# On the Performance of the Generalized Sidelobe Canceller in Coherent Situations

# Coherent 환경에서 Generalized Sidelobe Canceller의 동작

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## ABSTRACT

The convergence rate for the adaptive weights to reach the optimum value in an adaptive array system depends on the eigenvalue spread ratio of the autocovariance matrix. In this paper, we investigate how the eigenvalue spread ratio in a generalized sidelobe canceller (GSC) is affected by the various parameters. Futhemore, this paper derives expressions for the output power of GSC in coherent situations.

## 요 약

적응 어레이 시스템에서 최적해에 도달하기 위한 계수들의 수렴 속도는 공분산 행렬의 고유치 분포율에 의해 지배된다. 본 논문에서는 우선 GSC의 고유치 분포율을 구하였다. 또한 coherent 상황에서 GSC의 출력 파워 식을 유도하였다.

## I. Introduction

An adaptive array system automatically responds to a changing signal environment and improves signal-to-noise ratio (SNR) without prior knowledge of the interference. Frost[1] introduced a temporally adaptive procedure for minimizing output power while linearly constraining the weights to provide a prescribed steering point spectral filtering. As Griffiths and Jim explained in(2), the Generalized Sidelobe Canceller (GSC) can be viewed as an alternate implementation and extension of Frost's algorithm. The main advantage of the GSC is that we can easily use a variety of currently available adaptive multichannel filtering techniques. The convergence rate for the adaptive weights to reach the optimum value in an adaptive array system depends on the eigenvalue spread ratio (i.e., the ratio of the largest to smallest eigenvalue) of the autocovariance matrix. In this paper, we investigate how the eigenvalue spread ratio is affected by the class of the signal blocking matrix processor, the direction and the level of the interference.

Jablon[3] derived the expressions for the GSC Wiener solution, the steady stae output signal-to-interference-plus-noise ratio (SINR) in an incoherent environment,. However, the correlation

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between the desired signal and interference can severely degrade the performance of adaptive array systems, and can also cause a signal cancellation phenomena[4] in the case of narrowhand signals. Correlated sources occur primaril from multiple propagation paths (multipaths) or smart jamming. We derive an expression for the output power of the GSC in this coherent situation. Although only single interference is considered, the results presented could be extended to the case of multiple interferences. The numerical results are included.

## II. Eigenvalue Spread Ratio

Assume that the incident signals are narrowband in nature. The narrowband GSC is shown schematically in Fig.1, consisting of N omnidirectional equispaced elements. This beamformer consists of three parts, such as the conventional beamformer (delay-and-sum), signal blocking processor and adaptive noise canceller. The adaptive noise canceller receives the conventional beamformer output and signal blocking processor output signals as the primary input and the reference input signals, respectively. The weights are updated using the complex LMS (Least Mean Square) algorithm[5] because of its simplicity and efficiency. If we set s(k) as the desired signal, n(k) as the interference, and $\underline{w}$ as a zero mean uncorrelated white Gaussian noise vector of length N, the output vector $\underline{x}(k)$ of the elements at k-th time sample can be expressed as

$$\underline{x}(k) = s(k)\, \underline{1}_N + n(k)\, \underline{a} + \underline{w} \tag{1}$$

$$\underline{a} = [\ \exp(j2\pi f_o \tau_1)\ \exp(j2\pi f_o \tau_2)\ \ldots \exp(j2\pi f_o \tau_N)\ ]^T$$

$$\tau_n = \frac{(n-1)\,d}{f_o\,\lambda}\ (\sin\theta_n - \sin\theta_s) \qquad 1 \le n \le N$$

Here, $f_0$ is a signal center frequency, d is the

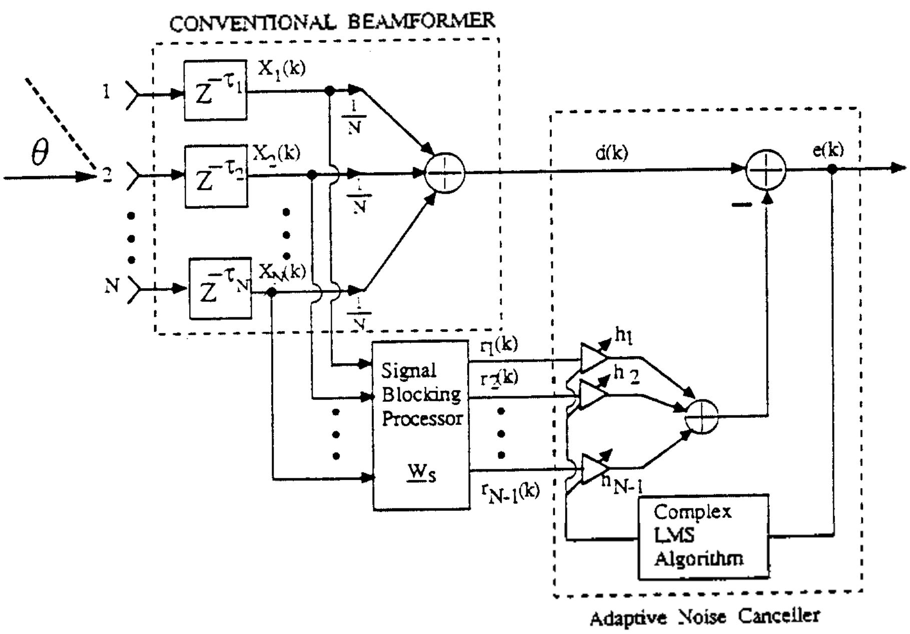


Fig.1. Schematic diagram of a narrowband GSC.

distance between neighboring elements, $\lambda$ is the signal wavelength, and $T_s$ is a sampling interval. And $\theta_s$ and $\theta_n$ are the incident angles of the desired signal and iterference. $\underline{1}_N$ is the all 1's vector of length N. $\tau_n$ is a presteering delay.

The signal blocking processor $\underline{W}_s$ removes the look direction signals using (N-1) by N transformation matrix such that each row is independent of thers and the sums of the row elements are given by zero. The output vector $\underline{r}(k)$ of the signal blocking processor is given by

$$\begin{aligned} \underline{r}(k) &= \underline{W}_S\, \underline{x}(k) \\ &= n(k)\, \underline{W}_S\, \underline{a} + \underline{W}_S\, \underline{w} \end{aligned} \quad (2)$$

The autocovariance matrix $\underline{R}_{rr}$ is given by

$$\begin{aligned} \underline{R}_{rr} &= E[\underline{r}(k)\, \underline{r}(k)^H] \\ &= \sigma_n^2\, \underline{W}_S\, \underline{a}\, \underline{a}^H\, \underline{W}_S^H + \sigma_w^2\, \underline{W}_S\, \underline{W}_S^H \end{aligned} \quad (3)$$

The superscript H denotes Hermitian transpose. $E^{(\cdot)}$ represents expectation. $\sigma_n^2$ and $\sigma_w^2$ are the interference and uncorrelated white noise variance, respectively. The convergence speed of the GSC is dominated by the eigenvalue spread ratio of $\underline{R}_{rr}$. From equation (3), we can know that the convergence rate of GSC is a function of the interference level, incident angle, the uncorrelated noise level, and signal blocking processor

If $\underline{W}_s$ is the orthonormal transformation marix ($\underline{W}_s\, \underline{W}_s^H = I$), then $\underline{R}_{rr}$ has a maximum eigenvalue and the N-2 same minimum eigenvalues.

$$\lambda_1 = (N-1)\, \sigma_n^2 + \sigma_w^2 \quad (4)$$

$$\lambda_j = \sigma_w^2 \qquad j = 2,3,\ldots,N-1 \quad (5)$$

Therefore, the eigenvalue spread ratio is given by

$$\begin{aligned} \frac{\lambda_{max}}{\lambda_{min}} &= \frac{(N-1)\, \sigma_n^2 + \sigma_w^2}{\sigma_w^2} \\ &= (N-1)\frac{\sigma_n^2}{\sigma_w^2} + 1 \end{aligned} \quad (6)$$

In equation(6), we can see that the convergence speed increases as the input interference-to-noise ratio and the number of elements decrease.

## Ⅲ. Output Power in the Coherent Situations

We calculate the Wiener solution to obtain an expression for the output power of the GSC in the coherent situations. Let $\rho$ denotes the correlation coefficient between the desired signal, then the interference is defined as

$$\begin{aligned} \rho &= E[s(k)^*\, n(k)] / \sigma_s\, \sigma_n \\ &= |\rho| \exp(j\phi) \end{aligned} \quad (7)$$

The asterisk denotes the complex conjugate. $\sigma_s^2$ is the desired signal variance. $\phi$ is the correlation phase. When $\rho$ lies on the unit circle, the two sources are fully coherent. If $\rho$ lies inside the unit circle, the two sourcess are only partially correlated, while $\rho=0$ implies uncorrelated sources. The conventional beamformer output d(k) is given by

$$d(k) = s(k) + \frac{1}{N}\, n(k)\, \underline{a}^T\, \underline{1}_N + \frac{1}{N}\, \underline{w}^T\, \underline{1}_N \quad (8)$$

The uncorrelated white noise is uncorrelated the desired signal and the intreference. Thus, the cross-correlation vecotr $\underline{r}_{rd}$ between the desired signal d(k) and the signal blocking processor output vector $\underline{r}(k)$ can be expressed as

$$\underline{r}_{rd} = E[\underline{r}(k)\, d(k)^*]$$
$$= (\sigma_s \sigma_n \rho + \frac{1}{N} \sigma_n^2 \underline{a}^H \underline{1}_N) \underline{W}_S \underline{a}^H \qquad (9)$$

Using the equation (3),(9) and Woodbury's identity[6], the Wiener solution for the GSC, denoted by the vector $\underline{H}_{opt}$ becomes

$$\underline{H}_{opt} = [\, h_1\, h_2 \ldots h_{N-1}\,]^T$$
$$= \underline{R}_{rr}^{-1}\, \underline{r}_{rd}$$
$$= \frac{\sqrt{SNR\ INR}\,\rho + INR\ \underline{a}^H \underline{1}_N}{1 + \beta\ INR} (\underline{W}_S \underline{W}_S^H)^{-1} \underline{W}_S \underline{a}$$
$$= m_o (\underline{W}_S \underline{W}_S^H)^{-1} \underline{W}_S \underline{a} \qquad (10)$$

where $\beta = \underline{a}^H \underline{W}_S^H (\underline{W}_S \underline{W}_S^H)^{-1} \underline{W}_S \underline{a}$

In equation(10), SNR is the input desired signal--to-noise ratio, and INR is the interference-to-noise ratio. Using the Wiener solution we can derive the output power $P_{out}$.

$$P_{out} = \frac{1}{2} E[\,|e(k)|^2\,]$$
$$= \frac{1}{2}\sigma_s^2 + \frac{1}{2}\sigma_s \sigma_n \Big( \frac{1}{N}\rho^* \underline{a}^H \underline{1}_N$$
$$+ \frac{1}{N}\rho\, \underline{a}^T \underline{1}_N - \rho^* m_o \beta - \rho\, m_o^* \beta \Big)$$
$$+ \frac{1}{2N^2} \sigma_n^2 |\underline{a}^T \underline{1}_N|^2 \Big(1 - \frac{N m_o^* \beta}{\underline{a}^T \underline{1}_N}\Big)$$
$$+ \frac{\sigma_w^2}{2N} (1 + N |m_o|^2 \beta) \qquad (11)$$

We can separate the output power into desired, interfering, desired plus interfering, and noise components.

## Ⅳ. Numerical Results

For the results presented in Fig.2 - Fig.5, the four-element linear array of one-half wavelength spacing is used. The desired signal is assumed to be broadside along the array. Although there are many types of signal blocking processors, we have decided upon using the Walsh function($\underline{W}_{S1}$)[7] and the difference function($\underline{W}_{S2}$) in our simulation. For example, the signal blocking processor of the four-element array is as follows.

$$\underline{W}_{S1} = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \qquad (12)$$

$$\underline{W}_{S2} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad (13)$$

Fig.2 shows the eigenvalue spread ratio as a function of the incident angle for two classes of signal blocking porcessor when input INR is 20 dB. The figure shows that the Walsh function has a smaller eigenvalue spread ratio than that of the difference function. This implies that the GSC wit the Walsh function has faster convergence speed than that the with the difference function. The reason is that Walsh function is an orthogonal function. The eigenvalue spread ratio reduces as it approaches the look direction because the maximum eigenvalue decreases due to the removal of the interference by the signal blocking processor at the look direction.

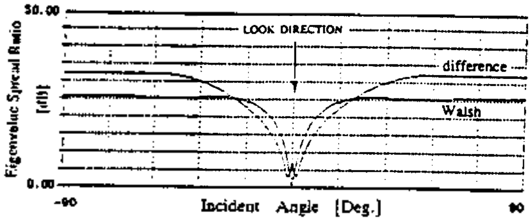

Fig.2. Eigenvalue spread ratio (N=4, input INR=20 dB)

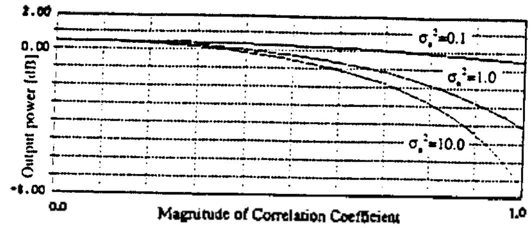

Fig.4. Output power (N=4, $\sigma_s^2$=2, $\sigma_w^2$=1)

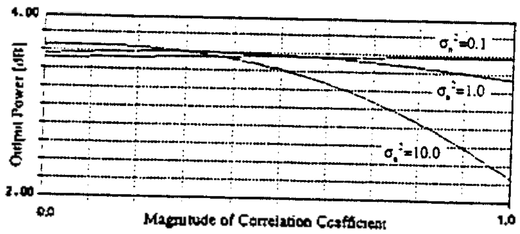

Fig.5. Output power (N=4, $\sigma_s^2$=2, $\sigma_w^2$=10)

Fig.3 shows the relationship between the eigenvalue spread ratio and the uncorrelated white noise level according to the differenct values of inerference levels. As the input INR increases, so does the eigenvalue spread ratio because the maximum eigenvalue is a linear combination of both the interference level and the uncorrelated white noise level.

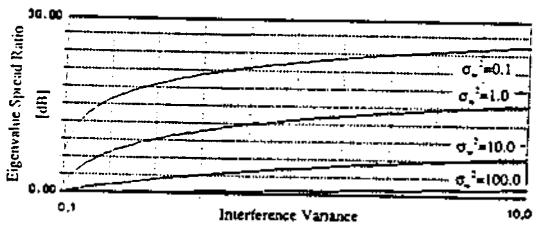

Fig.3. Eigenvalue spread ratio (N=4, Walsh function)

Fig.4 represents the output power as a function of the magnitude of correlation coefficient for various interference levels at a constant correlation phase. Here, the desired signal variance is two, while the uncorrelated white noise has unit variance. Also, the interference incident angle is 45°, and the correlation phase is assumed to be 45°. Fig.4 shows that the signal cancellation increases as the correlation coefficient increases, and also as the interference level increases.

By increasing the uncorrelated white noise level such that $\sigma_w^2$=10.0, the result is as shown in Fig. 5. Because the output power is dominated by the uncorrelated white noise component over the interference component, when the correlation coefficient is less than 0.3, the output power also increases as the level of interference increases. This is a result opposite fo Fig.4.

## V. Conclusions

The paper has shown how the eigenvalue spread ratio in the GSC is affected by the various paramenters. The numerical results show that the eigenvalue spread ratio decreases as the input interference-to-noise ratio and the number of element decrease.

We have derived an expression for the output power of the GSC in the presence of correlated interference. At lower levels of the uncorrelated wite noise, the output power decreases as the magnitude of the correlation and the level of interference increase. However, at high levels of the uncorrelated white noise, the output increases as the the correlation increases.

## ACKNOWLEDGEMENT

This study was supported in part by a research grant from Agency for Defense Development of KOREA.

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▲Il Whan CHA for a photograph and biography, see pp.52 of the December 1990 issue of this journal.

▲Dae Hee YOUN for a photograph and biography, see pp.52 of the December 1990 issue of this journal.