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◉ Technical Paper

On the Dynamic Analysis of Cables for ROV Implementation

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무인 잠수정 연결 케이블의 동적거동 해석에 관한 연구

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Key Words : Cables(케이블), Dynamic Tension(동적장력), ROV System(무인잠수정시스템), Finite Difference Scheme(유한차분법), Numerical Method(수치해석법).

요 약

본 논문은 케이블-ROV 시스템의 동적거동 규명을 위한 효과적인 해석방법에 대한 고찰이다. 해석방법에 대하여 일차적으로 연구하고 해석방법에 따른 소프트웨어를 개발하여 3차원 케이블-ROV 시스템 거동을 수치적으로 해석하였으며 같은 모델에 대한 거동을 실제 실험을 통하여 연구분석 하였다.

수치해석과 실험결과를 비교 검토하였고 여러가지 관련 문제점에 대해서도 연구하였다.

1. Introduction

ROV is the new technology equipment being made to replace manned systems, both manned submersibles and divers.

Principal task of ROV is to freely travel and steadily approach to an object in aqua-space and transmit video information of it to water surface by using tether cable, since electro-magnetic wave cannot be utilized in water, although it is very powerful means on land and in aero-space. Acoustic means is a possible substitute for wireless data transmission, but its capability is quite limited. Therfore, most of underwater vehicles have to be wired by tether cable.

Consequently, performance of ROV is strongly dependent upon its tether cable which is subject to hydrostatic and hydrodynamic forces in addition to gravity and inertia forces.

As the operational depth of ROV increases, the behavior of a tether cable becomes more compli-

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cated and the static/dynamic analysis of the cable become important for investigating the feasibility and safety of ROV.

For this study, the dynamic analysis scheme and related computer program of a cable is developed. The scheme is developed for the analysis of a single, nonlinear, three-dimensional and static/dynamic model of a submerged cable. The lumped mass method, developed by Walton and Polachek in 1959, with Houbolt integration algorithm and Newmark method are basically employed in the scheme and the related program.

The problem under consideration is the examination of the behavior of the tether cable under hydrodynamic loading, excitation caused by support vessel and the thrust of vehicle. Examples are presented to show the reliable performance of the related program and its applicability. Also experiments are performed and results are compared with theoretical ones. The scheme developed here may be very effective and precise for the dynamic analysis of a cable for ROV implementation.

2. Theory

2.1 Governing Equation of Motion

In the present mothod, the contineous distribution of the mass of cable is replaced by a discrete distribution of lumped masses at a finite number of points on the cable. That replacement amounts to idealizing the system as a set of point masses and non-mass linear springs. We call a lumped mass point as a node and a spring between two nodes as an element. The tension in the cable is assumed constant in an element. The external forces acting on the cable are weight, buoyancy and hydrodynamic forces. Those forces are also converted to concentrated loads at the nodes. The hydrodynamic forces are calculated at the element centroid and assumed constant along the element. For that reason, the length of each element must be taken sufficiently small[1].

As shown in Fig. 1, the governing equation of motion of j-th lumped mass can be written as follows ;

$$[M]_j \ddot{X}_j = C_j T_j - C_{j-1} T_{j-1} + F_j \quad \cdots\cdots (1)$$
$$(j=1, 2, \cdots\cdots, N+1)$$

where

- $[M]_j$: 3×3 mass matrix at j-th node. This matrix includes the added mass of water in addition to the cable mass.
- X_j : position vector of j-th node
- C_j : direction vector of j-th element(unit vector)
- T_j : tension of j-th element
- F_j : force vector at j-th node
- N : numbers of elements

The added mass of water is also lumped to the nodal points after transformation to the global coordinate system.

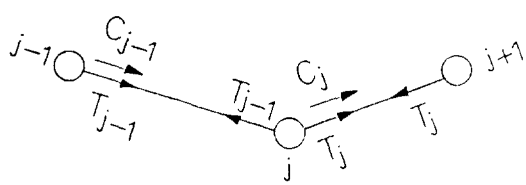

Fig. 1 Free body diagram of cable system.

The constraint equation on the length of j-th element is represented in equation (2). For simplicity, modulus of elasticity and diameter are assumed to be constant over the whole length of cable.

$$|X_{j+1} - X_j|^2 = (X_{j+1} - X_j) \cdot (X_{j+1} - X_j)$$
$$= l_j^2 (1 + T_j/EA)^2 \quad \cdots\cdots (2)$$
$$(j=1, 2, \cdots\cdots, N)$$

where

- $\cdot$: scalar product of two vectors

l_j : unstrained length of j-th element
E : modulus of elasticity of cable
A : cross-sectional area of cable

The equation (1) can be reduced as follows ;

$$\ddot{X}_j=(P_jT_j-Q_jT_{j-1}+R_j)/\Delta t^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where

Δt = time increment in numerical integration
$P_j=\Delta t^2\ [M]_j^{-1}C_j$
$Q_j=\Delta t^2\ [M]_j^{-1}C_{j-1}$
$R_j=\Delta t^2\ [M]_j^{-1}F_j$

On the other hand, in numerical integration scheme, the nodal displacements and velocities of the next time step (X_j^{n+1}, $\dot{X}_j^{n+1}$) can be expressed as follows using Newmark's constant-average-acceleration method[2].

$$X_j^{n+1}=X_j^n+\dot{X}_j^n\Delta t+(\ddot{X}_j^n+\ddot{X}_j^{n+1})\ \Delta t^2/4 \quad \cdots\cdots\cdots (4)$$

$$\dot{X}_j^{n+1}=\dot{X}_j^n+(\ddot{X}_j^n+\ddot{X}_j^{n+1})\ \Delta t/2 \quad \cdots\cdots\cdots\cdots\cdots\cdots (5)$$

Combining equations (3) and (4), the nodal displacements of next time step may be written as follows ;

$$X_j^{n+1}=X_j^n+\dot{X}_j^n\ \Delta t+\ddot{X}_j^n\ \Delta t^2/4 + (P_j^{n+1}T_j^{n+1}-Q_j^{n+1}T_{j-1}^{n+1}+R_j^{n+1})/4 \quad \cdots\cdots (6)$$

To abtain the tension of next time step T_j^{n+1}, we method [7]. It is assumed that T_j^{n+1} consists of two components as follows ;

$$T_j^{n+1}=\tilde{T}_j^{n+1}+\Delta T_j^{n+1}$$

where $\tilde{T}_j^{n+1}$ is the tentative value of the tension which is sufficiently close to the correct value and ΔT_j^{n+1} is the correction. Now, we define the following equation which is a function of cable tension of next time step. This equation is derived from equations (2) and (6).

$$\phi_j^{n+1}=-l_j^2(1+T_j^{n+1}/EA)^2+|X_{j+1}^{n+1}-X_j^{n+1}|^2 =\phi_j^{n+1}(T_{j-1}^{n+1},\ T_j^{n+1},\ T_{j+1}^{n+1})=0 \quad \cdots\cdots (7)$$

(j=1, 2, ……, N)

Expanding ϕ_j^{n+1} in a Taylor series about the $\{\tilde{T}_{j-1}^{n+1}, \tilde{T}_j^{n+1}, \tilde{T}_{j+1}^{n+1}\}$ and neglecting the higher order terms, we can obtain a system of N linear equations for the differential correction ΔT_j^{n+1} as follows ;

$$\tilde{E}_j^{n+1}\Delta T_{j-1}^{n+1}-\tilde{F}_j^{n+1}\Delta T_j^{n+1}+\tilde{G}_j^{n+1}\Delta T_{j+1}^{n+1}=-\tilde{\phi}_j^{n+1} \quad \cdots (8)$$

(j=1, 2, ……, N)

where

$$-\tilde{\phi}_j^{n+1}=l_j^2(1+\tilde{T}_j^{n+1}/EA)^2-|\tilde{X}_{j+1}^{n+1}-\tilde{X}_j^{n+1}|^2$$

$$\tilde{E}_j^{n+1}=\frac{\partial\tilde{\phi}_j^{n+1}}{\partial T_{j-1}^{n+1}}=\tilde{Q}_j^{n+1}\cdot(\tilde{X}_{j+1}^{n+1}-\tilde{X}_j^{n+1})/2$$

$$-\tilde{F}_j^{n+1}=\frac{\partial\tilde{\phi}_j^{n+1}}{\partial T_j^{n+1}}=-(\tilde{Q}_{j+1}^{n+1}+\tilde{P}_j^{n+1})\cdot(\tilde{X}_{j+1}^{n+1}-\tilde{X}_j^{n+1})/2 -2l_j^2(1+\tilde{T}_j^{n+1}/EA)/EA$$

$$\tilde{G}_j^{n+1}=\frac{\partial\tilde{\phi}_j^{n+1}}{\partial T_{j+1}^{n+1}}=\tilde{P}_{j+1}^{n+1}\cdot(\tilde{X}_{j+1}^{n+1}-\tilde{X}_j^{n+1})/2$$

(j=1, 2, ……, N)

and

$$\tilde{X}_j^{n+1}=X_j^n-\dot{X}_j^n\ \Delta t+\ddot{X}_j^n\ \Delta t^2/4 +(\tilde{P}_j^{n+1}\tilde{T}_j^{n-1}-\tilde{Q}_j^{n+1}\tilde{T}_{j-1}^{n+1}+\tilde{R}_j^{n-1})/4$$

2.2 Computational Procedure

The computational procedures of sloving the dynamic behavior of cable are as follows ;

(1) Calculate the configuration and tensions of cable when the cable is in a static equilibrium. A method of static calculation using the finite element model is described in Ref. (3) and (4). The results of the static calculation are transferred to the dynamic analysis program as the initial conditions of the cable.
(2) The prescribed forces and/or displacements of the next time step are applied to the cable conditons of the previous time step.
(3) Calculate the nodal displacements and tensions of the next time step by iteration. In

this step, the tensions of previous time step is used for the first approximation of the tentative value T_j^{n+1}. The corrections of tensions ΔT_j^{n+1} are obtained by solving the system of N linear equations derived from the equation (8).

(5) The iterative procedure from step (3) to step (4) is continued until sufficient convergence is obtained.

(6) Repeat the procedure from step (2) as one time step increased.

The flow chart of the related computer program is given in Fig. 2.

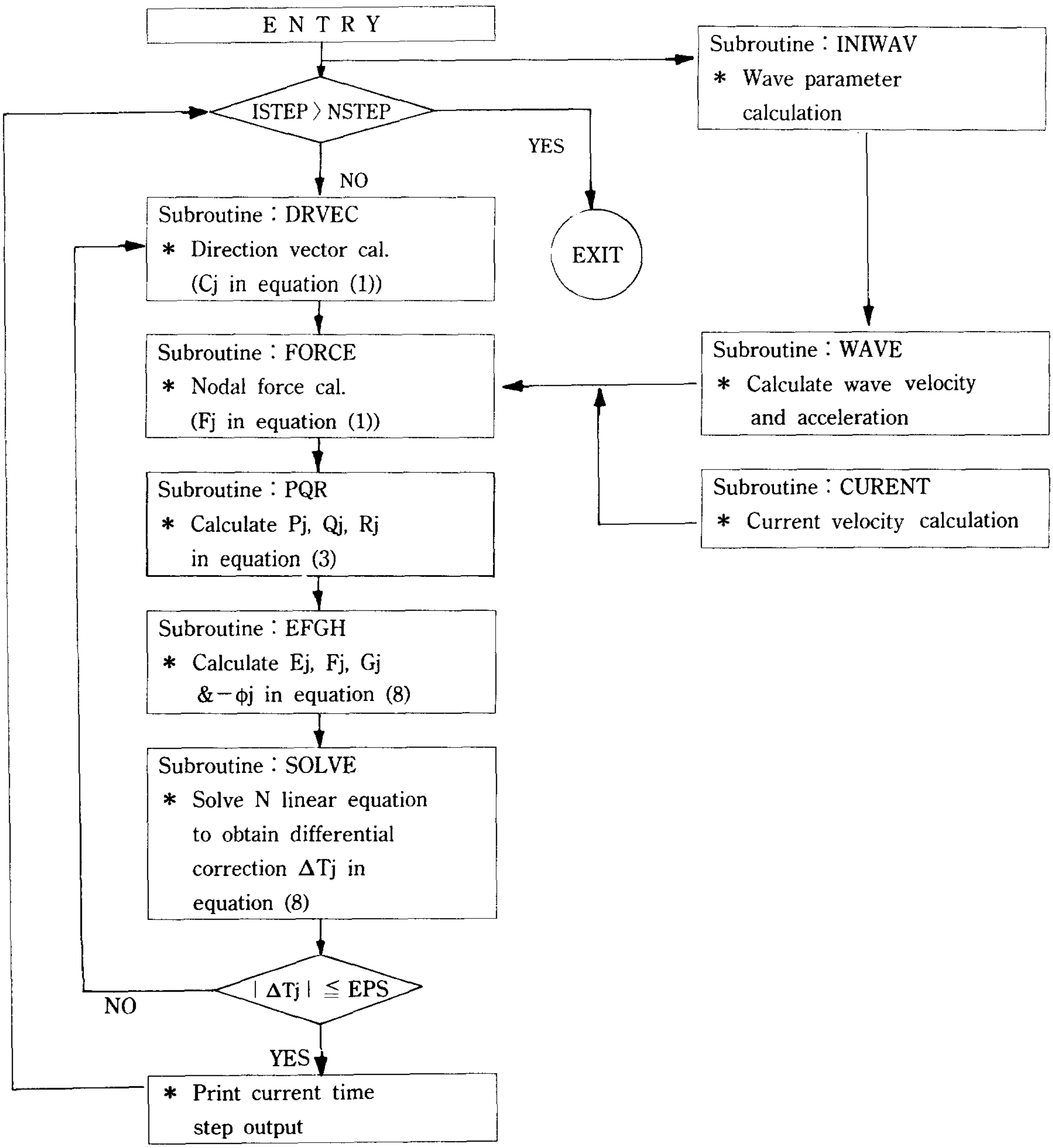

Fig. 2 Flow chart−related program

3. Numerical Examples and Experiments

In order to demonstrate the reliability and utility of the scheme developed here, examples are solved and the results are compared with those obtained by others. The computer program 'DS-CAP' is developed as mentioned before and used for solving the examples. Experiments using water tank are also performed and the results are compared with those obtained by the calculation.

Example 1

This example describes the motion of a sphere drug through water by a partially submerged cable suspended from a rotating surface crane. This problem is based on the example given by Huston [5]. The system is composed of a rotating surface crane with a boom dragging a cable attached to a submerged sphere. The boom makes a 90° turn in five seconds. Table 1 shows the necessary data for the example. The resultant motion of the sphere is shown in Fig. 3, Fig. 4 and Fig. 5. In Fig. 3 the marked symbols on the lines are given by 0.5 second time interval from t=0 to t=10 seconds. The calculated radial positions are proved to be well coincided with those by Huston.

Example 2

Tank experiments are performed with models of single rubber cable to check the reliable performance of the program. The experiments include anchor drop and buoy relaxation. The principal

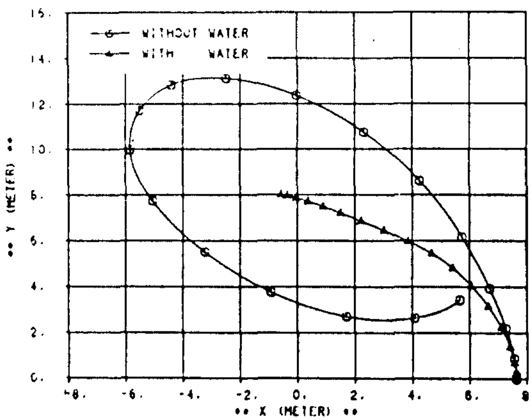

Fig. 3 Plane position of sphere, example 1

Table 1 Characteristics of model in Example 1

Cable	Length	15.24 *m*
	Diameter	25.4 *mm*
	Mass density	3.854 *kgf / m*
	Tension rigidity	inextensible
	Added mass coeff.	1.0
	Normal drag coeff.	1.2
	Tangential drag coeff.	0.062
	Numbers of elements	10 elements (equal length)
Sphere	Diameter	0.3048 *m*
	Mass	112.8 *kgf*
	Added mass coeff.	0.5
	Drag coeff.	0.5
Environments	Sea state	calm(no currents)
	Water surface	3.048*m* below the boom

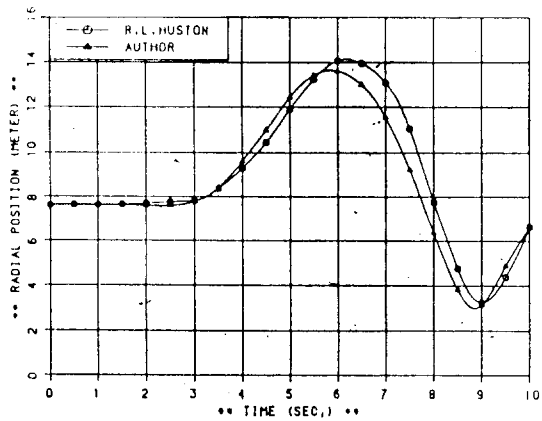

Fig. 4 Radial position of sphere without water, example 1

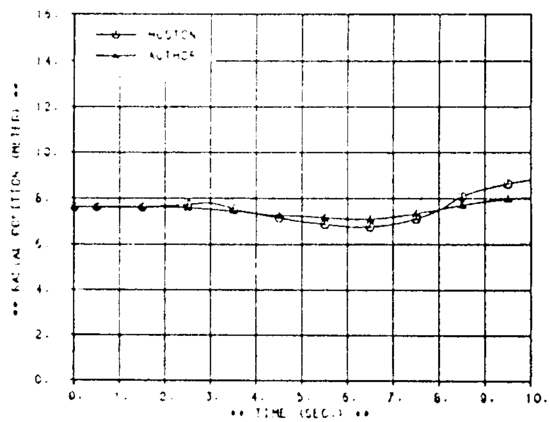

Fig. 5 Radial position of sphere with water, example 1

particulars of the cable, anchor and buoy are shown in Table 2 and the initial locations of the anchor and buoy are shown in Fig. 6, Fig. 7, respectively. The water density is 1.0 *gram/cm*^{3} and the depth of tank is 100 *cm*. The times are measured, which last from the moment of anchor release (or buoy release) to the moment of bottom contact (or water surface contact). Those measured values are compared with the analysis results in Table 3.

In the numerical calculation for dynamic behavior of the cable, the total length of the line is divided into 15 elements of equal length. The time increment Δt is taken by 0.01 seconds in both cases of anchor drop and buoy relaxation. The hydrodynamic cefficients used for the numerical calculations are assumed as follows[4~6].

C_M : added mass coefficient of cable 1.0

C_N : drag coefficient of cable in normal direction 1.27

C_T : drag coefficient of cable in tangential direction 0.062

C_{MO} : added mass coefficient of sphere 0.5

C_{DO} : drag coefficient of sphere 0.47

Fig. 8, Fig. 9 show the trajectories of the cable-anchor system and cable-buoy system, respectively by the experiments. Fig. 10, Fig. 11 show the trajectories of the cable-anchor system and cable-buoy system, respectively by the calculations. Fig. 12, Fig. 13 show the comparison between test results and calculation results for the case of cable-anchor system and cable-buoy system, respectively. The results show that the method developed here provides precise and effective ways for the dynamic analysis of the cable-ROV system, except the case where bending rigidity plays an nontrivial roles, i. e., in the case of rapid change of the curvature of the cable exists. In this example, two different integration algorithms, i. e., Houbolt method and Newmark method are employed respectively for the calculation of the behavior of the systems. Using PRIME 9650 computer, the calculations are performed. Table 4 shows the C. P. U. time used for the calculation with Houbolt method and Newmark method for anchor drop case and buoy release case respectively. The C. P. U. time used by Newmark method is far less than that used by Houbolt method.

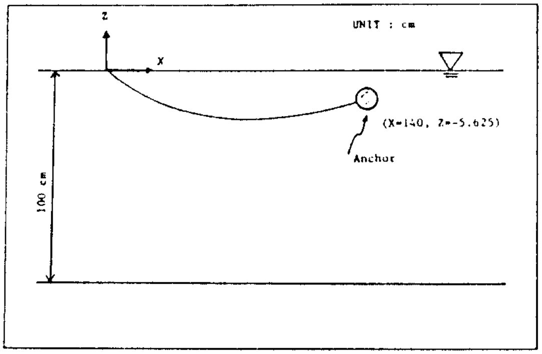

Fig. 6 Initial location of anchor

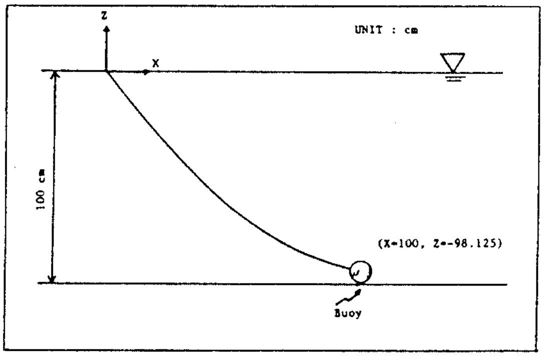

Fig. 7 Initial location of buoy

Table 2 Principal particulars of cable, anchor and buoy

Cable	Total length	150 *cm*
	Diameter	0.3 *cm*
	Mass density	0.112 *gram/cm*
	EA	9.807×10^7 *dyne*
Anchor (Sphere)	Diameter	3.75 *cm*
	Mass	34.9 *gram*
Buoy (Sphere)	Diameter	3.75 *cm*
	Mass	21.5 *gram*

Table 3 Comparison between analysis and experiments results

Case	Final stage reaching time(*sec*)	
	Analysis	Experiments
Anchor drop	2.50	2.35
Buoy release	3.10	3.00

Table 4 C. P. U. time comparison with Houbolt and Newmark Method

PRIME 9650 Computer (unit : *sec*)

Method / Case	Houbolt Method	Newmark Method
Anchor drop	167	131
Buoy release	151	110

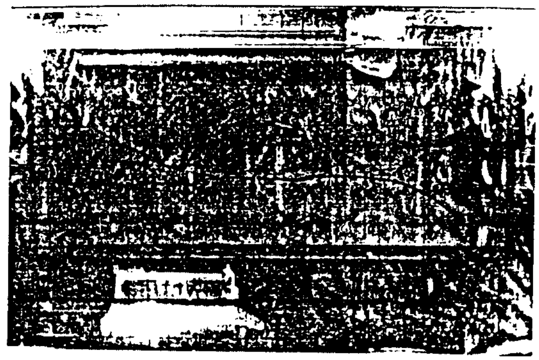

Fig. 8 Test trajectory of the cable-anchor system

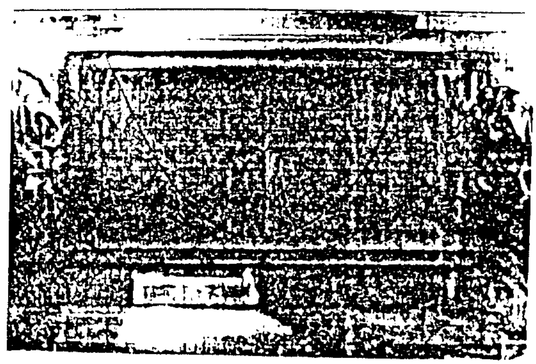

Fig. 9 Test trajectory of the cable-buoy system

4. Discussion

A nonlinear, three-dimensional, finite-segment model of a submerged multicomponent cable with ROV and the dynamic analysis scheme and the related program for the model have been developed. If the applied force system is known, the positon, velocity, acceleration, etc. of the member is

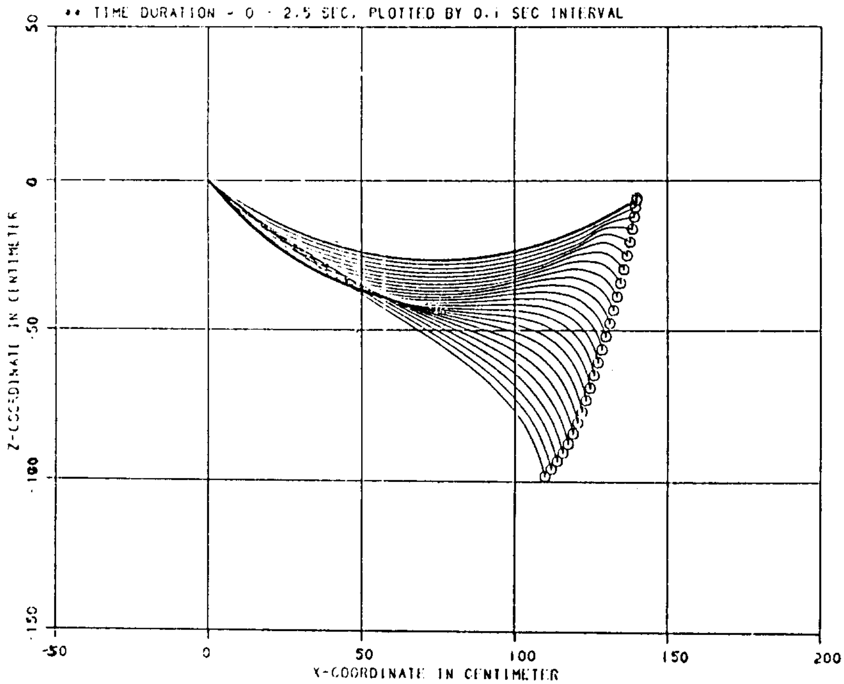

Fig. 10 Calculated trajectory of the cable-anchor system

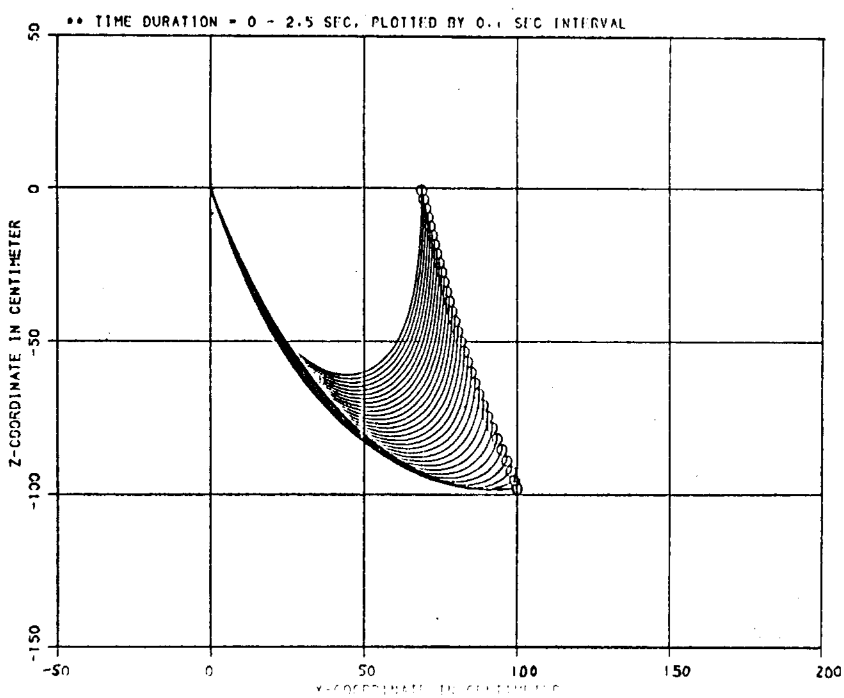

Fig. 11 Calculated trajectory of the cable-buoy system

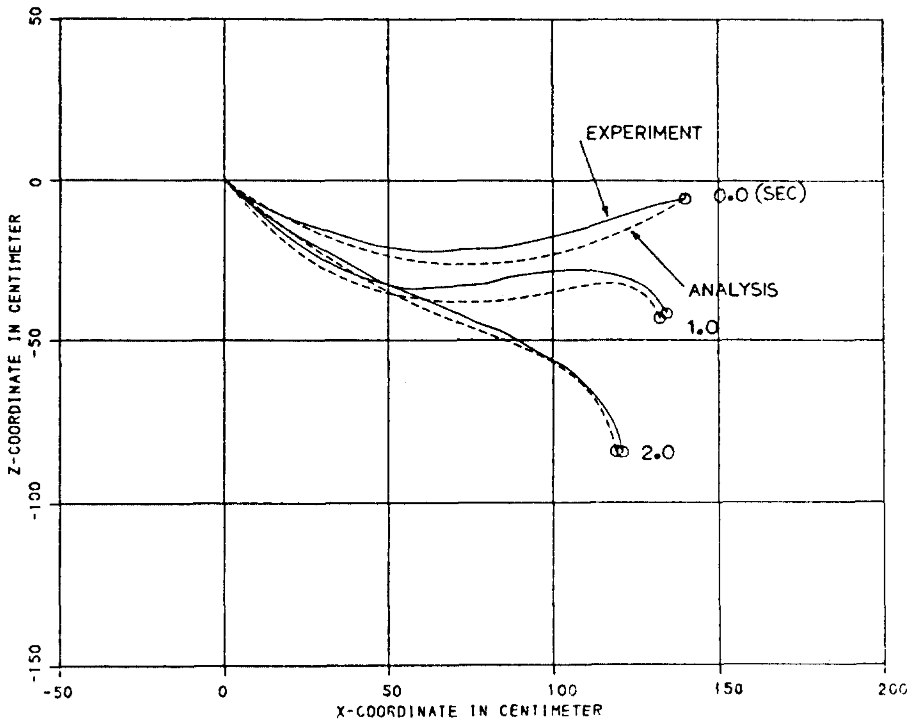

Fig. 12 Final location of anchor, cable-anchor system

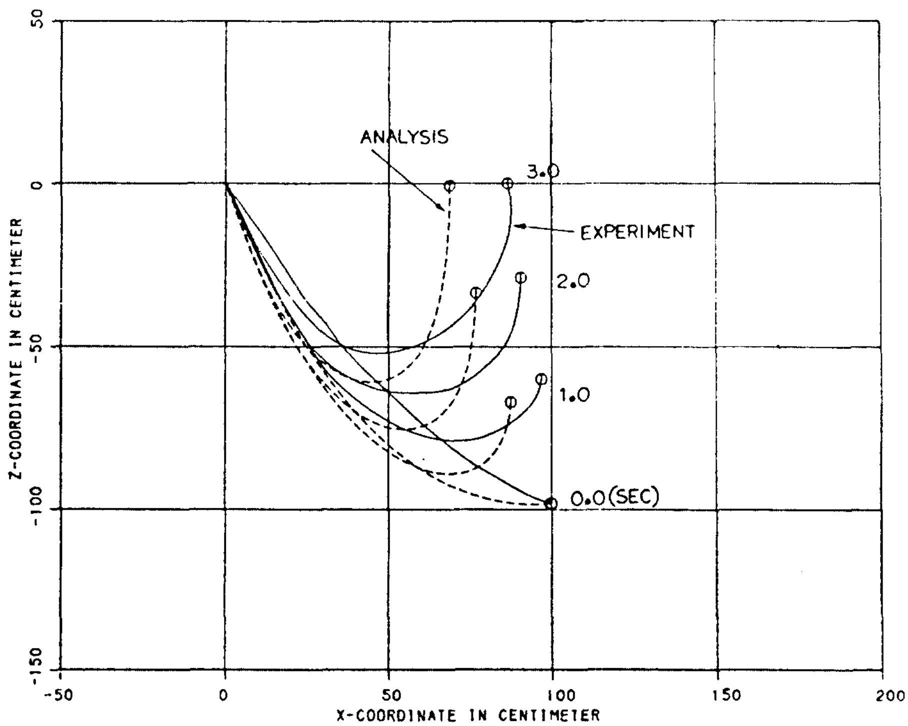

Fig. 13 Final location of buoy, cable-buoy system

known or if the member behavior is known, the required forces are determined. During the calculations the flexibility of the member is not considered, i. e., bending rigidity of the cable is not included, thus the behavior of the model which results in rapid change of the curvature of the cable turns out to be not to well coincide with the experimental results. Case by case, the bending rigidity of the cable should be incorporated for the better dynamic analysis of the system. Most problems seem to have no significant bending rigidity problems, however.

The computer program developed for the practical application of the scheme can be effectively modified to account for the accurate hydrodynamic effect of the ROV vehicle itself, for more sophisticated wave theories and so on. The numerical integration algorithms such as Houbolt method, Newmark method can be selectively incroporated with the analysis scheme in order to save the C. P. U. time. Tank experiments for the verification of the calculated results proved to be very benificial for the better understanding of the behavior of the cable-ROV system. Experiments in the current are also performed to account for the effects of the current, but after close examinations and doing plenty of the tests, the results will be disclosed. Experiments are thought to be essential for the seeking of the dynamic behavior of the cable-ROV system.

5. Summary

Since the performance of ROV is controlled by its tether cable in variable loading circumstances, the dynamic cable analysis problem is chosen as the primary subject. And the analysis scheme and the computer analysis program of a cable is developed. Example problems are demonstrated for the implementation of the cable-ROV system. Through the tank tests and numerical examples, the scheme and the related analysis program are proved to work well and it may be concluded that the method can be very effective and reliable for the dynamic analysis of the cable-ROV system behaviors.

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