System Identification on Flexure of SFRC

SFRC 휨거동에의 System Identification

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ABSTRACT

Flexural load−deflection relationships for steel fiber reinforced concrete(SFRC) are dependent on the tensile and compressive constitutive behaviors of the material, which may be refined in the presence of strain gradients under flexural loads. Considering the relatively large amount of flexural test results available for steel fiber reinforced concrete, and the relative ease of conducting such tests in comparison with direct tension tests, it seems to be important to obtain basic information on the tensile constitutive behavior of SFRC from the result of flexural tests. For this purpose "System Identification" technique was used for interpretating the flexural test data and it was successful in obtaining optimum sets of main parameters which explain the tensile constitutive behavior of SFRC under flexure.

요 약

강섬유 보강 콘크리트(SFRC)의 휨 거동은 재료의 인장 및 압축 응력−변형도에 의존하며 이때 이들은 휨 응력시 작용하는 strain gradient의 영향을 받게된다. SFRC의 경우, 휨 실험은 직인장 실험과 비교하여 볼 때 상대적으로 간편하며 또한 다수의 실험결과가 확보되어 있다. 따라서 이들 휨 실험 결과로부터 SFRC의 기본적 재료 성질인 인장응력−변형도를 유출하는 것은 중요하다고 하겠다. 본 연구의 목적을 위하여 휨 실험 data를 해석하기 위한 "System Identification"방법론이 사용되었으며 그 결과 휨 응력하에서의 SFRC의 인장거동을 설명하는 주요 변수들이 고찰되었다.

1. INTRODUCTION

Constitutive behaviors of a material in tension and compression determine material's flexural load−deflection relationships. Generally it is easier to conduct flexural tests of SFRC than to conduct direct tension tests and comparatively larger amount of data on the flexural test results is available. Flexural test data are interpreted in this study for the derivation of a basic tensile properties of SFRC under flexure. The analytical model for flexure developed previously by the author [1] has been used for this purpose together with "System Identifiation" technique. In "System Identification" the response of a system to a

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given input is known from experiments and a mathematical model is to be found which will describe the material behavior. Flexural responses of SFRC beam in terms of load-deflection relationships from experiments were simulated by iteratively adjusting the parameters in the constitutive models incorporated in the analytical model of SFRC under flexure. The derived values for the paramenters through "System Identification" are then compared with analytically, and experimentally obtained values. Some discussions are also made regarding the strain gradient effects on constitutive behavior of steel fiber reinforced concrete.

2. "SYSTEM IDENTIFICATION" FOR SFRC UNDER FLEXURE

In order to reflect the effects of the paramenters in the constitutive models on the overall flexural behavior of SFRC, an analytical model which can simulate both the physical flexural behavior of SFRC and constitutieve behavior of the material were established by the author in Ref.1. This model takes into account for a formation of one major crack and subsequent accumulation of the curvature at the critical section. Complete flexural load-deflection relationships were constructed by this model with cradk-opening considered through conducting a flexural analysis of the critical section, and using some assumptions regrading curvature distributions in the vicinity of the critical section. Satisfactory comparisons were obtained between test results and theoretical simulations based on the developed flexural model (refer to Ref. 1).

A mathematical form for error function is needed to measure the correlation between test results and predictions of the mathematical model for a given set of characteristic values. The error function should be able to quantify the differences in important flexural characteristics of SFRC. "System identification" deals with finding the location on the error surface with minimum error, the coordinates of which will be the desired optimizing paramenters. These optimized parameters can be interpreted as the values of representing the best correlation between the analytical and experimental results.

The characteristic material values in constitutive models are then adjusted until the best possible correlation is achieved between the predicted and measured responses of SFRC under flexure. Tensile and compressive constitutive models incorporated in flexural model of SFRC have ten material- and ten constitutive behavior-related paramenters for defining their complete shapes. Three important characteristic paramenters of the tesile constitutive behavior of SFRC were selected out of the ten material-related and ten constitutive behavior-related factors of SFRC, which will be described later, and these three parameters were kept constnat as "standard" values. The "standard" values of the factors have been chosen either on the basis of test results or considering practical ranges applicable to SFRC.

The error function (E) is defined to measure the correlation in overall flexural behavior between the experimentally measured and theoretically predicted load-deflection relationships. The characteristic values expressing the flexural behavior of SFRC are peak flexural load (P), flexural dutility (D), and flexural toughness (A). The differences in these characteristic values set the bases for computing the error between predicted and experimental flexural load-deflection relationships :

$$E = \sum_{i=1}^{3} \alpha_i \cdot e_i^2 \qquad (1)$$

where : α_i=weighing coefficient for each factor ;
$e_1=(P_e+P_t)/P_e$;
$e_2=(D_e+D_t)/D_e$;
$e_3=(A_e-A_t)/A_e$;
P=ultimate load (Fig. 1) ;
D=ductility
=P/P_r (Fig. 1) ; and
A=toughness
=area under load-deflection curve as defined in Fig.1.

Subscripts "e" and "t" in eq.(1) represent "experiment" and "theory", respectively. Considering equal contribution of each e_i to the total error (E) in terms of prepeak (with e_1), post-peak (with e_2) and overall behavior (with e_3) in load-deflection curve, values for weighing coefficients are given 1.0 for each error term.

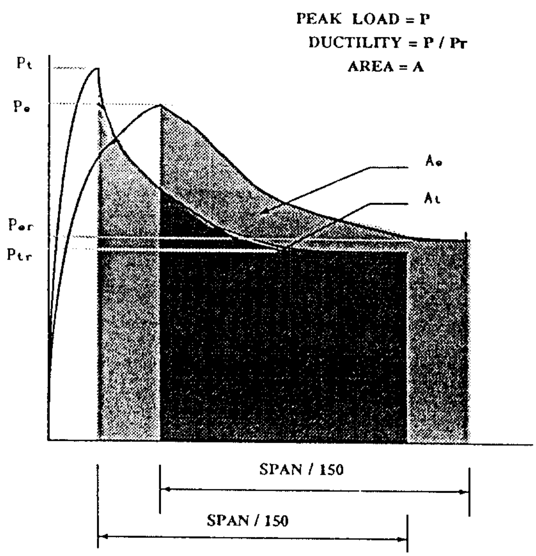


Figure 1. Definition of Three Different Criteria

The error fuunction derived above is an objective measure of how well the model fits the experimental data. The error function should be minimized in the N-parametric space. Nonlinear programming techniques can be used for this purpose. The nature of the present study suggests that the minimum point lies in the interior of the feasible region of the parameter space rater than on its boundary, and thus unconstrained nonlinar programmings suit this problem.

An iterative minimization algorithm was used in the related unconstrained nonlinear programming approach. The algorithm be able to converge to a stational point in the global sense and should also converge rapidly when it is in the neighborhood of a local minimum (Luenberger 1973). The iterative minimization approach adopted in this investigation is described below. Starting from the point in the parameter space selected after k steps (x_k), choose the next point as follows :

$$x_{k+1}=x_k+\zeta \cdot d \qquad (2)$$

where : d=direction vetor ; and
ζ=step length.

Individual methods vary in their choice of ζ and d and this choice in general determines the efficiency of the method. Calculation of the gradient numerically rather than analytically may be desirable or even necessary. As the calculation of partial derivatives of a given function is, in general, at least as complicated as calculation of function itself, a method which avoids the calucultion of derivatives has the possibility of being more efficient as well as having the advantage of being more convenient to use. One such method has been given by Powell 1964. The basic Powell's algorithm chosen for use in this study is presented below and it is modified further in this study to properly choose the direction vectors in order to avoid possible break down due to linear dependency of the direction vectors (refer to Walsh 1975 for details). The k^{th} iteration of this method starts with a current point x_k and n directions, $d_{k,j}$, $j=1,2,\cdots,n$. At the beginning, x_1 and d_1, j are assumed to be given.

1. Let $y_{k,o}=x_k$
2. Find β_j^* minimizing the function, $f(y_{kj-1}+\beta_j \cdot d_{k,j})$.
   Set $y_{k,j}=y_{k,j-1}+\beta_j^* \cdot d_{k,j}$ for $j=1, 2,\cdots,n$.
3. Let $\triangle_k=y_{k,n}-x_k$
4. Find β_n^* minimizing the function, $f(y_{k,n}+\beta_n \cdot \triangle_k)$.
   Set $x_{k+1}=y_{k,n}+\beta_n^* \cdot \triangle_k$.
5. Let $d_{k+1}=d_{k,j+1}$, $j=1, 2,\cdots, n-1$ and $d_{k+1,n}=\triangle_k$.
6. Go to step 1 and restart for $(k+1)^{th}$ step.

The k^{th} cycle which containes (n+1) subcycles for finding minimum along the given direction is schematically shown in Fig. 2 for n=1. In this figure, superscripts and subscripts represent the subcycle number and iteration number in a certain subcycle, respectively. In Powell's method, (n+1) line searches are needed to generate one conjugate direction. Therefore, in order to fined the global minimum point(assuming that the given function is quadratic and positive definite)a total of n(n+1) line searches are required. Since in the Powell's method, the error function is being approximated by a quadratic function, it seems to be appropriate to use quadratic line search. In the present study, the method of quadratic line search described by Powell (Powell 1964) has been used.

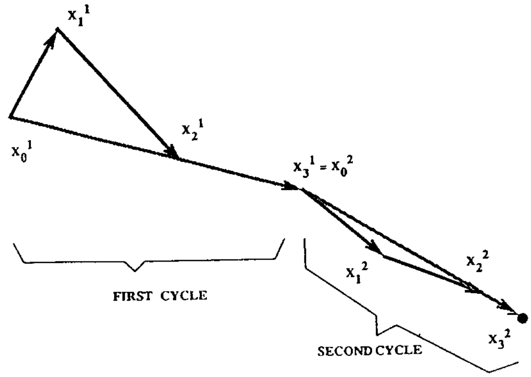
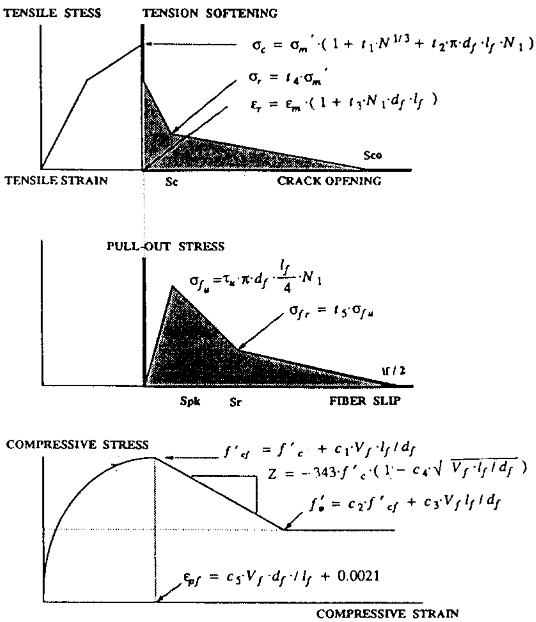


Figure 2. Main Theorems in Powell's Algorithm for n =2

The flexural model containes ten material−related and ten constitutive behavior−related factors (Fig.3). The variations in some of these factors have significant effects on the behavior of SFRC under flexure, while variations in other factors result in negligible effects on the flexural behavior of SFRC. Since it is not practical to optimize all these factors in the process of "System Identification", factors whose variations result in significant effects on the flexural behavior of SFRC need to be selected as the "System Identifiction" paramenters. Soroushian and Lee(Soroushian and Lee 1990) have examined on the basis of 2−k factorial design the influnece of each factor on the flexural peak load (P), flexural ductility (D), flexural toughness (A) and overall flexural behavior of SFRC which were described by combination of P, D and A defined earlier. It was observed that in the case of material−related factors, the fiber peak pull−out strength (τ_u), fiber diameter (d_f), fiber length (l_f), fiber volume fraction (V_f), matrix tensile strength (σ_m'), and fiber slip at reidual pull−out strength (S_r) are the most influential factors deciding the flexural behavior of SFRC (see Table 1). As far as the constitutive behavior−related factors are concerned, it was shown that their effects are negligible when compared with those of the material−related factors (Soroushian and Lee 1990).



* t's and c's are experimentally obtained coefficients. ctors

Figure 3. Factors considered in Flexural Analysis

Among the six influential material−related factors, those representing fiber dimension (i. e., dr and l_f) as well as the volume fraction of fibers (V_f) should be known inputs while analyzing some flexural test data obtained for SFRC. This further reduces the number of "System Identification" paramenters and leaves only three material−related factors to be entered as paramenters in "System Identification": fiber peak pull−out strength(τ_u), fiber slip at residual pull−out strength (S_r) and matrix tensile stregnth ($\sigma_{m'}$). It is worth mentioning that the tensile stregnth of SFRC can be determined once the values of these three factors are obtained through the analysis of flexural results using "System Identifiction".

Table 1. Results of 2−k Factorial Design

Factors	f−Values on Different Criteria (x 1000)			
	Peak Load	Ductility	Toughness	Overall Behavior
σ_m'	1349	38	20712	20.14
f'c	22	0.32	249	0.19
S_{cr}	183	1.39	1889	0.08
S_o	0.71	0.0	489	0.02
df	383	161	706830	77
lf	147	51	244490	34.4
Vf	244	91	425800	54.19
τ_u	343	231	865120	84
S_{pk}	18	0.33	662	0.087
S_r	2.4	145	195130	60

3. RESULTS OF "SYSTEM IDENTIFICATION"

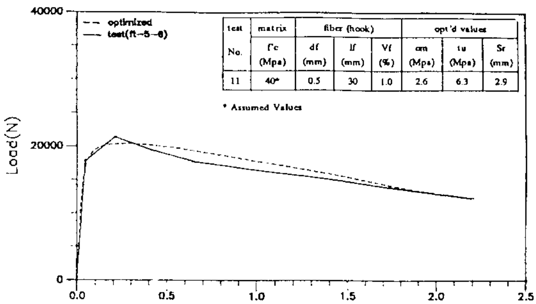
Table 2 summarizes conditions of the SFRC flexural tests considered for "System Identification", and also presents the optimized values of the three main parameters obtained from "System Identification". Fig.4 illustrates some typical comparisons between the experimentally obtained and theoretically optimized flexural load−deflection curves. Satisfactory correlations are observed in these figures.

From Table 2, the optimized values of three parameters are found to be larger than the values obtained from direct tension material tests (see the comparison presented in Table 3). The experimental data presented in Table 2 are the averages obtained from several direct tension and fiber pull−out test performed on materials comparable to those used in flex-

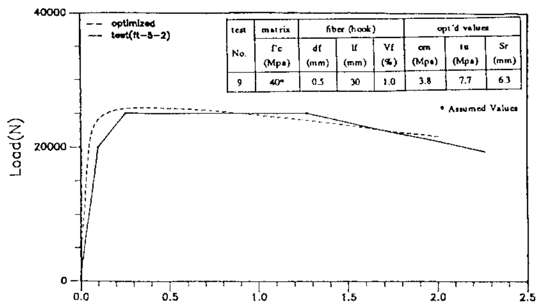
Table 2. Test Conditions and Optimized Values from "System Indentification."

Ref.	Test No.	Specimen			Fiber				f'c	Opt'd Values			Error	Itr. No.
		width	depth	length	type	df	lf	Vf		$\sigma m'$	τu	Sr		
Sakai and Nakamura, 1986	1	100	100	300	strt	0.56	30	0.01	(40)	5.032	6.174	2.72	0.000456	4
	2	100	100	300	strt	0.56	30	0.015	(40)	5.895	5.036	3.441	0.000024	3
	3	100	100	300	strt	0.56	30	0.02	(40)	7.132	4.413	3.441	0.000011	6
Soroushian and Ateff, 1989	4	100	100	300	strt	0.56	30	0.01	34.6	3.332	3.831	2.198	0.010869	6
	5	100	100	300	strt	0.56	30	0.015	34.6	4.649	3.933	3.121	0.000273	3
	6	100	100	300	strt	0.56	30	0.01	48	3.032	5.0	3.0	0.026306	2
	7	100	100	300	strt	0.56	30	0.01	24.7	2.564	2.752	2.56	0.000494	7
Cho and Kobayashi, 1982	8	100	100	300	hook	0.5	30	0.01	(40)	3.444	9.291	3.085	0.000231	3
	9	100	100	300	hook	0.5	30	0.01	(40)	3.381	7.73	6.247	0.000967	4
	10	100	100	300	hook	0.5	30	0.01	(40)	3.695	5.371	2.957	0.004180	2
	11	100	100	300	hook	0.5	30	0.01	(40)	2.57	6.25	2.887	0.003061	3

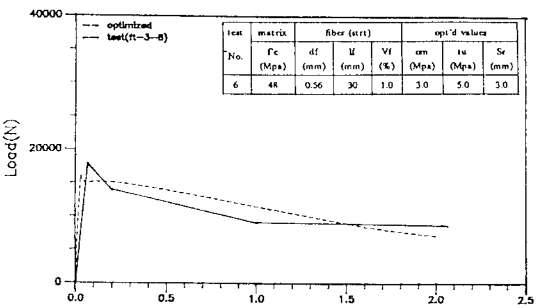
Values in parenthesis are assumed ones.



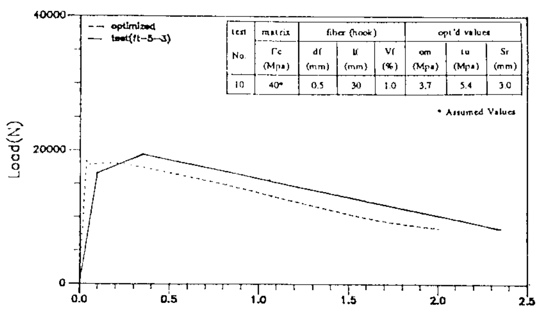
(a) Test Results from Soroushian and Ateff, 1989.



(c) Test Results from Soroushian and Ateff, 1989.



(b) Test Results from Sakai et al. 1986.



(d) Test Results from Soroushian and Ateff, 1989.

Figure 4. Comparisons between Experimentally Obtained and Theoretically Optimized Flexural Load–Deflection Curves.

Table 2. Comparisons of the Tension Test Results with the Optimized Values of Parameters in Analysis of Flexural Test Results using "System Identification"

Ref.	Test	Fiber				σm'	Ratios		
	No.	type	df	lf	Vf	(0.332 · $\sqrt{f'c}$)	σmo / σm'	τuo / τm'	Sro / Sr
Sakai and Nak-	1	strt	0.56	30	0.01	2.1	2.4	2.35	0.97
amura, 1986	2	strt	0.56	30	0.015	2.1	2.8	1.92	1.23
	3	strt	0.56	30	0.01	2.1	3.4	1.68	1.23
Soroushian and	4	strt	0.56	30	0.01	1.95	1.7	1.45	0.97
Afteff, 1989	5	strt	0.56	30	0.015	1.95	2.4	1.50	0.11
	6	strt	0.56	30	0.01	2.30	1.32	1.90	1.07
	7	strt	0.56	30	0.01	1.65	1.55	1.05	0.91
Cho and Koba-	8	hook	0.5	30	0.01	2.1	1.63	2.07	1.10
yashi, 1982	9	hook	0.5	30	0.01	2.1	1.82	1.72	2.23
10	hook	0.5	30	0.01	2.1	1.76	1.20	1.05	
	11	hook	0.5	30	0.01	2.1	1.22	1.35	1.03

ural tests. The matrix tensile strength (σ_m') and performance of fibers obtained from the analysis of flexural test results may be improved in comparison with those obtained from direct tension and pull−out tests due to the strain gradient effects under flexural loading condition, which generally lead to improved tensile performance of the material (Swamy et al., 1974). The improvements in pull-out performance in flexural test specimens over those obtained from single fiber pull−out tests may also be attributed to the positive effects of fiber reinforcement at the surrounding matrix (noting that single fiber pull−out tests are generally conducted using non−fibrous surrounding matrices) in flexural test specimens. Swamy et al. 1974, using an analysis of experimental data, has also reported increase in pull−out strength under flexure when compared with pull−out strength under tension.

Large variations in the values of parameters(τ_u, σ_m' and S_r) obtained from "System Identification" in Table 2 suggest that the highly variable (and unreliable) measurements of flexural deflections in the pre−peak region have some influence on the analysis of flexural test data using the "System Identification" approach. These variations may also partly result from the fact that some flexural test results reported in the literature were not accompanied by reliable informations on basic material properties and thus some assumptions had to be made on these properites through the course of "System Identification".

4. CONCLUSION

The following conclusions are made from this study :

(1) The improvements in pull−out performance in flexural tests over those obtained from single fiber pull−out tests (where fibers are generally pulled out of non−fibrous matrices) may be attributed to the positive effects of reinforcements of the surrounding matrix in flexural test specimens.

(2) The matrix tensile stregnth (σ_m') and pull−out performance of fibers obtained from the analysis of flexural test results were superior to those obtained from direct tension and pull−out tests. This may be attributed to

the positive effect of strain gradient under flexural loads.

(3) Large variations were observed in the values of paramenters(τ_u, σ_m' and S_r) obtained from "System Identification." This could result from both unreliable measurements of flexural deflections in the pre-peak region in some test results reported in the literature, and also from the lack of information on some basic material properites for flexural tests conducted by other investigators.

5. 감사의 글

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