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自己回歸-移動平均모델에 의한 시스템 파라미터 推定

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Estimation Of System Parameters With Arma Model

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抄 錄

自己回歸-移動平均모델에 의하여 시스템의 파라미터를 추정할 수 있는 벡터채널 원형 격자필터(vector channel circular lattice filter)의 알고리즘을 제시하였다. 이 알고리즘은 스칼라 연산만으로 이루어져 계산이 간단한 장점이 있다. 3자유도 시스템의 시뮬레이션 결과로부터 격자필터의 성능을 검증하였으며, 1자유도 팔의 고유진동수와 감쇠비를 추정하였다.

1. Introduction

Recently there have been increasing interests in adaptive identification and control of flexible structures. Adaptive identification is the basis for output prediction algorithms that are used in adaptive control. The recursive least-squares method is used widely for adaptive parameter identification. This method, however, has one serious limitation for identification of flexible structures : it is based on a fixed-order model. Large flexible structures have many, theoretically infinitely many, modes of vibration, of which different numbers may be excited at different times.

A least-squares lattice filter is an algorithm for least-squares parameter estimation that is recursive in both time and order. The order recursive property allows the lattice filter to identify the number of substantially excited modes of a flexible structure as well as the parameters of a digital input/output model. The lattice filter is more efficient than the standard least-squares algorithm for large

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orders, and it is numerically stable. Lee, Morf and Friedlander[1] first derived the time update equations for the lattice and studied its applications in signal processing and control. Montgomery and Sundararajan[2-3] used lattice filters in adaptive identification and control of a flexible beam. Jabbari and Gibson[4-7] presented adaptive parameter identification results for a complex flexible structure with many closely packed natural frequencies.

Pagano[8] has found a one-to-one relationship between multivariate autoregressions an scalar periodic autoregressions, and proposed an estimation method for the former model based on the latter which involves smaller number of parameters. He derived a set of fundamental Yule-Walker type equations for estimating the parameters. Sakai [9] derived a Levinson-type algorithm for solving the YW equations, and showed a circular lattice structure of the process. Sakai et. al.[10-12] modified the algorithm to an adaptive form for on line time and order recursive computation by using the geometric approach. Scalar periodic lattice filters consist of calculations of scalar quantities for analyzing a multivariate autoregressive system, thus completely avoid matrix manipulations accompanying the usual multivariate processing methods.

For single-input single-output (SISO) systems, the input is considered a separate output and, using imbedding, a two-channel lattice is implemented. The ARMA models of SISO systems have many attractive characteristics : The minimal order ARMA for such systems has order n and is unique, and the eigenvalues can be calculated easily. When there are multiple outputs for the system, however, almost all of these desirable properties disappear. The order of the ARMA contains no information about the order of the state space representation of the system. Another problem is the calculation of the system eigenvalues, that is, identifying the extraneous eigenvalues is difficult, if not impossible.

Vector channel lattice filter uses an input/output representation of the form

$$Y(t) + \sum_{j=1}^{n} Y(t-j) A_j = W(t) \qquad (1)$$

where $Y(t)$ is $m \times p$ matrix and A_j's are $p \times p$ matrices. The $W(\cdot)$ is uncorrelated with zero mean matrix. This model has the same uniqueness and minimality properties, and order determination and eigenvalue calculations are performed similarly. Fitting the model to a vector channel lattice is accomplished in the following way. Let there be moutputs and one input. The first channel, then, will be the actual output channel of the measurement vector $[y_1(t) \cdots y_m(t)]^T$. The second channel will be the m-vector $[u(t)\ 0 \cdots 0]^T$. The third channel is $[0\ u(t)\ 0 \cdots 0]^T$, and finally, the $(_{m+1})$ channel becomes $[0 \cdots 0\ u(t)]^T$. We extend the scalar periodic lattice filter to the vector channel case,- and call it vector channel circular lattice filter. Numerical examples are given to show its performance to estimate the natural frequencies of a 3-mass system, and the natural frequency and damping ratio of one degree-of-freedom arm are estimated.

2. Vector Channel Circular Lattice Filter

We say that a process $z(\cdot)$ is a periodic autoregression of period p and order $(n_1,\cdots,n_p)$ if for all integers t,

$$z(t)+\sum_{j=1}^{n_t}\alpha(t,j)z(t-j)=v(t), \qquad (2)$$

where $v(\cdot)$ is uncorrelated with zero mean and $E\{v^2(t)\}=\sigma^2(t)$, $n_t=n_{t+p}$, $\sigma^2(t)=\sigma^2(t+p)$ and $\alpha(t,j)=\alpha(t+p,j)$, $j=1,\cdots,n_t$. We denote the autocovariance of $z(\cdot)$ by $R(s,t)=E\{z(s)z(t)\}$.

Let us consider the following m-variate p-channel nth order AR process

$$Y(t)+\sum_{j=1}^{n}Y(t-j)A_j=W(t), \qquad (3)$$

where Y(t) is m×p matrix and A_j's are p×p matrices. $W(\cdot)$ is uncorrelated with zero mean matrix and $\mathrm{cov}(w_i(t))=W_i$, where $W(t)=[w_1^T w_2^T \cdots w_m^T]^T$. We define Y(t) as

$$Y(t)=\begin{bmatrix} z_1(1+p(t-1)) & z_1(2+p(t-1)) & \cdots z_1(p+p(t-1)) \\ z_2(1+p(t-1)) & z_2(2+p(t-1)) & \cdots z_2(p+p(t-1)) \\ \vdots & \vdots & \vdots \\ z_m(1+p(t-1)) & z_m(2+p(t-1)) & \cdots z_m(p+p(t-1)) \end{bmatrix} \qquad (4)$$

Then, since z_i's have the same coefficients, for $i=1,\cdots,m$,

식 (5), (6), (7)

Thus we get

$$Y(t)L+Y(t-1)A'_1+\cdots+Y(t-n)A'_n=V'(t), \qquad (8)$$

where

식 (9)～(11)

$$V'(t)=\begin{bmatrix} v_1(1+p(t-1)) & v_1(2+p(t-1)) & \cdots v_1(p+p(t-1)) \\ v_2(1+p(t-1)) & v_2(2+p(t-1)) & \cdots v_2(p+p(t-1)) \\ \vdots & \vdots & \vdots \\ v_m(1+p(t-1)) & v_m(2+p(t-1)) & \cdots v_m(p+p(t-1)) \end{bmatrix} \qquad (12)$$

Comparing equation (8) with equation (3), it follows that

$$A_j=A_j'L^{-1} \qquad j=1,\cdots,n$$
$$W(t)=V'(t)L^{-1}, \qquad t=0,\ \pm1,\cdots \qquad (13)$$
$$W_j=(L^{-1})^T D_j L^{-1} \qquad j=1,\cdots,m$$

with

$$L_{kj}=\alpha(j,j-k), \qquad j>k$$
$$\alpha(j,0)=1 \quad \text{and} \quad \alpha(j,k)=0 \quad \text{for } k<0$$
$$A_{r,kj}'=\alpha(j,p^{r+j-k}), \qquad r=1,\cdots,n \qquad (14)$$
$$D_i=\mathrm{diag}(\sigma_i^2(1),\cdots,\sigma_i^2(p))$$

This implies that, if $Y(\cdot)$ and $z_i(\cdot)$ are related by

$$y_{ij}(t)=z_i(j+p(t-1)),\ i=1,\cdots,m\ j=1,\cdots,p \qquad (15)$$

then $Y(\cdot)$ is an AR process of order n with positive definite W_i, $i=1,\cdots,m$, if, and only if, $z_i(\cdot)$ is a periodic autoregression of period p and order $(n_1,\cdots,n_p)$ with positive $\sigma_i(1),\cdots,\sigma_i(p)$ and $n=\max_j[(n_j-j)/p]+1$, where $[x]=k$ for $k\leq x<k+1$. Note that L matrix is a unit upper triangular matrix, and inversion of L is quite easy.

Suppose a set of data $\{Y(1),\cdots,Y(N)\}$ is available. Then we can form a data set $\{z_i(1),\cdots,z_i(N_p)\}$, $i=1,\cdots,m$, and it satisfies the following equations,

$$z_i(t)+\sum_{j=1}^{n_t}\alpha(t,j)z_i(t-j)=v_i(t), \quad i=1,\cdots,m \qquad (16)$$

Multiplying both sides by $z_i(t-q)$ and taking expected values yields,

$$E\{z_i(t)z_i(t-q)\}+\sum_{j=1}^{n_t}=E\{\alpha(t,j)z_i(t-j)z_i(t-q)\}$$
$$=E\{v_i(t)z_i(t-q)\}. \qquad (17)$$

This leads to the following modified Yule-Walker equations for $\alpha(k,j)$,

$$R_i(k,k-q)+\sum_{j=1}^{n_k}\alpha(k,j)R_i(k-j,k-q)=\delta_{q0}\sigma_i^2(k)$$
$$k=1,\cdots,p,\ i=1,\cdots,m,\ q\geq 0. \qquad (18)$$

Adding the above m equations results in

$$\sum_{i=1}^{m}R_i(k,k-q)+\sum_{j=1}^{n_k}\alpha(k,j)\sum_{i=1}^{m}R_i(k-j,k-q)=\delta_{q0}\sum_{i=1}^{m}\sigma_i^2(k),\ k=1,\cdots,p,\ \sum_{i=1}^{m}q\geq 0. \qquad (19)$$

When k-1 becomes 0, from the cyclic

property, the subscript k-1 must be replaced by p.

Define the qth order jth channel forward and backward linear prediction errors for ith measurement by

식 (20), (21)

for $i=1, \cdots, m$, $j=1, \cdots, p$. The predictor coefficients $\alpha(j, q, r)$, $\beta(j, q, r)$, $r=1, \cdots, q$, are determined by minimizing $\sum_i E[\varepsilon^2(i, j+pk, q)]$ and $\sum_i E[\eta^2(i, j+pk, q)]$ with respect to $\alpha(j, q, r)$, $\beta(j, q, r)$, respectively. That is,

식 (22) (23)

follow where

$$\alpha_k(j)=[1 \;\; \alpha(k, j, 1) \;\cdots\; \alpha(k, j, j)]^T, \tag{24}$$

$$\beta_k(j)=[\beta(k, j, j) \;\cdots\; \beta(k, j, 1) \;\; 1]^T, \tag{25}$$

and the (s, t) element of $R(k, j)$ is $\sum_i R_i(k-s+1, k-t+1)$, $1 \leq s, t \leq j+1$.

The order update recursions for $\varepsilon(i, j, q+1)$, $\eta(i, j, q+1)$ can be obtained in the same way as in [12]. In our case it becomes

$$\varepsilon_T(i, j, q+1)=\varepsilon_T(i, j, q)+\alpha_T(j, q+1, q+1)\eta_T(i, j-1, q) \tag{26}$$

$$\eta_T(i, j, q+1)=\eta_T(i, j, q)+\beta_T(j, q+1, q+1)\varepsilon_T(i, j, q) \tag{27}$$

where the subscript T denotes the value evaluated at time T and

식 (28), (29)

Since

$$\sigma_T^2(j, q)=\sum_i E[\varepsilon_T^2(i, j, q)], \tag{30}$$

$$\tau_T^2(j-1, q)=\sum_i E[\eta_T^2(i, j-1, q)], \tag{31}$$

$$\triangle_T(j, q)=\sum_i E[\varepsilon_T(i, j, q)\eta_T(i, j-1, q)], \tag{32}$$

the time update recursions, also, can be given as follows

식 (33), (34), (35)

where μ is the forgetting factor and $0 < \mu \leq 1$. The order update recursion for $\gamma_T(j, q+1)$ becomes

식 (36)

The predictor coefficients $\alpha(k, j+1, r)$, $\beta(k, j+1, r)$ are calculated by

$$\alpha(k, j+1, r)=\alpha(k, j, r)+\alpha(k, j+1, j+1)\beta(k-1, j, j+1-r) \tag{37}$$

$$\beta(k, j+1, r)=\beta(k-1, j, r)+\beta(k, j+1, j+1)\alpha(k, j+1, j+1-r) \tag{38}$$

for $r=1, \cdots, j$

The necessary and sufficient condition for the stability of a lattice filter is[9]

$$0 < \alpha_T(j, q+1, q+1)\beta_T(j, q+1, q+1) < 1 \tag{39}$$

but it is not necessary that $|\alpha_T(j, q+1, q+1)| < 1$ and $|\beta_T(j, q+1, q+1)| < 1$.

3. Numerical Examples

The forced vibrations of a structure are modeled as the response of a finite dimensional linear system. The equations of motion can be given as

$$M\ddot{x}(t)+D\dot{x}(t)\;K\times(t)=Eu(t) \tag{40}$$

where $x(t)$ is the n-dimensional generalized displacement vector and $u(t)$ is the m-dimensional generalized force vector. The mass matrix M, the damping matrix D and the stiffness matrix K are real symmetric n×n matrices with M positive definite and D and K nonnegative. In the structural identification applications with which we are concerned, the input $u(t)$ is constant on sampling intervals, and at the beginning of each sampling interval we have m linear measurements of the state vector.

A description of a 3-mass system will be as in Fig. 1. Their state-space models are given as

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{41}$$

$$y(t)=Cx(t)+w(t) \tag{42}$$

where

$$x(t)=[x_1\ \dot{x}_1\ x_2\ \dot{x}_2\ x_3\ \dot{x}_3]^T \qquad (43)$$

식 (44), (45)

The numerical values for this study are chosen as follows;

$m_1=m_2=m_3=1$

$k_1=k_2=k_3=k_4=100$

$c_1=c_2=c_3=c_4=0.5$

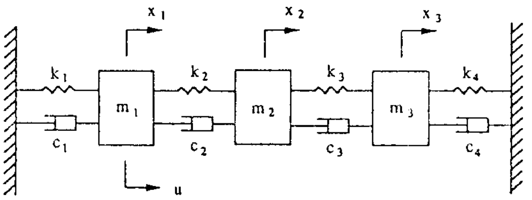

Fig. 1. A 3-mass system

This system has three natural frequencies at 7.654, 14.142, and 18.478 rad/sec. The mass 1 is excited by step input of 200 units. The displacements of masses 1 and 2 are measured every 0.05 seconds (20Hz). The measurements have random noise which has standard deviation of $\sigma_w=0.05$. Fig. 2 shows the estimation results of lattice filter for modal frequencies from the measurement of displacement of mass 1. It is observed that the result converges to the three modal frequencies very quickly. As shown in Fig. 3, measurement of displacement of mass 2 gives good estimate of the modal frequencies except the 2nd flexible mode. It is not observable, since mass 2 is located at a nodal point of the 2nd flexible mode. The estimation result of vector channel circular lattice filter with measurements of masses 1 and 2 is given in Fig. 4. It can estimate the three modal frequencies including the 2nd mode which is not observable with measurement of displacement of mass 2.

Next, we generated a random signal for measurement noise with standard deviation of $\sigma_w=0.1$, and estimated the modal frequencies. Fig. 5, 6, and 7 show estimation results from

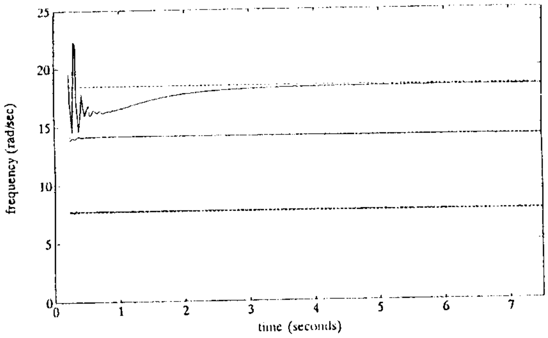

Fig. 2. Estimation result from the displacement of mass 1

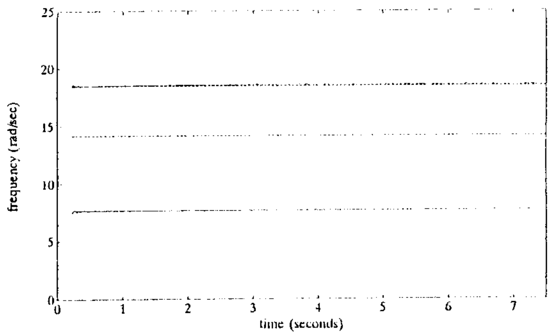

Fig. 3. Estimation result from the displacement of mass 2

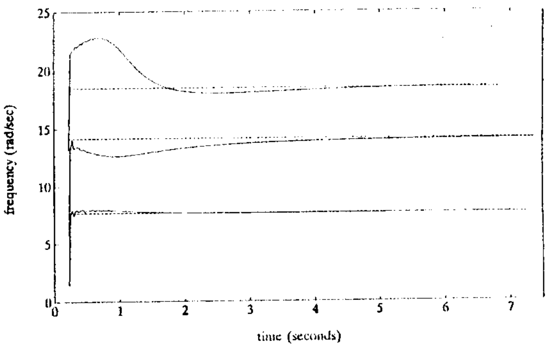

Fig. 4. Estimation result from the displacement of masses 1 and 2

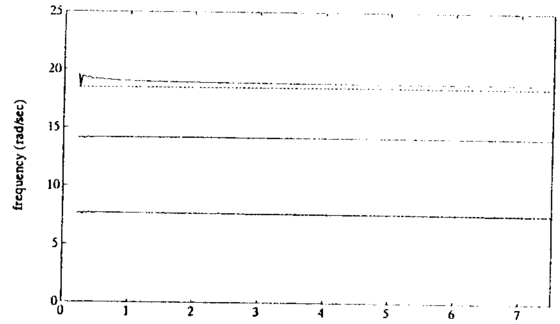

Fig. 5. Estimation result from the displacement of mass 1

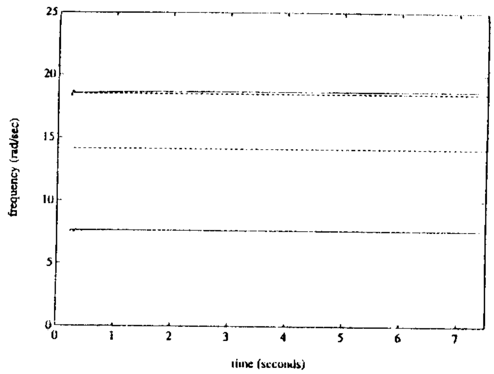

Fig. 6. Estimation result from the displacement of mass 2

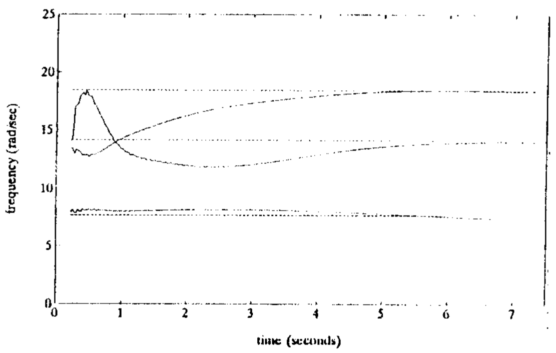

Fig. 7. Estimation result from the displacement of masses 1 and 2

the measurement of displacement of mass 1, mass 2, and masses 1 and 2, respectively. They show similar characteristics with the corresponding results obtained before, with a little slower convergence. But it does not give any problems in practical applications.

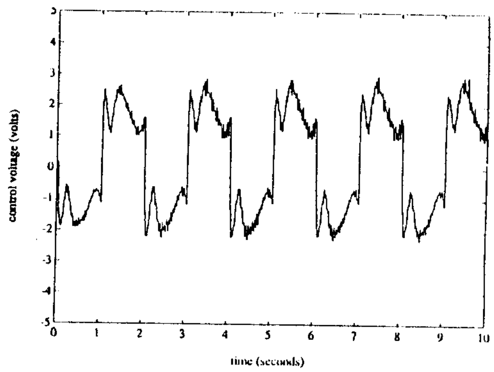

Fig. 8. Control signal of one degree-of-fredom arm

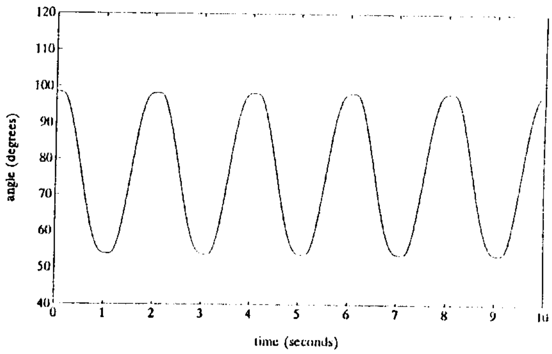

Fig. 9. Output angle of one degree-of-freedom arm

4. Experiments

The experimental one degree-of-freedom arm uses a pneumatic cylinder to drive a rotary joint via cables. A position encoder measures the joint angle, and a differential pressure transducer measures the pressure difference between the two sides of the cylinder. The equation of motion for the arm is

$$J\ddot{\theta} + mgl\cos\theta = Fr \qquad (46)$$

where J is the moment of inertia of the arm about the pivot, mgl $\cos\theta$ is the torque due to gravity, F is the force in the cable, and r is the moment arm. This is a nonlinear equation, and it is linearized to implement a control system. The the coefficients of the linearized equation are either not known or vary during the motion of the arm. It is necessary to identify the system parameters in order to effectively control the arm. The control signal and the output angle of the arm are measured every 0.005 seconds, and are shown in Fig. 8 and Fig. 9, respectively. Identification results for the natural frequency and the damping ratio are shown in Fig. 10 and Fig. 11. The natural frequency of the arm is found to be about 7.61 rad/sec at the vertical position, and the lattice filter gives values around 7.76 rad/sec. Damping ratio is hard to determine, and it is found to be between 0.05 and 0.19. The estimated result from the lattice filter is around 0.1, which is quite acceptable.

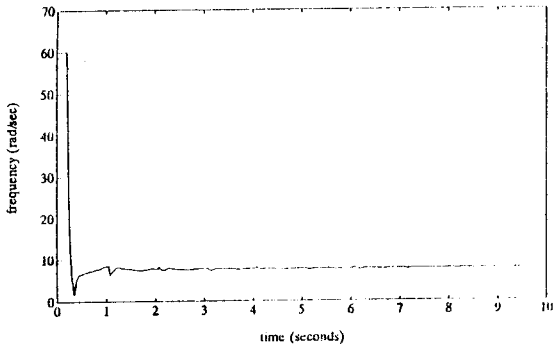

Fig. 10. Estimation result for natural frequency

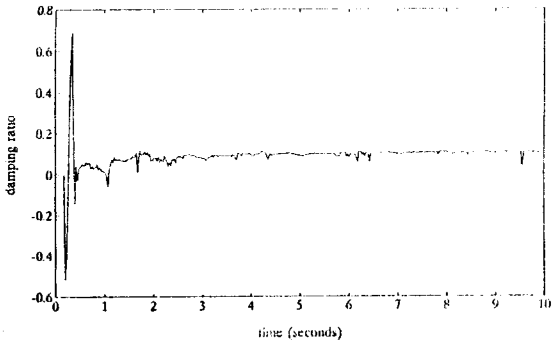

Fig. 11. Estimation result for damping ratio

5. Conclusion

Vector channel circular lattice filter is introduced, which consists of calculations of scalar quantities only. Numerical examples of estimating modal frequencies of 3-mass system show its performance. It shows a good performance for estimating the modal frequencies in the presence of measurement noise. Scalar circular lattice filter converges quickly, and vector channel lattice filter identifies modal frequencies even when some of them is unobservable for one measurement. Experimental results demonstrated the ability of the lattice filter to identify the natural frequency and the damping ratio of one degree-of-freedom arm.

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