{\mu}{4\omega_1}\left[\xi^2\frac{\beta^2}{4\mu^2}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} - 1\right]^{1/2} + O(\varepsilon^2) \quad (25)$$

(Case Ⅲ) Ω is near $\omega_p - \omega_q$

In this case the results can be obtained from those above by simply changing the sign of ω_q. For this case unstable solutions occur only when f_{pq} and f_{qp} have different signs. The transition curves can be obtained of the form :

$$\frac{\Omega}{2\omega_1} = \frac{\omega_p-\omega_q}{2\omega_1} \pm \frac{\mu}{4\omega_1}\left[-\xi^2\frac{\beta^2}{4\mu^2}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} - 1\right]^{1/2} + O(\varepsilon^2) \quad (26)$$

such that

$$-\frac{(\varepsilon\beta\hat{\xi})^2}{4\mu^2}\frac{f_{pq}f_{qp}}{\omega_p\omega_q} - 1 \geq 0.$$

3.2 Numerical Method

The numerical method to construct the stability chart is based on the Floquet-Liapunov theorem for a system of linear differential equations with periodic coefficients. In order to describe this method briefly, let's consider a dynamic system whose equation of motion is described as the form :

$$\ddot{u}_i + \sum_{j=1}^{k} f_{ij}(t) \ u_i = 0, \ i = 1, \ 2, \ \cdots, \ k \qquad (27)$$

where $f_{ij}(t+T) = f_{ij}(t)$. It is convenient to express Eq. (27) as a set of $2k$ first order differential equations by defining state vectors :

$$x_n = u_n, \ n = 1, \ 2, \ \cdots\cdots, \ k$$

$$\dot{x}_n = u_n, \ n = k+1, \ k+2, \ \cdots\cdots, \ 2k \qquad (28)$$

Hence Eq. (27) can be written in the compact form :

$$\{\dot{x}(t)\} = [A(t)]\{x(t)\} \qquad (29)$$

where $\{x\}$ is a $(n\times 1)$ vector and $[A(t)]$ is a $(n \times n)$ matrix continuous in time, and periodic with period T but generally nonsymmetrical. Then, it is well known fact that the fundamental matrix $[\phi(t)]$ have the following relation :

$$[\phi(t+T)] = [\phi(t)][C], \qquad (30)$$

where the matrix $[C]$ is sometimes referred to as the monodromy matrix of the fundamental matrix $[\phi(t)]$.

Substituting the equation, $t = 0$, into Eq. (30) yields

$$[\phi(T)] = [\phi(0)][C]. \qquad (31)$$

(1) Evaluation of the Transition Matrix

Letting $[\phi(0)] = [I]$, $[C] = [\phi(T)]$. Then $[C]$ is constructed by numerically solving n IVP of Eq. (31) with the initial condition $x_{ij}(0) = \delta_{ij}$. Above method involves n numerical integrations, so it requires a considerable amount of computation. To overcome this sort of defficiency, the efficient numerical scheme which consists of single integration pass was proposed by Friedmann, *et. al.* [12].

This numerical itegration scheme of evaluating $[\phi(T)]$ is based on the fourth order Runge–Kutta numerical scheme with Gill's coefficients. The calculation procedure of $[\phi(T)]$ can be summarized as follows :

$$[\phi(T)] = (\prod_{n=1}^{N}[K(T-nh)] \qquad (32)$$

where h is the stepsize calculated by T/N and

$$[K(t_i)] = [I] + \frac{h}{6}\{[A(t_i)] + 2(1-1/\sqrt{2})[E(t_i)] + 2(1+1/\sqrt{2})[F(t_i)] + [G(t_i)]\} \qquad (33)$$

$$[E(t_i)] = [A(t_i+0.5h)]([I]+0.5h[A(t_i)]) \qquad (34)$$

$$[F(t_i)] = [A(t_i+0.5h)]([I]+(1/\sqrt{2}-0.5)h[A(t_i)] + (1-1/\sqrt{2})h[E(t_i)]) \qquad (35)$$

$$[G(t_i)] = [A(t_i+h)]([I]+(h/\sqrt{2})[E(t_i)] + (1+1/\sqrt{2})h[F(t_i)]) \qquad (36)$$

(2) Stability Criterion

Inroducing the transformation

$$[\phi(t)] = [\phi(t)][B] \qquad (37)$$

where $[B]$ is a constant nonsingular matrix. Introducing Eq. (37) into (30) and post-multiplying the result by $[B]$, we obtain

$$[\phi(t+T)] = [\phi(t)][B]^{-1}[C][B] \qquad (38)$$

Since the matrix $[B]^{-1}[C][B]$ is similar to the matrix $[C]$, we conclude that the solutions of Eq(29) are asymptotically stable as $t\rightarrow\infty$ if the modulus of the eigenvalues of $[C]$ are all less than unity, bounded for all t if all of them are unity and unbounded as $t\rightarrow\infty$ if any of them are greater than unity. The eigenvalues of $[C]$ is often called the characteristic multipliers.

(3) Modification of the Stability Criterion for the Numerical Computation

Let's examine the behavior of $[\phi(t)]$ as time increases. Theoretically, Floquet–Liapnov theorem says that

If $|\lambda_i| > 1$, then Unbounded solution.

If $|\lambda_i| = 1$, then Neutrally stable solution.

If $|\lambda_i| < 1$, then Asymptotically stable solution.

However, there is some difficulty in numerical computation to find the characteristic multiplier whose modulus is exactly one. This motivates us to modify the stability criterion as follows :

If $|\lambda_i| > 1+\varepsilon^{ms}$, then Unbounded solution.

If $1-\varepsilon^{ms} \leq |\lambda_i| \leq 1+\varepsilon^{ms}$, then Neutrally stable solution.

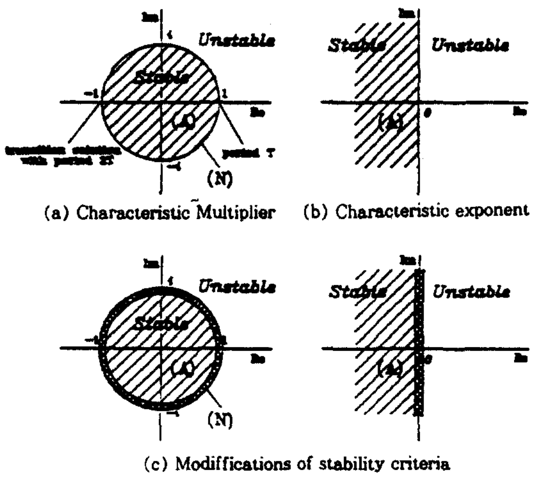
If $|\lambda_i| < 1-\varepsilon^{ms}$, then Asymptotically stable solution.

The suggested way to select the error bound ε^{ms} from the numerical experience can be stated as follows :

First, choose the error bound ε^{ms} in order not to have any asymptotically stable solutions in an undamped system. Then, use this value in the damped system.

4. Results and Discussions

To make comparisions between the perturbation method and the numerical method, the following examples whose results may be available in the literature are selected : The columns with four different boundary conditions, and subjedted to a periodic axial force $P(t) = P_1 \cos \Omega t$.

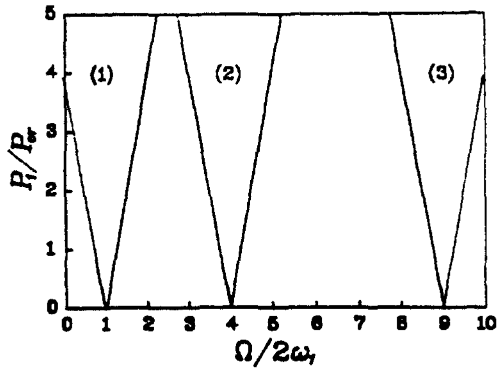


(a) Characteristic Multiplier (b) Characteristic exponent

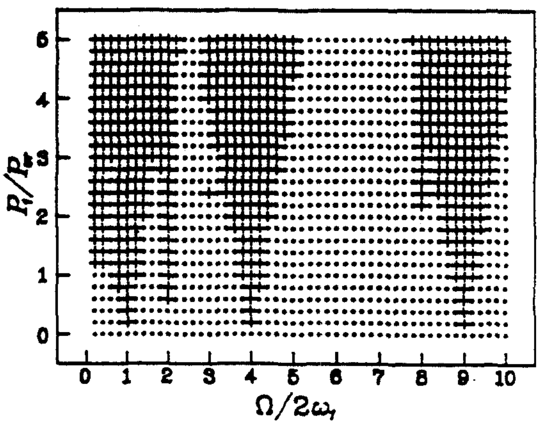
(c) Modiffications of stability criteria

Fig. 2 Relationship between stability and the location of the characteristic multipliers

Fig. 3 shows the instability regions of a simply supported columns. Fig. 3(a) is the result by perturbation method and Fig. 3(b) is the result by the numerical method. In the figures P_1/P_{cr} $(=\xi)$ is taken as the ordinate and $\Omega/2\omega_1$ is taken as the abscissa. It is noted that the value of P_{cr} is different for different boundary conditions. The solid line in Fig. 3(a) shows the transition curve seperating stability from instability. Inside these lines and the marked regions in Fig. 3(b) the response of the beam increases and becomes unbounded with an increase of time. The symbol (p), p=1, 2, 3, 4, implies the parametric resonance of the pth mode. From this figure, we can find that any



(a) Perturbation



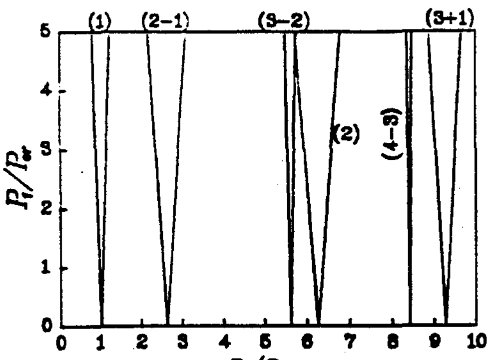
(b) Numerical simulation

Fig. 3 Instability regions of a simply supported column under the parametric excitation combination resonance does not occur and only the parametric resonances $\Omega \simeq 2\omega_p$, $(p=1, 2, \cdots)$ occur. This is due to the fact that the integral of Eq. (7a) is identically zero except for $n = m$, and Eq. (7) become decoupled.

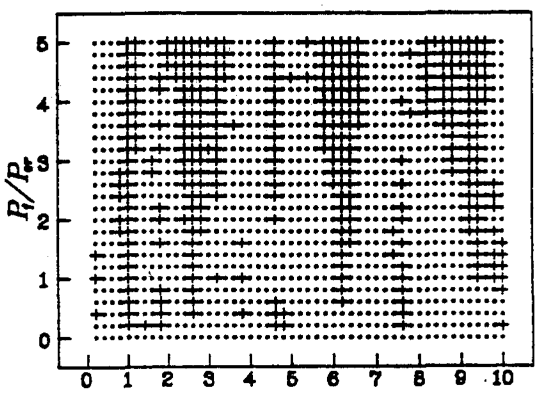
Fig. 4 shows the instability regions of a clamped−free column. While for a simply supported column only parametric resonances occur, in this case not only the parametric resonances but also the combination resonances such that $\Omega \simeq \omega_2-\omega_1$, $\Omega \simeq \omega_3-\omega_2$, $\Omega \simeq \omega_4-\omega_3$ and $\Omega \simeq \omega_3+\omega_1$ occur. It is noted that the regions of the first and fourth regions of combination resonance in the figure are far more dominant than the regions of the parametric resonances.

Fig. 5 shows the instability regions of a clamped−simply supported column. Combination resonances of sum type such as $\Omega \simeq \omega_p+\omega_q (p, q=1, 2, \cdots$ and $p \neq q)$ are all possible but a difference type of combination resonance of such as $\Omega \simeq \omega_p-\omega_q$ is not possible. The dominance of the combination resonances is very weak relative to that of the parametric resonances.

Fig. 6 shows the instability regions of a clamped−clamped column. It is noted that the types of combination resonances are same as

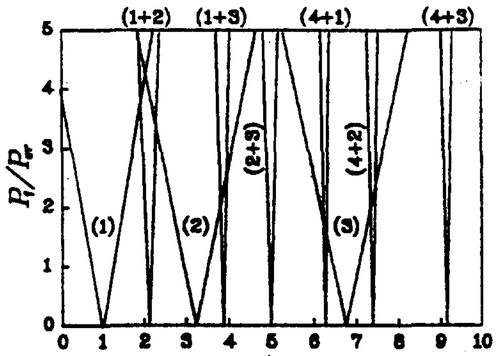


(a) Perturbation

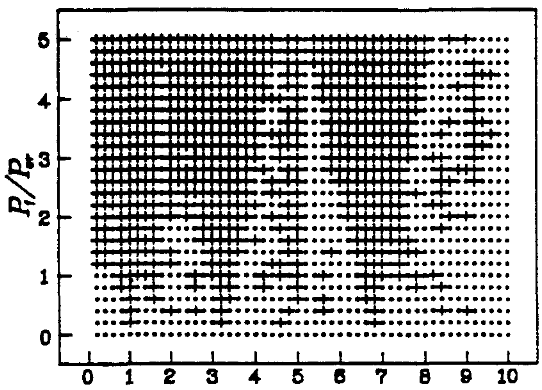


(b) Numerical simulation

Fig. 4. Instability regions of a clamped−free column under the parametric excitation



(a) Perturbation

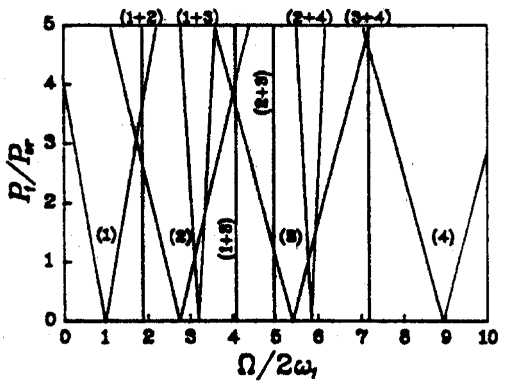


(b) Numerical simulation

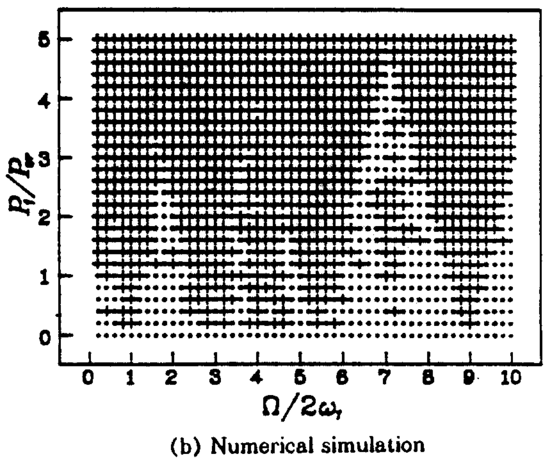
Fig. 5 Instability regions of a clamped simply supported column under the parametric excitation those of the clamped-simply supported column and their dominance is so weak that there is some difficulty to find them in the numerical result.

It is noted that, regadless of boundary conditions, the system may be stable even though the excitation amplitude is five times the Euler static buckling load corresponding to boundary conditions in some excitation frequency. In dynamic stability, if the parametrically exciting frequency is far from resonant condition, this phenomena can be occured.

From the numerical experience, the following state can be made :



(a) Perturbation



(b) Numerical simulation

Fig. 6 Instability regions of a clamped-clamped column under the parametric excitation

The instability regions found by both methods are generally in good agreement, provided that the proper error bound be selected as suggested in this paper. However, there are some differences between the two.

(1) In the case of nearly static excitation, the result of the numerical method shows unstable solution when P_t/P_{cr} is greater than 1, while that of the perturbation does not. In this region, we have to note that the validity of the perturbation method must be checked.

(2) As mentioned ealier, in the results of the perturbation method, we may even find the instability regions which are as weak or narrow as a single line, but in the result of the numerical method there is some difficulty to find the corresponding regions.

But, there is a big advantage to use numerical method even in the case of the large periodic load.

⊙ The effects of viscous damping

Fig. 7(a)-(e), which are obtained by the numerical method, show that the effects of viscous damping on the instability regions of simply suported column. The dimensionless damping coefficient used in this figures are (a) $\mu=1$, (b) $\mu=3$, (c) $\mu=5$, (d) $\mu=7$ and (e) $\mu=9$. Fig.7(f) is the result obtained from Eqs. (25) and (26) by the perturbation method.

According to a literature[1], the damping may destablize the system when modal damping is assumed and following condition is satisfied :

$$A_{pq} < 4\mu_p\mu_q(\mu_q+\mu_p)^2(\mu_q-\mu_p)^{-2}, \quad (39)$$

where μ_p is p^{th} modal damping. However, in the case of constant viscous damping, it has the effects of stablizing the system. Also it has a trend to lift up and reduce the instaility regions.

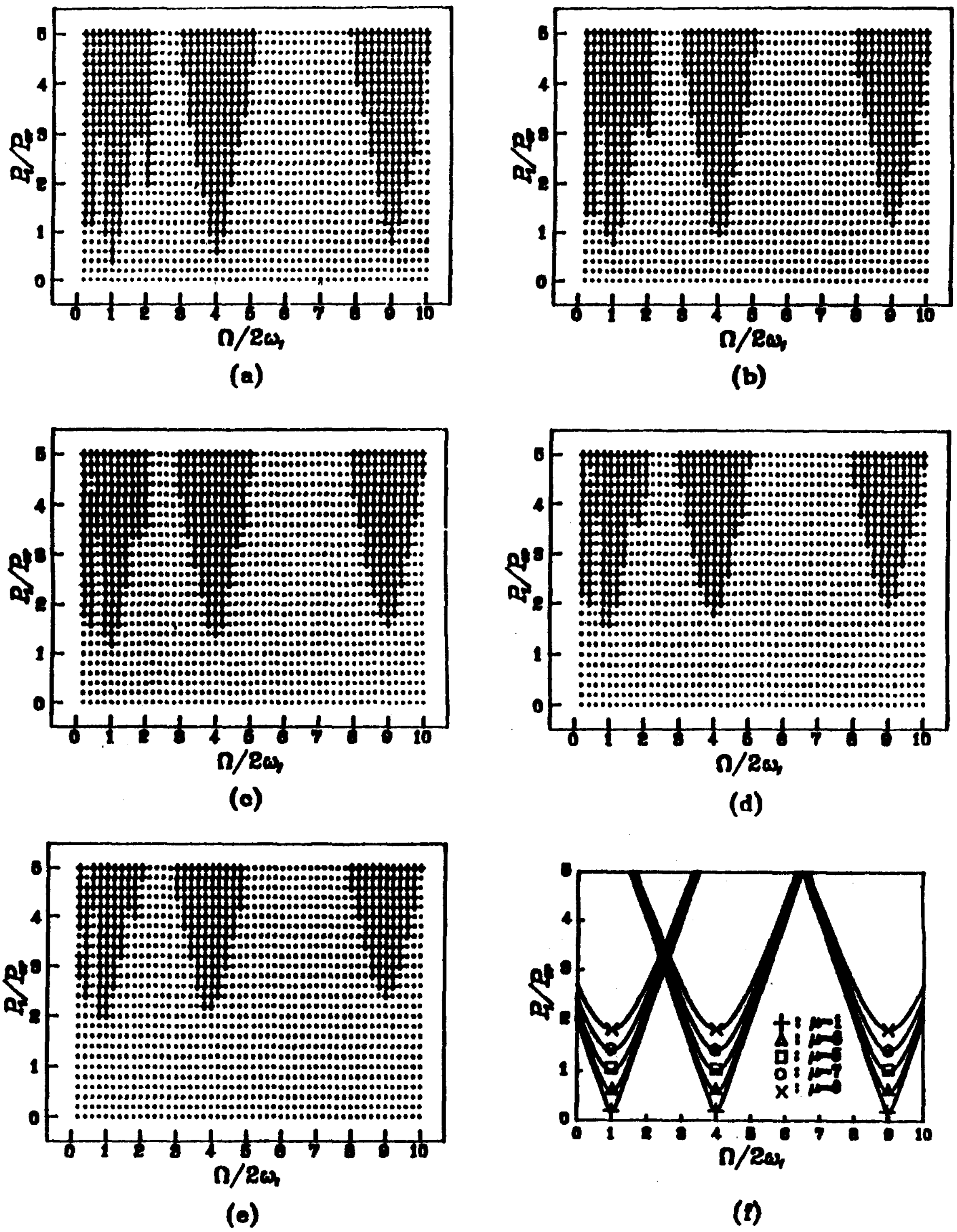


Fig. 7 Effects of viscous damping on the instability regions of a simply supported column
(a) $\mu=1.0$ (b) $\mu=3.0$ (c) $\mu=5.0$ (d) $\mu=7.0$ (e) $\mu=9.0$ (f) by perturbation method

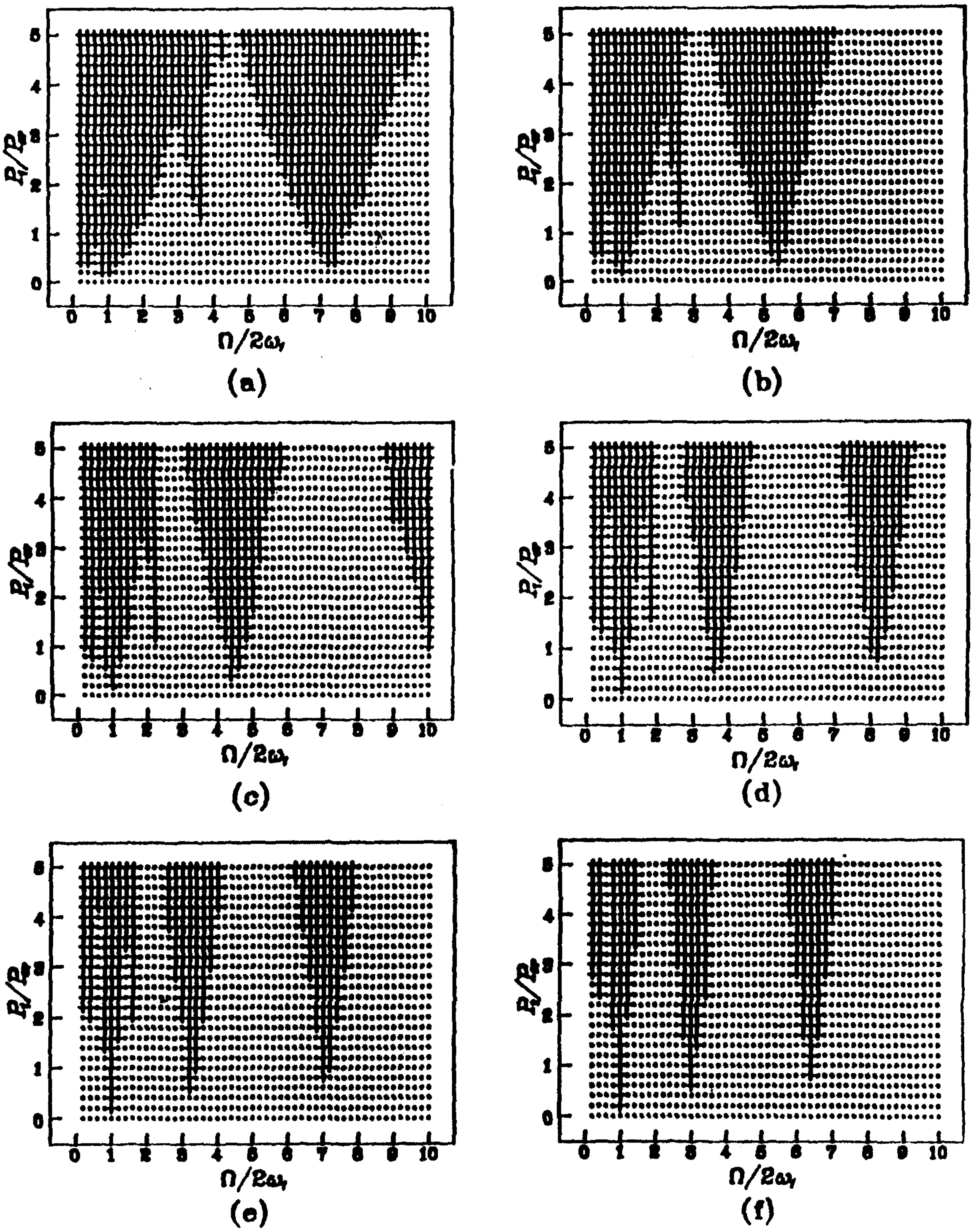


Fig. 8 Effects of mean tension on the instability regions of a simply supported column—by numerical method
(a) $\sigma=0.75$ (b) $\sigma=0.5$ (c) $\sigma=0.25$ (d) $\sigma=-0.25$ (e) $\sigma=-0.75$ (f) $\sigma=-1.25$

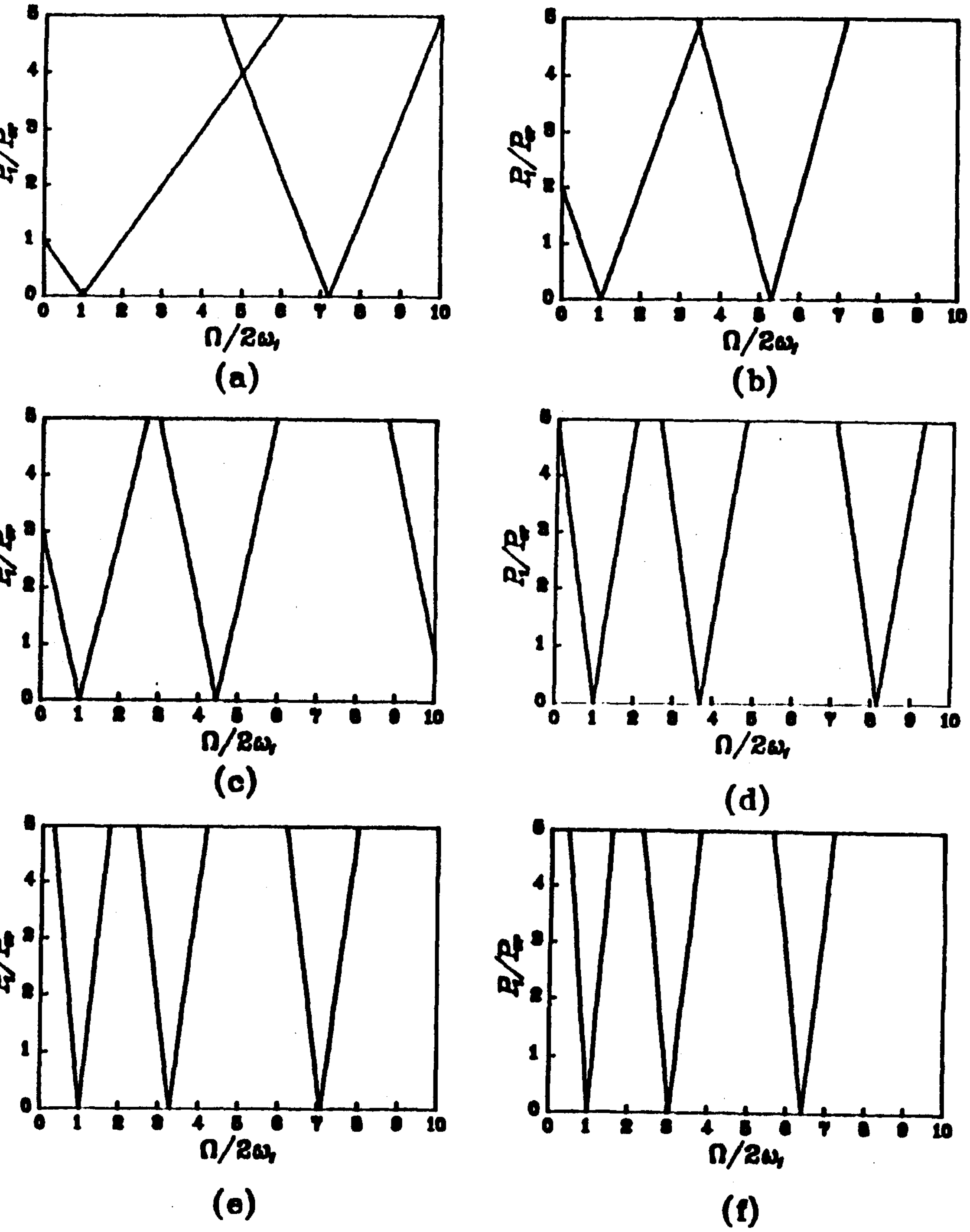


Fig. 9 Effects of mean tension on the instability regions of a simply supported column—by perturbation method

(a) $\sigma=0.75$ (b) $\sigma=0.5$ (c) $\sigma=0.25$ (d) $\sigma=-0.25$ (e) $\sigma=-0.75$ (f) $\sigma=-1.25$

⊙ The effects of mean tension

Fig. 8 and 9 show the variations of instability regions with changing direction and incresing magnitude of mean tension, which are obtained by numerically and analytically. The mean tension parameters, σ ($=P_0/P_{cr}$) used here are (a) $\sigma=0.25$, (b) $\sigma=0.5$ (c) $\sigma=0.75$, (d) $\sigma=-0.25$, (e) $\sigma=-0.75$ and (f) $\sigma=-1.25$, where the positive sign represents that the mean force is compressive force and the negative one represents that the mean tension is tensile force.

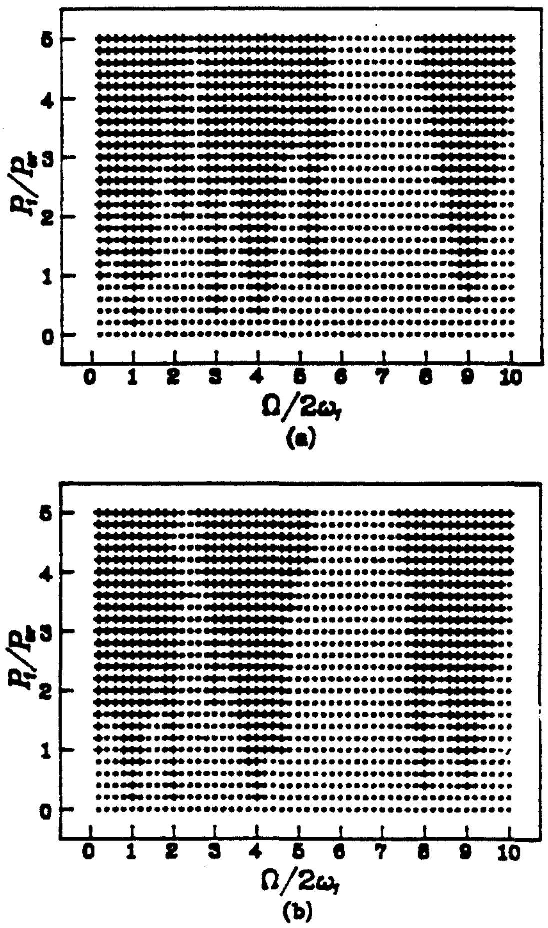


Fig. 10 Effects of multi－frequency input on the instability regions of a simply supported column,
(a) $P(t) = P_1(\cos \Omega t + \cos \Omega t)$
(b) $P(t) = P_1(\cos \Omega t + \cos 2\Omega t)$

We can see that the position eminating instability regions are varying with changing direction and magnitude of mean tension. While the system becomes more stable when the tension is applied, the system becomes more unstable when the compressive force is applied. This is because the natural frequencies increase with the increase of mean tension.

⊙ The effects of multi－frequency excitation

We can find the exciting period T of the multi－frequency excitation, unless one of the excitation frequencies is not a rational number. Then, we can use numerical method to calculate the transition matrix.

Fig. 10 shows the instability regions of a simply supported column subjected to a periodic tangential multi－frequency excitation. Frequencies used in this example $P(t) = P_1(\cos\Omega_1 t + \cos\Omega_2 t)$ are (a) $\Omega_1=\Omega$ and $\Omega_2=3\Omega$ and (b) $\Omega_1=\Omega$ and $\Omega_2=2\Omega$. Fig. 10 (a) shows the unstable regions when the value $\Omega/2\omega_1$, is equal to 1/3, 1, 4/3, 3, 4, 16/3, 9 etc. Also Fig. 10 (b) shows the unstable regions when the value $\Omega/2\omega_1$ is equal to 1/2, 2, 4, 9/2, 8, 9 etc. In this example, there is no combination resonance and only the parametric resonance occurs. There-fore, each parametric excitation affects the stability of the system separately.

5. Conclusions and Recommendations

From the analytical and numerical results, the following conclusions can be made :

(1) The accuate stability boundary can be obtained by using the perturbation method when the parametric excitation is relatively small. However, the numerical method can be effectively used to obtain the stability chart even in the case of the large parametric excitation and the multi－frequency excitation,

provided that the proper error bound be selected as suggested in the present paper.

(2) The constant viscous damping has the effect of stabilizing the system, such that it has a trend to lift up and reduce the unstable regions.

(3) The mean tension has the effect to stabilizing the system, while the mean compressive force does not. Due to the change of the natural frequencies of the system, the unstable regions shift and change their area according to the magnitude of the mean tension.

(4) The results of the present work can be directly applicable to the dynamic stability analysis of the supporting leg of Tension Leg platforms.

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References

(1) Nayfeh, A. H. and Mook, D. T., "Nonliear Oscillations", New York : John Wiley & Sons, 1979

(2) Bolotin, V. V., "The Dynamic Stability of Elastic Systems", Holden Day, Inc, 1964

(3) Bolotin, V. V., "Nonconservative Problems of the Theory of Elastic Stability". Pergamon Press LTD, 1963

(4) Iwatsubo, T., Sugiyama, Y. and Ishihara, K., "Stability and Non-stationary Vibration of Columns under Periodic Load", *J. of Sound and Vibration*, vol. 23, pp. 245-257, 1972.

(5) Iwatsubo, T. and Saigo, M., "Parametric Instability of Clamped-Clamped and Clamped-Simply Supported Columns under Periodic Axial Load", *J. of Sound and Vibration*, vol. 30(1), pp. 65-77, 1973

(6) Iwatsbo, T., Sugiyama, Y. and Ogino, S., "Simple and Combination Resonances of Columns under Periodic Axial Loads", *J. of Sound and Vibration*, vol. 33, pp. 211, 1974.

(7) Nayfeh, A. H. and Mook, D. T., "Parametric Excitations of Linear System Having Many Degress of Freedom", *J. Acoust. Soc. Am.*, vol. 62, No. 2., pp. 375-381, *August*, 1977

(8) Hsu, C. S., "On the parametric excitation of a dynamic system having multiple degree of freedom", *J. appl. Mech.* vol. 30, pp. 368, 1963

(9) Hsu, C. S., "Impulsive Parametric Excitation : Theory", *Journal of Applied Mechanics, Trans. of ASME.*, pp. 551-558, *June*, 1972

(10) Hsu, C. S. and Cheng, W. H., "Applications of the Theory of Impulsive Parametric Excitation and New Treatments of General Parametric Excitation Problems", *Journal of Applied Mechanics, Trans. of ASME.*, pp. 78-86, *March*, 1973

(11) Hsu, C. S., "On approximating a general linear periodic system", *J. Math. Analysis and Applications*, vol. 45, pp. 234-251, 1974

(12) Friedmann, P., Hammond, C. E. and Woo, T. H., "Efficient Numerical of Periodic Systems with Application to stability Problems", *Int. J. for Num. in Eng.*, vol. 11, pp. 1117-1136, 1977

(13) Kim, Y. C., "Nonlinear Vibrations of Long Slender Beams", ph. D. Thesis, Massachusetts Institute of Technology, MA, May, 1983

(14) Kim, Y. C., "Natural Frequencies and Critical Bucking Loads of Marine Risers", *J. of Offshore Mech. and Arctic Eng., ASME*, vol. 110, pp. 2-8, Feb., 1988