

# 치환방법을 이용한 주파수 도약 확산 시스템의 주파수 도약 패턴 발생 방법

正會員 任 春 植\* 正會員 河野 隆二\* 正會員 今井 秀樹\*

## Methods of Generating Hopping Patterns Based on Permutation Frequency Hopping SSMA System

Choon Sik YIM\* Ryuji KOHNO\* Hideki IMAI\* *Regular Members*

**要 約** 본 논문은 비주기 주파수 도약 확산 시스템(FH/SSMA)에 있어서 주파수 도약 패턴의 요소간, 치환을 이용한 새로운 주파수 도약 패턴 발생 방법을 제안하고, 종래 방법과 비교 검토한다. 제 1방법은 Latin Square에 의한 발생법을 이용하였다. 제 2방법은 유한체의 요소를 이용한 치환 방법을 설계하였다. 제 3방법에서는 유한체의 기준 집합 요소의 치환 base를 이용하여 발생법을 설계하였다. 평가를 위하여 주기/비주기성의 시간-주파수 변환에 의한 도약 패턴간의 히트에 대해서 검토하였다. 검토 결과, 종래의 RS계열을 이용한 패턴 보다 비주기성의 시간-주파수에 의한 히트면에서 제 2발생 방법이 좋은 결과를 얻었다.

**ABSTRACT** This paper proposes the generation of several classes of frequency hopping patterns, which are derived by permutation, for an asynchronous frequency hopping spread-spectrum multiple access system (FH/SSMA). The first class of hopping patterns is obtained by using a Latin square. The second class of hopping patterns is derived by generalizing the first class which is designed by using a permutation technique. The third class of hopping patterns is designed by using a rotational base of elements. We evaluate the hit property of the proposed classes of hopping patterns when these patterns are mutually shifted in an FH/SSMA system. Compared to the Reed-Solomon sequences generated by the conventional method, the sequence obtained by the permutation technique can reduce the number of hits among hopping frequencies in asynchronous time/frequency shift.

### I. Introduction

In code-division multiple access(CDMA) based on a spread spectrum techniques, multiple access capability is due primarily to spreading codes, i.e. hopping pattern quite different from the traditional time and frequency division multiple access methods. Two of the most common forms of SSMA are

frequency hopped(FH) SSMA and direct-sequence(DS) SSMA. In FH/SSMA, there is no requirement for time or frequency coordination between the transmitters. A unique frequency hopping pattern is assigned to each user, so that the RF signal form given to the transmitter is hopped from slot to slot by changing the carrier frequency according to its own hopping pattern. Errors of decoded data may occur whenever the RF signals of different users simultaneously occupy the same frequency slot. In such a case, we say a

\*横浜國立大學 電子情報工學科  
Dept. of Elec. Eng., Faculty of Engineering, Yokohama  
National Univ.  
論文番號: 91-128(接受1991. 8. 14.)

hit occurs. Variety type of FH/SSMA techniques for reducing the number of hits have been considered. In particular, it is the most important problem in designing sets of hopping patterns that can minimize the number of hits and improve multiple access capability. The technique of designs of hopping patterns with a fixed number of hits using a polynomial equation has been discussed by Mersereau and Seay<sup>(1)</sup>. The technique of designs using permutation vectors have been considered by Copper and Nettleton<sup>(2)</sup>. Furthermore, the problem of designing patterns using rook placement has been discussed by Golomb and Posner<sup>(3)</sup>, and so on<sup>(4)(5)</sup>. This paper extends the previous techniques to the presence of simultaneous time and frequency uncertainties to decrease number of hits among users in an FH/SSMA system. We propose methods to generate three classes of hopping patterns based on permutation. In the following section, we describe the model of an FH/SSMA system. In section III, we define the hit for time/cyclic shifts in the sequence of hopping patterns. In section IV, we propose methods to generate new classes

of hopping patterns which are : the first method using a Latin square, the second method using permutataion techniques, and the third method using rotational base of element. Finally, we evaluate the correlation characteristics for these hopping patterns.

## II. System model

This paper is concerned with the application of FH/SSMA to a system in which  $Q$  transmitter receiver pairs wish to communicate over a common channel. A source generates a message, independent of messages generated by other users. Fig.1 shows a block diagram of an FH/SSMA system with a frequency synthesizer, where users are transmitting individual signals  $S_q(t)$ ,  $q=1, 2, \dots, Q$  continuously in time over the same frequency band. Each transmitter possesses M-ary FSK(MFSK) modulator followed by a mixer. Incoming binary data are encoded by the encoder to possess error correcting capability and the output codewords are then mapped into signal points by the MFSK modulator.

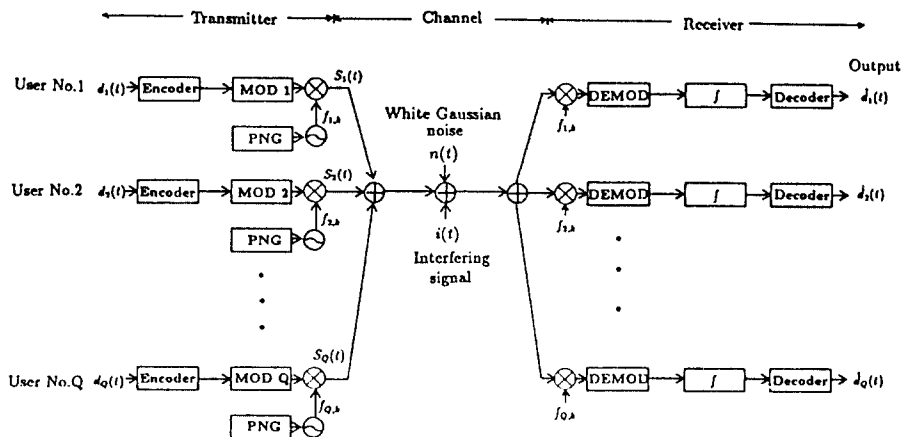


Fig.1 System block diagram for FH/SSMA.

Next, the carrier frequency of the resulting modulated signal is periodically switched or hopped by a frequency synthesizer. This second modulator chopping the carrier frequency from one frequency to another according to a pre-determined hopping pattern is called a frequency hopper.

Fig.2 shows the transition of modulated signals in MFSK(M=4) and that of hopped carrier frequency. At the frequency de-hopper, which performs the inverse operation of the frequency hopper, received signals are de-hopped to recover the MFSK signal. Then, the MFSK signal is demodulated to recover binary data. Moreover, the data is decoded to detect or correct errors at the decoder. In a primitive FH/SSMA system, both the encoder and a decoder for error correction can be omitted.

### III. Hit in Time and Cyclic Shifts of Hopping Patterns

The frequency-hopped signals are generally sinusoids produced by switching the carrier frequency among a finite set of frequencies according to each transmitter's own periodical hopping pattern. A hit may occur in an FH/SSMA system when a signal is hopped to a frequency slot that is occupied by another user at the same time causing an error in demodulation. Therefore, a decoded error due to the hit is a major problem in FH/SSMA. When a hopping pattern is compared symbol by symbol with the other patterns used in an FH/SSMA system, the number of coincident symbols or hits between any two patterns should be as small as possible. When users of an FH/SSMA system hop their carrier frequencies asynchronously with each other, the number of hits should be minimized not only among the original patterns but also among their shifted patterns. We define two types of shifts of hopping patterns : time shift and cyclic shift.

Let a set of finite hopping frequency vectors  $F_i$  ( $i=0, 1, \dots, Q-1$ ) for FH/SSMA be

$$F = \{F_0, F_1, F_2, \dots, F_{Q-1}\} \tag{1}$$

The  $i$ -th hopping frequency vector  $F_i$  which the  $i$ -th user employs can thus be represented as a p-tuple

$$F_i = (f_{x_{i,0}}, f_{x_{i,1}}, \dots, f_{x_{i,p-1}}) \tag{2}$$

where  $f_{x_{i,n}}$  is the frequency assigned to the  $(n \bmod p)$ -th time chip of the  $i$ -th user. If a set practical hopping frequencies  $f_0, f_1, \dots, f_{p-1}$

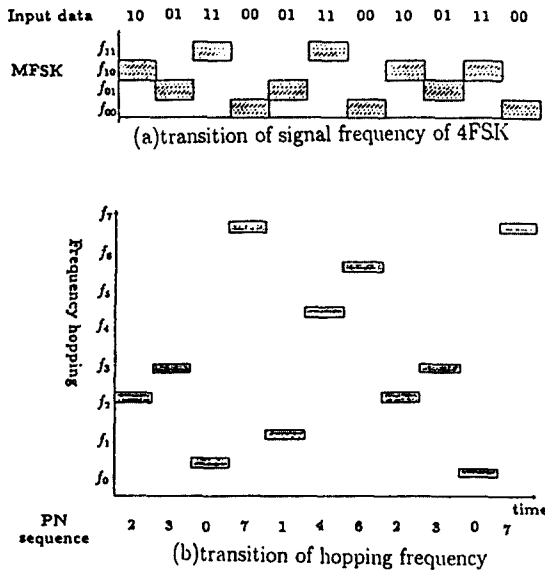


Fig.2 Data flow of transmitted signal for an M-ary FH/SSMA system.

is given,  $f_{i,n}$  must be a hopping frequency of that set, i.e.  $X_{i,n} \in \{0, 1, 2, \dots, p-1\}$ . Then we define the hopping pattern of the  $i$ -th user by

$$X_i = (X_{i,0}, X_{i,1}, \dots, X_{i,p-1}) \quad (3)$$

Let us consider coincidence or hit of the shifted hopping patterns of the  $i$ -th and the  $j$ -th users,

$$\begin{aligned} X_i &= (X_{i,0}, X_{i,1}, \dots, X_{i,p-1}) \\ X_j &= (X_{j,0}, X_{j,1}, \dots, X_{j,p-1}) \end{aligned} \quad (4)$$

If an element of  $X_i$  is coincident with an element of  $X_j$  cyclically shifted by  $l$  ( $l=0, 1, \dots, p-1$ ) such as,

$$X_{i,n} = X_{j,(n+l) \bmod p}, \quad (5)$$

then we call this coincidence "a hit in cyclic shift of hopping pattern". If an element of  $X_i$  is coincident with an element of the  $X_j$  non-cyclically shifted by  $l$  ( $l=0, 1, 2, \dots, p-1$ ) such as

$$X_{i,n} = X_{j,(n+l)} \quad \text{for } 0 \leq n \leq p-1-l, \quad (6)$$

we call this coincidence "a hit in time shift of hopping patterns". The hit in time and cyclic shifts represent cross correlation among the hopping pattern.

#### IV. Three Classes of Hopping Patterns

##### 1. Hopping Pattern Based on a Latin Square

Latin square<sup>(1)</sup> of size  $N$  is an  $N \times N$  matrix based on some set of  $N$  symbols, with the property that every row and column contains each symbol exactly

Table 1: Example of Latin square.

|       | n column |   |   |   |   |   |
|-------|----------|---|---|---|---|---|
| m row | 2        | 4 | 3 | 0 | 5 | 6 |
|       | 4        | 6 | 5 | 2 | 0 | 1 |
|       | 3        | 5 | 4 | 1 | 6 | 0 |
|       | 0        | 2 | 1 | 5 | 3 | 4 |
|       | 5        | 0 | 6 | 3 | 1 | 2 |
|       | 6        | 1 | 0 | 4 | 2 | 3 |

once. If a cell of a Latin square satisfies the following condition

$$Cell(m, n) = Cell(n, m), \quad (7)$$

where  $0 \leq m, n \leq N-1$ , we call it an orthogonal Latin square.

##### 1.1 Method to Generate the Hopping Pattern

A method to generate hopping patterns combining a Latin square and the conventional method used to generate hopping patterns given by Copper and Nettleton<sup>(2)</sup> is given as follows.

- (a) Select a prime number  $q$  which is denoted by GF( $q$ ) on finite field set. Then  $p = q - 1$
- (b) Take the smallest primitive root  $\alpha$  of  $q$ . Then, generate the permutation vector  $\Pi_q$  which is derived by  $\alpha^i$  as shown as

$$\Pi_q = \{\alpha^0, (\alpha^1), (\alpha^2), \dots, (\alpha^{q-1})\}, \quad (8)$$

where the operation  $(\alpha^i)$  denotes the modulo  $q$  power of  $\alpha$ ; i.e.,

$$(\alpha^i) = \alpha - \lfloor \alpha^i / q \rfloor q \quad (9)$$

where  $\lfloor y \rfloor$  denotes the largest integer  $\leq y$ .

- (c) Generate the  $i$ -th hopping pattern  $X_i$  defined by (3) as shown as

$$X_i = \Pi_q + \Pi_q(i), \quad (10)$$

Where the elements of this hopping pattern  $X_i$  are equal to the sum of respective element of  $\Pi_q$  and  $\Pi_q(i)$ .  $\Pi_q(i)$  denotes the  $(i+1)$ -th element of permutation vector  $\Pi_q$ . Therefore, we can rewrite (10) as

$$X_i = \{1 + \alpha^i, \alpha^1 + \alpha^i, \alpha^2 + \alpha^i, \dots, \alpha^{p-1} + \alpha^i\}, \quad (11)$$

**Example 1** Let us select a prime number  $q=7$ . we obtain the smallest primitive root  $\alpha=3$  of  $q$ . The permutation vector  $\Pi_q$  can be generated as

$$\Pi_q = \{1, 3, 2, 6, 4, 5\}. \quad (12)$$

Using (10), we obtain the following hopping patterns in the form of a Latin square,

$$\begin{aligned} X_1 &= \{2, 4, 3, 0, 5, 6\} \\ X_2 &= \{4, 6, 5, 2, 0, 1\} \\ X_3 &= \{3, 5, 4, 1, 6, 0\} \\ X_4 &= \{0, 2, 1, 5, 3, 4\} \\ X_5 &= \{5, 0, 6, 3, 1, 2\} \\ X_6 &= \{6, 1, 0, 4, 2, 3\} \end{aligned}$$

### 1.2 Correlation Property of the Hopping Patterns

When one hopping pattern in the class defined by (10) is shifted in time by any integer with respect to any other pattern in the class, there will be, at most, one time chip within a period  $p$ .

**Proof** An element of  $X_i$  and a shifted element in time of  $X_j$  are written by

$$\begin{aligned} X_{i,k} &= \Pi_q(k) + \Pi_q(i) \\ X_{j,k+t} &= \Pi_q(k+t) + \Pi_q(j) \end{aligned} \quad (13)$$

If  $X_{i,k}$  equals to  $X_{j,k+t}$ , then

$$(\Pi_q(k) - \Pi_q(k+t)) + (\Pi_q(i) - \Pi_q(j)) = 0 \quad (14)$$

Assume that element  $X_{i,k}$  equals another element shifted in time by a different  $t$  such as

$$X_{j,k+t} = \Pi_q(k+t) + \Pi_q(j). \quad (15)$$

Then, from (14) and (15)

$$\begin{aligned} X_{i,k} - X_{j,k+t} & \\ &= (\Pi_q(k) - \Pi_q(k+t)) + (\Pi_q(i) - \Pi_q(j)) \\ &= \Pi_q(k+t) - \Pi_q(k+t) = 0. \end{aligned} \quad (16)$$

Since every element of  $\Pi_q$  is different by the definition of (8),  $t$  must be equal to  $l$ .

## 2. Hopping Pattern Based on Permutation

### 2.1 Method to Generate the Hopping Pattern

A method to generate hopping patterns based on the permutation method is described as follows.

- (a) Let elements in a finite field with the order  $p$ ,  $GF(p)$  be  $(e_0=0, e_1, e_2, \dots, e_{p-1})$ .
- (b) Generate the  $i$ -th hopping pattern  $X_{i(s)}$ ,

$$X_{i(s)} = \{X_{0,s}^i, X_{1,s}^i, \dots, X_{p-1,s}^i\}, \quad (17)$$

where  $X_{j,s}^i$  denotes  $j$ -th element of  $i$ -th hopping pattern  $X_{i(s)}$  which in turn is defined by

$$X_{j,s}^i = i(e_j + e_s) \bmod p, \quad (18)$$

where  $X_{j,s}^i \in GF(p)$  and  $i, j, s \in GF(p)$ . We may rewrite eq(17) by

$$X_{i(s)} = \{i(e_0 + e_s), i(e_1 + e_s), \dots, i(e_{p-1} + e_s)\}. \quad (19)$$

If we use  $e_t (t \neq s)$  instead of  $e_s$ , a different set of hopping patterns can be obtained.

### 2.2 Correlation Property of the Hopping Patterns

Two hopping patterns  $X_{i(s)}$  and  $X_{j(s)} (i \neq j)$  defined by (19) have hit only one hit within a period  $p$  in a sense of the cyclic shift.

**Proof** If  $i \neq k$ , it will be confirmed that  $(ke_l - ie_s) / (i - k) \notin GF(p)$ . Then, there is an element of  $GF(p)$ ,  $e_j$  such as

$$(ke_l - ie_s) / (i - k) = e_j. \tag{20}$$

Moreover, (20) can be rewritten as

$$i(e_j + e_s) = k(e_l + e_l). \tag{21}$$

This means that the  $j$ -th elements of  $X_{i(s)}$  and  $X_{k(s)}$  are coincident. Assume that the  $l$ -th ( $l \neq j$ ) elements of  $X_{i(s)}$  and  $X_{k(s)}$  are coincident. Then, we can obtain the following equation from (18),

$$i(e_l + e_s) = k(e_l + e_l). \tag{22}$$

From (21) and (22), we can derive the following equation,

$$(i - k)(e_l - e_s) = 0. \tag{23}$$

Therefore,  $l = j$  because  $i \neq k$ .

## 3. Hopping pattern Based on Rotational Base

### 3.1 Method to Generate the Hopping Pattern

A method to generate hopping pattern using rotational base element is described below.

- (a) Select a prime number  $p$ . Let a vector of elements in  $GF(p)$  be  $(e_0, e_1, e_2, \dots, e_{p-1})$  in the same way as described in Section 2.
- (b) The  $j$ th element  $X_{i,j}$  of  $X_i$  is defined by,

$$X_{i,j} = e_{[j(i+j)] \bmod p}, \tag{24}$$

where  $X_{i,j} \in GF(p)$  and  $i, j \in GF(p)$ .

Therefore, we may write the  $i$ -th hopping pattern  $X_i$  as,

$$X_i = \{e_0, e_{i+1}, e_{2(i+2)}, \dots, e_{(p-1)(i+p-1)}\}. \tag{25}$$

**Example 2** Select a prime number  $p=7$ . The vector of elements in  $GF(7)$  is  $(0, 1, 2, 3, 4, 5, 6)$ . We can obtain from (24) and (25) the following hopping patterns,

- $X_0 = \{0, 1, 2, 3, 4, 5, 6\}$
- $X_1 = \{0, 2, 4, 6, 1, 3, 5\}$
- $X_2 = \{0, 3, 6, 2, 5, 1, 4\}$
- $X_3 = \{0, 4, 1, 5, 2, 6, 3\}$
- $X_4 = \{0, 5, 3, 1, 6, 4, 2\}$
- $X_5 = \{0, 6, 5, 4, 3, 2, 1\}$

### 3.2 Correlation Property of the Hopping Patterns

Two hopping patterns  $X_i$  and  $X_j (i \neq j)$  have only one hit within a period  $p$  in the sense of the cyclic shift.

**Proof** If  $i \neq k$ , it will be confirmed that

$$e_{[j(i+j)] \bmod p} - e_{[j(k+j)] \bmod p} \notin GF(p). \tag{26}$$

We assume that  $e_s$  is the  $s$ th element of  $GF(p)$ . We may rewrite (26) as follow

$$e_{[j(i+j)] \bmod p} - e_{[j(k+j)] \bmod p} = e_s. \tag{27}$$

If it is shifted  $n$  times, we assume that two elements selected are equal, that is,

$$e_{[j(i+n+j)] \bmod p} - e_{[j(k+n+j)] \bmod p} = 0. \tag{28}$$

From (27) and (28), we obtain

$$e_{[l(k+l)] \bmod p} - e_{[j(k+j)] \bmod p} = e_s. \quad (29)$$

Therefore, if  $l=j$ , One element is shifted in time by any integral multiple that  $X_l$  equals  $X_k$ .

### V. Performance Evaluation of Hopping Patterns

In FH/SSMA, the number of hits should be as small as possible whenever two or more spread spectrum signals hopped by different patterns are transmitted simultaneously with the same set of hopping frequencies. In order to evaluate the performance of the FH/SSMA system using the previously mentioned classes of hopping patterns, we should obtain the hit property<sup>(1)</sup> between any two hopping patterns  $X_i, X_j$  with  $m$  ( $m=0, 1, 2, \dots, p-1$ ) shift frequencies of  $X_i$  as shown in the definition of (4). For example, the number of hits is shown in Fig.3 when time and frequency are shifted by increments of one between the two patterns derived from the second method using permutation is varied in a synchronous chip time.

Hence, we use  $p=23$  distinct hopping signal. The  $x$  axis is denoted by time shift  $t$  ( $0 < |t| < p$ ) on the midpoint 0. The  $y$  axis represents the frequency shift  $f_s$  on the midpoint 0. By inspection Fig.3, we see that the number of hits never exceeds two in this example. As previously mentioned, Fig.4 shows the number of hits in which two hopping patterns are shown with an arbitrary time delay  $\Delta t$  ( $0 < |\Delta t| < T$ ).

Obviously, we see that the number of hits is no larger than 2 in Fig.4. The comparison of hit properties of the proposed patterns and

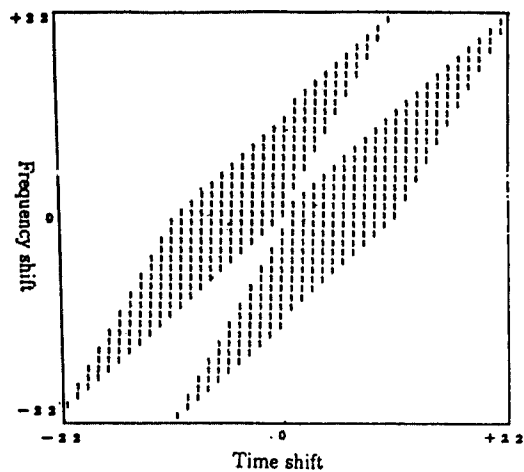


Fig.3 Number of hits between  $f_i$  and time-frequency shifts  $f_j$  based on method using permutation with chip synchronous for the 2nd method ( $p=23$ ).

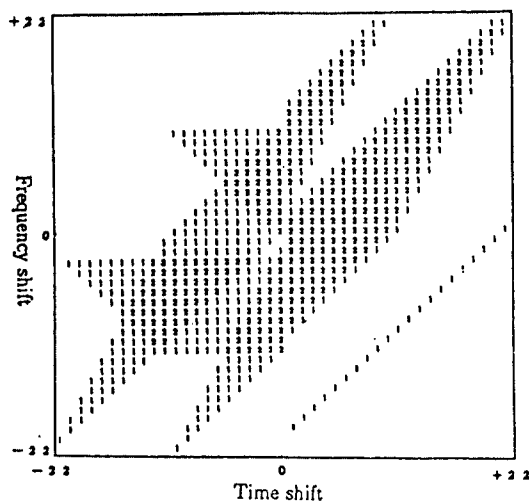


Fig.4 Number of hits between  $f_i$  and time-frequency shifts  $f_j$  based on method using permutation with chip asynchronous for the 2nd method ( $p=23$ ).

conventional patterns of RS code sequences is illustrated in Table 2.

It is noted that the method using permutation produced hopping patterns which performed better than conventional RS code sequences under the same condition.

Table 2. Proposed sequence property in comparison with other sequence ( $p$  : prime number)

|   | Ref. [1] | meth od 1 | meth od 2 | meth od 3 |
|---|----------|-----------|-----------|-----------|
| Length of sequence  | $p-1$    | $p-1$     | $p$       | $p$       |
| No. of patterns   | $p(p-1)$ | $p(p-1)$  | $p(p-1)$  | $p(p-1)$  |
| No. of hits with synchronous time / freq. shift ( $q=23$ )  | $\leq 2$ | $\leq 1$  | $\leq 1$  | $\leq 1$  |
| No. of hits with asynchronous time / freq. shift ( $q=23$ ) | $\leq 4$ | $\leq 8$  | $\leq 2$  | $\leq 2$  |

## VI. Conclusion

In this paper, we have proposed the number of generation and presented correlation properties of three hopping patterns in a finite field and evaluated correlation property. The first method using a Latin square generates hopping patterns with the same property as RS code sequences, and the second method using permutation have better performance than the first method. Finally, the hopping patterns of the third method possess the

same correlation property of those of the second method.

Therefore, the FH /SSMA system using hopping patterns generated by the second or third method can reduce the number of hits and improve the capacity or number of simultaneously accessing users.

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任 春 植(Choon Sik YIM) 正會員

1952년 4월 3일생

1975년 2월 : 한국항공대학 통신공학과 졸업 공학학사 취득

1986년 2월 : 동대학 대학원 전자공학과 졸업 공학석사

1991년~현재 : 일본 요코하마 국립대학 박사과정 재학중

1980년~현재 : 한국전자통신연구소(ETRI) 선임 연구원  
주관심 분야 : 디지털 이동통신, 스펙트럼 확산통신, 채널 코딩 이론 및 적응 어레이 안테나 등임



河野 隆二(Ryuji KOHNO) 正會員

1979년 : 요코하마 국립대학 공학부 전자정보공학과 졸업

1981년 : 동대학 대학원 석사과정 졸업

1984년 : 동대학 대학원 박사과정 수료 공학박사 학위취득 동년 동양대학 강사(동대 전기과)

1984~1985년 : 캐나다 토론토대학 객원 연구원

1986년 : 동양대학 조교수

1985년~현재 : 요코하마국립대학 조교수 (공학부 전자정보공학과)

주관심분야 : 부호계연, 적응디지털 신호처리, 스펙트럼 확산 통신방식, 네트워크의 신뢰성등의 연구 종사



今井 秀樹(Hideki IMAI) 正會員

1966년 : 동경대학 공학부 전자공학과 졸업

1971년 : 동대학대학원 박사과정 수료, 공학박사 동년 요코하마 국립대학 강사(공학부 전기과)

1972년 : 동대학 조교수

1984년~현재 : 동대학 교수

주관심분야 : 부호이론과 응용, 암호 알고리즘, 디지털 신호처리, 데이터 압축 및 통신방식의 연구 종사