

# 불확실한 로봇 시스템의 제어와 파라미터 추정을 위한 반복 학습제어

## Control and Parameter Estimation of Uncertain Robotic Systems by An Iterative Learning Method

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**Abstract-** An iterative learning control scheme for exact-tracking control and parameter estimation of uncertain robotic system is presented. In the learning control structure, the control input converges globally and asymptotically to the desired input as iteration increases. Since convergence of parameter errors depends only on the persistent exciting condition of system trajectories along the iteration independently of the time-duration of trajectories, it may be achieved with system trajectories with small duration. In addition, the proposed learning control schemes are applicable to time-varying parametric systems as well as time-invariant systems, because the parameter estimation is performed at each fixed time along the iteration. In the parameter estimator, the acceleration information as well as the inversion of estimated inertia matrix are not used at all, which makes the proposed learning control schemes more feasible.

### 1. Introduction

A lot of work in the design of advanced control systems for a class of robotic manipulators has been performed in recent years. The use of an adaptive control system such as a model reference adaptive system is attractive in the sense that it takes care of time varying nature of the robot

manipulation. That is, good performance of manipulator in many of dynamic operating environments can be achieved by adapting the system to payloads/tool changes as well as parameter changes such as aging, etc., In addition, the parametric uncertainties involved in imperfect modeling-link length, mass, inertia, elasticities and backlashes of gear train and frictional nonlinearities etc. -can be coped with via usage of adaptive control systems.

Nevertheless, most of the adaptive control techniques suffer from restrictive assumptions or approximations of one kind or another, i.e., decou-

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pling assumption for joint motions[5], linearization of robot dynamics[18], slow variation of robot model[18, 14].

These assumptions fail immediately if the manipulator moves faster or is equipped with direct drive actuators. In this case, the nonlinear coupling dynamic terms of the manipulator becomes dominant in high-speed operation and these terms should be taken into account in control system designs.

Other model based control schemes which exploit the full nonlinear dynamics have been developed by using hyperstability technique[6, 2]. Major drawbacks of these schemes are : gravity loadings are neglected and a priori knowledge of the range of variations of the robot parameters is required.

More recently, the adaptive schemes based on the cancellation of nonlinear terms have been reported in the literature. These include the works of Craig, Hsu and Sastry([4]), Middleton and Goodwin([9]), and Slotine and  $L_1$ ([15]). Common to these control schemes is a reparametrization technique which is utilized to make the robot dynamics linear with respect to unknown parameters. If the reference trajectory is bounded, these adaptive controllers are known to be globally convergent.

However, these control schemes have disadvantages in that they generally require acceleration measurement and/or filtering techniques and the invertibility of the estimated inertia matrix([4]). Slotine and  $L_1$ [16] relaxed these restrictions and proved the system stability under the ideal model of the robot system. Obviously, there is no guarantee that the same conclusion can be applied to the more practical models which include joint frictions and disturbances.

Though it is widely known in these adaptive schemes that the persistent exciting(*PE*) condition of the system trajectory[4] is required for convergence of the parameter estimation, most of the system trajectories rarely satisfy the *PE* condition mainly because of their finite duration and boundedness. Moreover, most adaptation algorithms being used in robot control techniques assume time invariance of the system parameters. But,

when the system contains time-varying parameters due to slow variation or step changes during the on-line operation, the tracking error as well as parameters do not converge, so that global stability of these controllers may not be ensured.

These shortcomings in a large class of the adaptive control schemes motivate us to introduce the iterative learning control methodology. In this connection, lots of learning control schemes have been reported in the literature. Some of them are simple in that they use only input-output relationships of uncertain system in adjusting the input ([1, 3, 7, 10]). The others use parameter estimation in addition to input adaptation to provide an estimated system model([8, 11, 12]).

The purpose of this paper is to present a new scheme for the iterative learning control of mechanical manipulators which guarantees global stability of the system dynamics without acceleration measurement or estimation. By incorporating a parameter estimator in the domain of iteration sequence, exact-tracking of uncertain systems is achieved for all  $t \in [0, t_f]$ . The parameter estimator in this learning control scheme does not use any system acceleration and inversion of the estimated inertia matrix, so that no introduction of filter and no invertibility assumption of the estimated inertia matrix is needed[9, 4, 16]. In addition, the parameter estimation error will be shown to be bounded for all  $t \in [0, t_f]$  and for all  $j$  of iteration in the parameter estimator.

More significantly, since we do not use the time-invariance of system parameters in the stability proof or in parameter estimation, we expect that this learning control scheme may be effectively applied to the systems with time-varying parameters once the tracking and parameter errors are restricted in stable region at the first iteration. Moreover, if the *PE* condition is satisfied in the domain of iteration sequence for each  $t \in [0, t_f]$ , the convergence of system parameters (of time constant or time-varying) is guaranteed, even when the system trajectories are monotone and/or the duration ( $t_f$ ) of system trajectories is so small that the *PE* condition[4] is not satisfied.

## 2. Notations and Definitions

In the above and in the sequel, by exact-tracking we mean that the system trajectories follow the desired command trajectories precisely for all  $t \in [0, t_f]$  with  $t_f (< \infty)$  being the final time of operation. We will use the notation  $\equiv$  instead of  $\cong$  as defined as.

$R_+$  will denote the set of nonnegative real numbers, and  $R^n$  will represent the  $n$ -dimensional vector space over  $R$  endowed with the Euclidean or  $l_2$  norm

$$|x| = \sqrt{\sum_{i=1}^n x_i^2}$$

For a  $n \times l$  matrix  $B$  over  $R$ ,  $\|B\|$  will be the corresponding induced norm

$$\|B\| = \sqrt{\lambda_{\max}(B^T B)}$$

where  $\lambda_{\max}(\cdot)$  denotes the largest eigenvalue.

It is understood that  $C^2[0, t_f]$  represents a set of at least twice differentiable functions for  $t \in [0, t_f]$ .

In a Lyapunov arguments, we will mean by the error set  $\Omega$  of the stable region that

$$\Omega = \{z, \theta \mid \dot{W}(z, \theta) \leq 0\},$$

where  $\dot{W}$  stands for the time-derivative of a candidate Lyapunov scalar function with  $z \in R^n$ ,  $\theta \in R^l$  of tracking and parameter errors.

The persistent exciting (PE) condition of a matrix function  $Y^j : R_+ \rightarrow R^{n \times l}$  along the iteration  $j$  is defined as follows.

**Definition:** A matrix function  $Y^j : R_+ \rightarrow R^{n \times l}$  is persistently exciting (PE) along the iteration if there exist positive constants  $\alpha_1, \alpha_2$  and a positive integer  $N$  such that

$$\alpha_1 I \leq \sum_{i=j}^{j+N} Y^{iT}(t) Y^i(t) \leq \alpha_2 I$$

for each  $t \in [0, t_f]$ .

### 3. Main Results

Consider a robotic system model of  $n$  rigid bodies, with possible joint frictions, described by the equation at the  $j$ th iteration

$$\Sigma : \begin{aligned} D^j(q^j(t)) \ddot{q}^j(t) + F^j(q^j(t), \dot{q}^j(t)) + T_a = T^j, \end{aligned}$$

where  $D^j \in R^{n \times n}$  is a symmetric positive definite

matrix and  $F^j \in R^n$  torques or forces of centripetal plus Coriolis and gravitational and frictional etc., in the generalized joint coordinates  $q^j \in R^n$ , and  $T^j \in R^n$  a joint torque vector and  $T_a \in R^n$  an unknown but bounded disturbances vector due to unmodeled dynamics.

Suppose that a desired joint trajectory  $q_d(t) \in C_2[0, t_f]$  for each joint is required to be tracked for all  $t \in [0, t_f]$  by the system  $\Sigma$ .

Hence, our control problem is of exact-tracking which will be solved by using a learning control method in the sequel.

In order to achieve exact-tracking via iterative learning, we let at the  $j$ th iteration

$$T^j = E^j + H^j \tag{1}$$

Here,  $E^j$  is defined as follows :

at  $j=1$

$$E^1 = D^1 \ddot{q}_d + F^1 + aD^1 \dot{e}^1 + E\delta_0, \tag{2}$$

at  $j \geq 2$

$$E^j = D_e^j \ddot{q}_d + F_e^j + aD^j \dot{e}^j + E\delta, \tag{3}$$

where

$$\begin{aligned} D_e^j &\equiv D^j(q^j) - D(Dq_d) \\ F_e^j &\equiv F^j(\hat{q}, \dot{q}^j) - F(q_d, \dot{q}_d) \\ E\delta &\equiv L\dot{z} \\ z^j &\equiv \dot{e}^j + ae^j \\ e^j &\equiv q_d - q^j \end{aligned}$$

for a positive constant  $a$ . The symmetric positive definite matrix  $L$  is chosen for all  $t \in [0, t_f]$  such that at  $j=1$

$$D\delta_0 \equiv (2L - \dot{D}^1) > 0,$$

at  $j \geq 2$

$$D\delta \equiv ((2 - \beta)L - \dot{D}^j) > 0$$

for a positive constant  $\beta (0 < \beta < 2)$ . The learning rule for the feedforward input sequence  $\{H^j\}$  is given by

$$H^{j+1} = H^j + \beta E\delta, \tag{4}$$

for  $j=1, 2, \dots$ . As an initial condition, we let  $z^j(0) = 0$  for all  $j$  and  $H^1 = 0$  for all  $t \in [0, t_f]$ .

Then, we have the following results.

**Result 1:** With the set of learning control schemes (2), (3), (4), the system  $\Sigma$  is globally convergent in such a way that for all  $t \in [0, t_f]$  and for all  $j$

- i)  $V^{j+1}(t) \leq V^j(t)$
- ii)  $\lim_{j \rightarrow \infty} e^j(t) = 0$
- iii)  $\lim_{j \rightarrow \infty} \tilde{U}^j(t) = 0$ ,

where the performance index functional  $V^j$  is defined as

$$V^j(t) \equiv \int_0^t \tilde{U}^{jT}(\tau) R \tilde{U}^j(\tau) d\tau$$

$$\tilde{U}^j \equiv T_d - H^j$$

with  $R \equiv L^{-1}$ . Here, the desired input torque  $T_d$  corresponding to the desired trajectories  $\{q_d, \dot{q}_d, \ddot{q}_d\}$  is not known apriori because of the unknown  $T_a$  but can be written in the form

$$T_d \equiv D(q_d) \ddot{q}_d + F(q_d, \dot{q}_d) + T_a \tag{5}$$

**Proof :** Let

$$\tilde{U} \equiv \tilde{U}^{j+1} - \tilde{U}^j \tag{6}$$

Then, from the update law (4) and the definition of  $\tilde{U}^j$ , it becomes

$$\tilde{U} = -\beta E \delta \tag{7}$$

Applying the control input (1) to the system  $\Sigma$  yields at the first iteration and at  $j \geq 2$

$$D^1 \dot{z}^1 + Lz^1 = T_a$$

$$D^j \dot{z}^j + Lz^j = \tilde{U}^j \tag{8}$$

At  $j=1$ , let a candidate Lyapunov function be

$$W(t) \equiv \frac{1}{2} \dot{z}^T D^1 z^1$$

Differentiating  $W(t)$  along the error system (8), we obtain

$$\dot{W}(t) \leq -\frac{1}{2} \dot{z}^T (D \delta z^1 - 2T_a)$$

Hence, the system remains stable in the error set  $\mathcal{Q}$  for all  $t \in [0, t_f]$ .

At  $j \geq 2$ , if we let

$$\Delta V \equiv V^{j+1} - V^j$$

then from (6), (7) and the error equation (8)

$$\Delta V = \int_0^t (\tilde{U}^T R \tilde{U} + 2\tilde{U}^T R \tilde{U}^j) d\tau$$

$$= \int_0^t (\beta^2 z^{jT} L z^j - 2\beta z^{jT} (D^j \dot{z}^j + Lz^j)) d\tau$$

Integration by part yields

$$\Delta V = -\beta z^{jT} D^j z^j - \int_0^t (\beta z^{jT} D \delta z^j) d\tau \leq 0$$

Hence, i) follows for all  $t \in [0, t_f]$  and the equality holds only when  $z^j = 0$ . Further, since the bounded and monotone sequence  $\{V^j\}$  converges to a fixed value,  $\Delta V \rightarrow 0$  as  $j \rightarrow \infty$  implying  $z^j \rightarrow 0$  such that

$$\Delta V \leq -\beta z^{jT} D^j z^j \leq 0$$

In view of the definition of  $z^j$  (an input to a strictly stable first-order filter), this implies ii). Finally, since  $z^j \rightarrow 0$  for all  $t \in [0, t_f]$  means that  $\dot{z}^j \rightarrow 0$  for all  $t \in [0, t_f]$ , the feedforward input sequence  $\{H_j\}$  converges to  $T_d$  for all  $t \in [0, t_f]$  in the equation (8). Hence, iii) follows to complete the proof.  $\Delta \Delta \Delta$

**Remark 1 :**

1) Note that  $T_d$  is bounded for the bounded signal  $\dot{q}_d$  and bounded disturbance  $T_a$  for all  $t \in [0, t_f]$ . With this fact, i) implies that  $V^j$  is bounded for all  $t \in [0, t_f]$

$$V^j(t) \leq V^1 = \int_0^t T_d^T R T_d dt < \infty$$

2) Without  $T_a$ , the system  $\Sigma$  will be globally stable in the sense of Lyapunov at  $j=1$ .

3) In the stability proof, we have implicitly assumed that  $T_a$  is periodic at each iteration.

When the dynamical system  $\Sigma$  contains uncertainty in system parameters, the error input torque  $E_j$  is modified so that estimated parameters can be used in the learning controller. That is,

at  $j=1$

$$E^1 = \tilde{D}^1 \ddot{q}_d + \tilde{F}^1 + a \tilde{D}^1 \dot{e}^1 + E \delta_0 \tag{9}$$

at  $j \geq 2$

$$E^j = \tilde{D}_e^j \ddot{q}_d + \tilde{F}_e^j + a \tilde{D}^j \dot{e}^j + E \delta \tag{10}$$

where  $(\hat{\cdot})$  denotes an estimated system such that

$$\tilde{D}_e^j \equiv \tilde{D}^j(q^j) - \hat{D}(q_d)$$

$$\tilde{F}_e^j \equiv \tilde{F}^j(q^j, \dot{q}^j) - \hat{F}(q_d, \dot{q}_d)$$

Then, the error equation will be

$$D^1 \dot{z}^1 + Lz^1 = \tilde{D}^1 \ddot{q}_d + \tilde{F}^1 + a \tilde{D}^1 \dot{e}^1 + T_a$$

$$D^j \dot{z}^j + Lz^j = \tilde{D}_e^j \ddot{q}_d + \tilde{F}_e^j + a \tilde{D}^j \dot{e}^j + \tilde{U}^j \tag{11}$$

where

$$\tilde{D}^j \equiv D^j - \hat{D}^j$$

$$\tilde{F}^j \equiv F^j - \hat{F}^j$$

$$\begin{aligned} \tilde{D}_e^j &\equiv D_e^j - \hat{D}_e^j \\ \tilde{F}_e^j &\equiv F_e^j - \hat{F}_e^j \end{aligned}$$

By reparametrizing the error system, we obtain

$$\begin{aligned} D^1 \dot{z}^1 + Lz^1 &= Y^1(q^1, \dot{q}^1, \dot{q}_a, \ddot{q}_a) \theta^1 + T_a \\ D^j \dot{z}^j + Lz^j &= Y^j(q^j, \dot{q}^j, q_a, \dot{q}_a, \ddot{q}_a) \theta^j \\ &\quad + \tilde{U}^j, \end{aligned} \tag{12}$$

where  $Y^j \in R^{n \times l}$  denotes a regression matrix and  $\theta^j \in R^l$  a parameter error vector such that

$$\theta^j \equiv \theta - \theta^j$$

for the uncertain system parameter vector and its estimate  $\theta^j$ . Let the parameter vector  $\theta$  be estimated in such a way that for all  $t \in [0, t_r]$  at  $j=1$  and at  $j \geq 2$

$$\begin{aligned} \hat{\theta}^1 &= \beta^1 Y^{1T} z^1 \\ \hat{\theta}^{j+1} &= \hat{\theta}^j + \beta S^{-1} Y^{jT} z^j \end{aligned} \tag{13}$$

where we set the normalizing gain matrix  $S$  properly so that the feedback gain matrix  $L$  to be chosen below may be not too large. For example,  $S$  may be given such as for a positive constant  $\lambda$

$$S = \lambda I + Y^{1T} Y^1.$$

Then, by choosing a symmetric positive definite matrix  $L$  so that it satisfies

at  $j=1$

$$D_0^1 \equiv (2L - \dot{D}^1) > 0,$$

at  $j \geq 2$

$$D_0^j \equiv ((2 - \beta)L - \dot{D}^j - \beta Y^j S^{-1} Y^{jT}) > 0$$

for all  $t \in [0, t_r]$ , we again obtain a convergent learning control scheme with parameter estimation.

**Results 2:** Suppose that the iterative learning scheme consists of the learning control input(1) and an update law(4) for the feedforward input and the parameter estimator(13). Then the uncertain system  $\Sigma$  is globally convergent in the sense that

- i)  $V^{j+1}(t) \leq V^j(t)$
- ii)  $\lim_{j \rightarrow \infty} e^j(t) = 0$
- iii)  $\lim_{j \rightarrow \infty} \tilde{U}^j(t) = 0,$

where

$$V^j(t) \equiv \int_0^t (\tilde{U}^{jT}(\tau) R \tilde{U}^j(\tau) + \theta^{jT} S \theta^j) d\tau.$$

for all  $t \in [0, t_r]$  and all  $j$ .

**Proof:** At  $j=1$ , note that  $V^1$  is bounded for the bounded  $\ddot{q}_a$  and  $\theta^1$  for all  $t \in [0, t_r]$ . Actually, let a candidate Lyapunov function be

$$W(t) \equiv \frac{1}{2} (z^{1T} D^1 z^1 + \frac{1}{\beta^1} \theta^{1T} \theta^1)$$

for a positive constant  $\beta^1$ . Differentiating  $W(t)$  along the error system(12) and the parameter estimator(13) leads to

$$\dot{W}(t) \leq -\frac{1}{2} z^{1T} (D_0^1 z^1 - 2T_a)$$

implying that the system remains stable in the region of the error set  $\Omega$  for all  $t \in [0, t_r]$ .

At  $j \geq 2$ , let

$$\theta \equiv \theta^{j+1} - \theta^j. \tag{14}$$

Then, it becomes from(13)

$$\theta = -\beta S^{-1} Y^{jT} z^j. \tag{15}$$

If we define

$$\Delta V \equiv V^{j+1} - V^j,$$

then from the error equation(12) and parameter estimator(14, 15), we obtain

$$\begin{aligned} \Delta V &= \int_0^t (\tilde{U}^T R \tilde{U} + 2\tilde{U}^T R \tilde{U}^j + \theta^T S \theta \\ &\quad + 2\theta^T S \theta^j) d\tau = \int_0^t (\beta^2 z^{jT} L z^{jT} L z^j \\ &\quad - 2\beta z^{jT} (D^j \dot{z}^j + Lz^j - Y^j \theta^j)) d\tau \\ &\quad + \int_0^t (\beta^2 z^{jT} (Y^j S^{-1} Y^{jT}) z^j \\ &\quad - 2\beta z^{jT} Y^j \theta^j) d\tau = \int_0^t (\beta^2 z^{jT} L z^j \\ &\quad - 2\beta z^{jT} (D^j \dot{z}^j + Lz^j)) d\tau + \int_0^t \\ &\quad \beta^2 z^{jT} (Y^j S^{-1} Y^{jT}) z^j d\tau. \end{aligned}$$

Interating by part yields

$$\Delta V = -\beta z^{jT} D^j z^j - \int_0^t (\beta z^{jT} D_0^j z^j) d\tau \leq 0.$$

Following a similar reasoning as in the proof of Result 1 and using the fact that ii) implies  $Y^j \rightarrow 0$  as  $j \rightarrow \infty$  for all  $t \in [0, t_r]$  completes the proof. **Remark 2:** Notice that in the learning control scheme above,

1) acceleration mesurement as well as an assumption of invertibility of the estimated inertia matrix are not required at all.

2) Exact-tracking and feedforward input learning are achieved simultaneously, so that an inverse dynamics control may be applied after perfect learning.

3) In addition, since we do not use time-invariance of the unknown system parameters except at the first iteration, we expect that the learning control scheme may be applied to an uncertain time-varying parameteric system by restricting the tracking and parameter errors in the error set  $\Omega$  of the stable region at the first iteration.

In Result 2, i) implies that the parameter errors of learning controller remain bounded for all  $j$  and all  $t \in [0, t_f]$ . In order for the parameter estimation to converge in the learning controller, persistent excitation of system trajectories will be required along with iteration number  $j$ .

Actually, as for convergence of parameter errors for the systems  $\Sigma$ , we have the following result.

**Result 3:** Let  $Y^j : R_+ \rightarrow R^{n \times l}$  be PE along the iteration. Then, the parameter estimation by using the estimators(13) for the systems  $\Sigma$  will be globally asymptotically convergent in the learnign controller. That is,

$$\lim_{j \rightarrow \infty} \theta^j = 0$$

for all  $t \in [0, t_f]$ .

**Proof:** First, note that the parameter estimator (13) can be written in the form

$$\theta^{j+n} = \theta^j - \beta S^{-1} Y^{jT} z^j. \tag{16}$$

Applying the recursive relation in the parameter estimator, we obtain for  $1 \leq n \leq N$

$$\theta^{j+n} = \theta^{j+N+1} + \beta \sum_{i=j+n}^{j+N} S^{-1} Y^{iT} z^i. \tag{17}$$

Further, multiplying  $Y^{j+n-1}$  on both sides of the parameter equation(16) at the  $(j+n-1)$ th iteration yields

$$Y^{(j+n-1)} \theta^{(j+n)} = Y^{(j+n-1)} \theta^{(j+n-1)} - \beta Y^{(j+n-1)} S^{-1} Y^{(j+n-1)T} z^{(j+n-1)} \tag{18}$$

for  $1 \leq n \leq N+1$ .

Now, let for  $1 \leq n \leq N+1$

$$S_N^{n-1} \equiv Y^{(j+n-1)} \theta^{(j+n-1)} - \beta \sum_{i=j+n-1}^{j+N} S^{-1} Y^{iT} z^i.$$

Then, it is obvious from the convergence results that for each  $t \in [0, t_f]$

$$\lim_{j \rightarrow \infty} S_N^{n-1} = 0 \tag{19}$$

for  $1 \leq n \leq N+1$ . In view of (17) and (18), it becomes for  $1 \leq n \leq N+1$

$$S_N^{n-1} = Y^{(j+n-1)} \theta^{(j+N+1)}. \tag{20}$$

Let  $S_N$  be a finite series of  $N+1$  scalar terms such that

$$S_N \equiv \sum_{n=1}^{N+1} S_N^{(n-1)T} S_N^{(n-1)}.$$

Then, from (20), we have

$$S_N = \theta^{(j+N+1)T} \sum_{i=j}^{j+N} (Y^{iT} Y^i) \theta^{(j+N+1)}. \tag{21}$$

Applying the PE condition of  $Y^j$  to the equation (21), it becomes

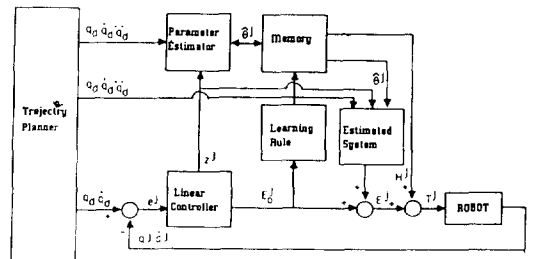
$$0 \leq \alpha_1 \theta^{(j+N+1)T} \theta^{(j+N+1)} \leq S_N \leq \alpha_2 \theta^{(j+N+1)T} \theta^{(j+N+1)}. \tag{22}$$

Combining this inequality with the fact(19), we have the desired result.  $\Delta\Delta\Delta$

**Remark 3:**

1) Result 3 implies that time-varying parameters as well as time-invariant ones converge in the learning controller in the domain of the iteration sequence.

2) Note also that since the PE condition for an ordinary adaptive system[4] is rarely satisfied due to the finite duration( $t_f$ ) of the system trajectories, the convergence of parameter estimation is not



**Fig. 1** Schematic Diagram of the Parameter Adaptive Learnign Condroller

guaranteed with short duration and/or monotone system trajectories. On the contrary, in the domain of the iteration sequence, the convergence of parameter estimation of uncertain systems is guaranteed and independent of the length of system trajectories once the PE condition is assumed. Hence, it is expected that PE condition in the domain of the iteration sequence is much more likely to be satisfied than in the time domain.

### 4. Simulation Results

The feasibility of the learning control scheme is tested with a simple two degree of freedom manipulator.

In the following,  $m_i, I_i, l_i, l_{ci}(i=1, 2)$  represent the mass, inertia (about  $z$ -axis out of the page passing through the center of mass), length, and distance from the previous joint to the center of mass.  $q_i$  denotes the joint angle for  $i=1, 2$ .

The dynamic motion of this manipulator is described by differential equations  $\Sigma$  with the following entries ([17])

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$F = [f_1 \ f_2]^T$$

$$T_a = [T_{a1} \ T_{a2}]^T$$

$$T = [T_1 \ T_2 \ \dots \ T_{12} \ T_{13} \ T_{14} \ T_{15} \ T_{16} \ T_{17} \ T_{18} \ T_{19} \ T_{20} \ T_{21} \ T_{22} \ T_{23} \ T_{24} \ T_{25} \ T_{26} \ T_{27} \ T_{28} \ T_{29} \ T_{30}]^T$$

$$d_{12} \equiv d_{21} = m_2(l_{c2}^2 + l_1 l_{c2} c_2) + I_2$$

$$d_{22} \equiv m_2 l_{c2}^2 + I_2$$

$$f_1 \equiv -m_2 l_1 l_{c2} s_2 \dot{q}_1 - m_2 l_1 l_{c2} s_2 (\dot{q}_1 + \dot{q}_2) + (m_1 l_{c1} + m_2 l_1) g c_1 + m_2 l_{c2} g + k_1 \dot{q}_1 + p_1 \text{sign}(\dot{q}_1)$$

$$f_2 \equiv m_2 l_1 l_{c2} s_2 \dot{q}_1 + m_2 l_{c2} c_{12} g + k_2 \dot{q}_2 + p_2 \text{sign}(\dot{q}_1)$$

where  $c_i \equiv \cos(q_i)$ ,  $s_i \equiv \sin(q_i)$  and  $c_{ik} \equiv \cos(q_i + q_k)$ .  $g$  denotes the gravity acceleration constant and  $k_i, p_i$  stand for friction coefficients (kviscous and coulomb).

Defining parameters  $\theta_1, \dots, \theta_{13}$  as

$$\theta_1 \equiv m_1 l_{c1}^2, \theta_2 \equiv m_2 l_1^2, \theta_3 \equiv m_2 l_{c2}^2,$$

$$\theta_4 \equiv m_2 l_1 l_{c2}$$

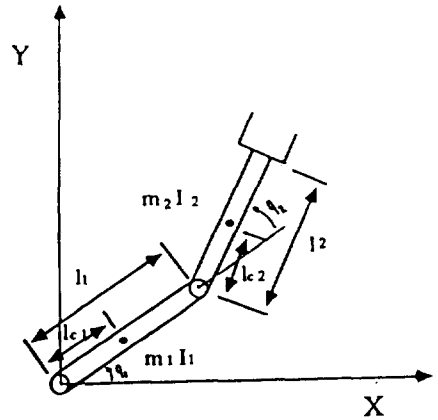


Fig. 2 Schematic Diagram of the Manipulator used in Dynamic Computer Simulation

where  $y_{ik}$  is defined as at  $j=1$

$$y_{11} = \ddot{q}_1 + a \dot{e}_1, y_{12} = y_{11}$$

$$y_{13} = \ddot{q}_1 + a \dot{e}_1 + \ddot{q}_2 + a \dot{e}_2$$

$$y_{14} = c_2(y_{11} + y_{13}) - s_2(2\dot{q}_1 + \dot{q}_2)\dot{q}_2$$

$$y_{15} = y_{11}, y_{12} = y_{13}, y_{17} = c_1, y_{18}$$

$$= c_1, y_{19} = c_{12}$$

$$y_{1,10} = \dot{q}_1, y_{1,11} = 0.0, y_{1,12} = \text{sign}(\dot{q}_1),$$

$$y_{1,13} = 0.0$$

$$y_{21} = 0.0, y_{22} = 0.0, y_{23} = y_{13}, y_{29}$$

$$= c_2 y_{11} + s_2 \dot{q}_1^2$$

$$y_{25} = 0.0, y_{26} = y_{15}, y_{27} = 0.0, y_{28}$$

$$= 0.0, y_{29} = c_{12}$$

$$= 0.0, y_{29} = c_{12}$$

$$y_{2,10} = 0.0, y_{2,11} = \dot{q}_2, y_{2,12}$$

$$= 0.0, y_{2,13} = \text{sign}(\dot{q}_2),$$

and at  $j \geq 2$

$$y_{11} = a \dot{e}_1, y_{12} = y_{11}, y_{13} = a(\dot{e}_1 + \dot{e}_2)$$

$$y_{14} = (c_2 - c_{a2})(2\ddot{q}_1 + \ddot{q}_2) + c_2(y_{11} + y_{13})$$

$$- 2(s_2 \dot{q}_1 \dot{q}_2 - s_{a2} \dot{q}_1 \dot{q}_2) - (s_2 \dot{q}_2^2 - s_{a2} \dot{q}_2^2)$$

$$y_{15} = y_{11}, y_{16} = y_{13}, y_{17} = c_1 - c_{d1}, y_{18} = y_{17}$$

$$y_{19} = c_{12} - c_{d12}, y_{1,10} = \dot{e}_1, y_{1,11} = 0.0$$

$$y_{1,12} = \text{sign}(\dot{q}_1) - \text{sign}(\dot{q}_{d1}), y_{1,13} = 0.0$$

$$y_{21} = 0.0, y_{22} = 0.0, y_{23} = y_{13}$$

$$y_{24} = (c_2 - c_{d2})\ddot{q}_1 + c_2(a \dot{e}_1)$$

$$+ (s_2 \dot{q}_1^2 - s_{d2} \dot{q}_1^2)$$

$$y_{25} = 0.0, y_{26} = y_{16}, y_{27} = 0.0, y_{28}$$

$$= 0.0, y_{29} = y_{19}$$

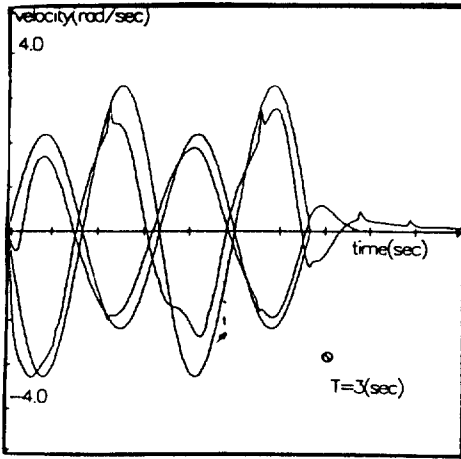


Fig. 3 Trajectories of the system at the first iteration

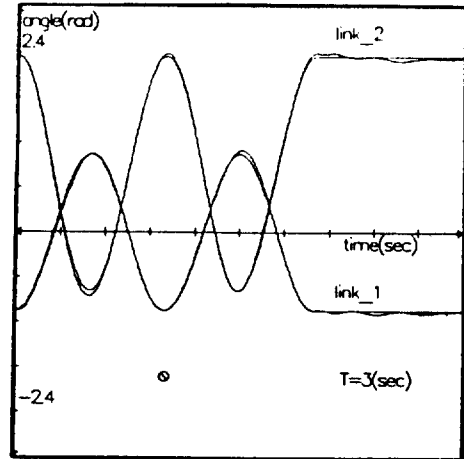


Fig. 5 Trajectories of the system after the 10th iteration

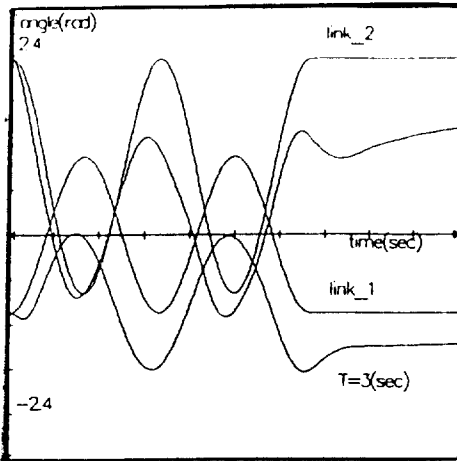


Fig. 4 Velocity trajectories of the system at the first iteration

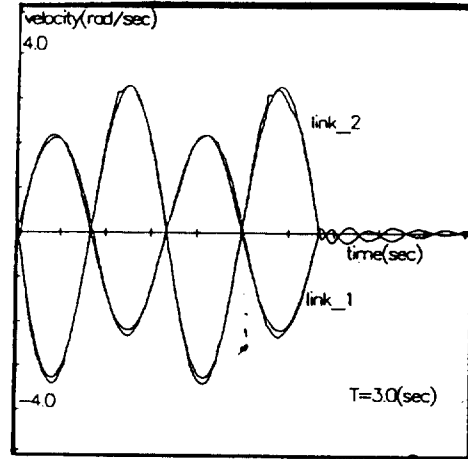


Fig. 6 Velocity trajectories of the system after the 10th iteration

As an example, let's assume  $m_1 = m_2 = 10.0 \text{Kg}$ ,  $l_1 = l_2 = 1.0 \text{m}$ ,  $l_{c1} = l_{c2} = 0.5 \text{m}$ . The link inertia  $I_1 = I_2 = 0.83 \text{Kg m}^2$  and friction coefficients  $k_i = 1.0$ ,  $p_i = 4.0$ . For the desired joint trajectories, we choose

$$\begin{bmatrix} q_{a1}(t) \\ q_{a2}(t) \end{bmatrix} = \begin{bmatrix} -\frac{\pi}{3} \cos\left(\frac{2\pi}{3}t\right) \\ \frac{\pi}{2} \cos\left(\frac{2\pi}{3}t\right) + \frac{\pi}{4} \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_{a1}(t) \\ \dot{q}_{a2}(t) \end{bmatrix} = \begin{bmatrix} -\frac{2\pi^2}{9} \sin\left(\frac{2\pi}{3}t\right) \\ -\frac{\pi^2}{3} \sin\left(\frac{2\pi}{3}t\right) \end{bmatrix}$$

$$\begin{bmatrix} \ddot{q}_{a1}(t) \\ \ddot{q}_{a2}(t) \end{bmatrix} = \begin{bmatrix} -\frac{4\pi^3}{27} \cos\left(\frac{2\pi}{3}t\right) \\ -\frac{2\pi^3}{9} \cos\left(\frac{2\pi}{3}t\right) \end{bmatrix}$$

for  $t \in [0, 6]$ .  $T_a$  represents biased disturbances, whose components are :

$$T_{a1} = 20 + 3.0 \sin(3000.0t)$$

$$T_{a2} = 20 + 2.0 \sin(3000.0t)$$

This model may be considered as a description of biased nonlinear characteristic of actuator drives (for example, actuator input drift).



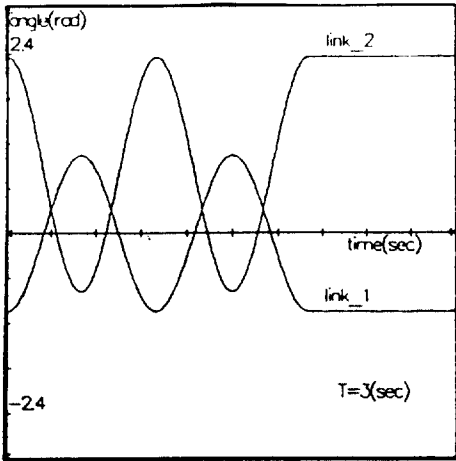


Fig. 7 Trajectories of the system after the 50th iteration

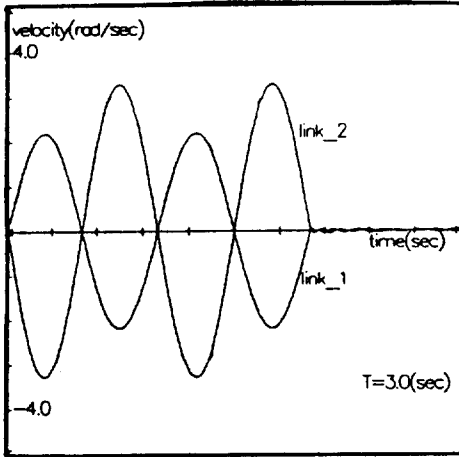


Fig. 8 Velocity trajectories of the system after the 50th iteration

The feedback gains were set  $a=3.0$  and

$$L = \begin{bmatrix} 30 & 0 \\ 0 & 20 \end{bmatrix}$$

We also assume the training factor  $\beta^1 = \beta = 0.6$  for each joint and sampling period is 5.0 (ms). In the parameter estimator, the gain matrix was chosen simply as  $S = I$ . As a physical constraint, we have assumed that actuator input torques were limited with maximum value of 250 (Nm) for joint 1 and 220 (Nm) for joint 2, respectively.

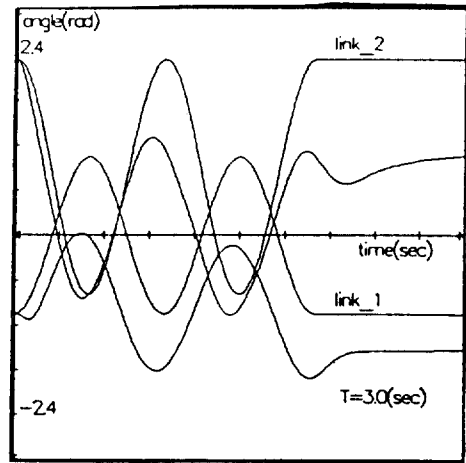


Fig. 9 Trajectories of the system at the first iteration with payload change during on-line operation

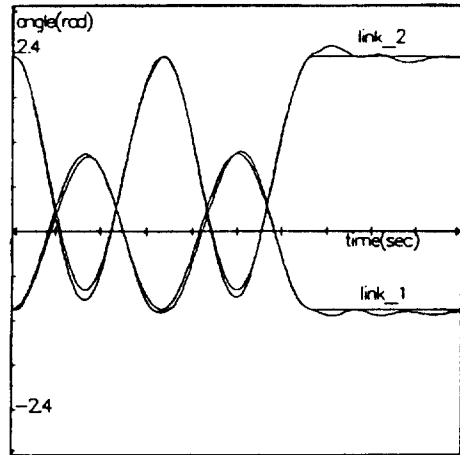
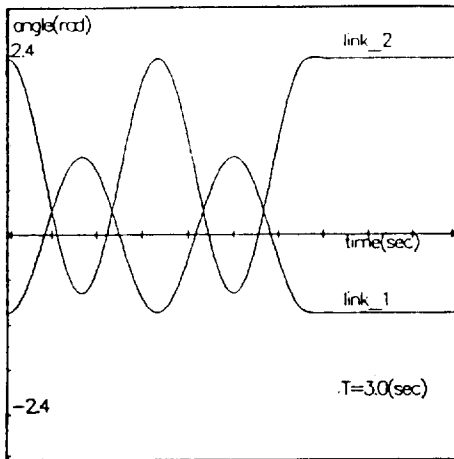
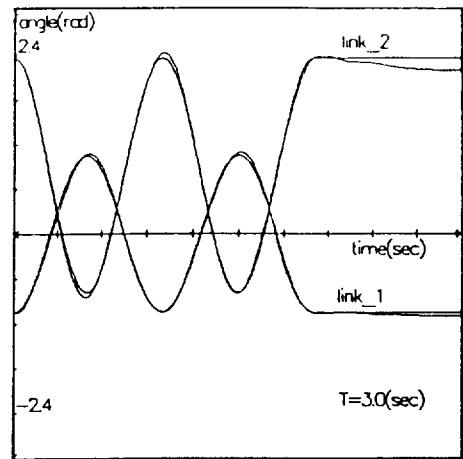


Fig. 10 Trajectories of the system after the 10th iteration with payload change during on-line operation

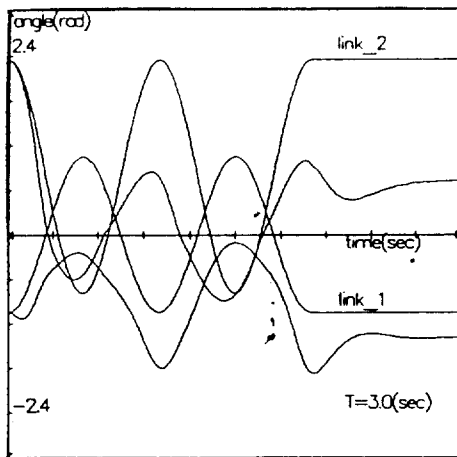
First, we considered in the simulation the case where there is no disturbances or payload changes during the on-line operation of tracking so that parameters may be considered as time-invariant for all  $t \in [0, 6]$ . Figure(3), (4) show the actual trajectories of the system without payload along with the desired trajectories when the learning control input does not activated (i.e.,  $H^1 = 0.0$  at  $j=1$ ). Figure (5)-(8) show that the trajectories of



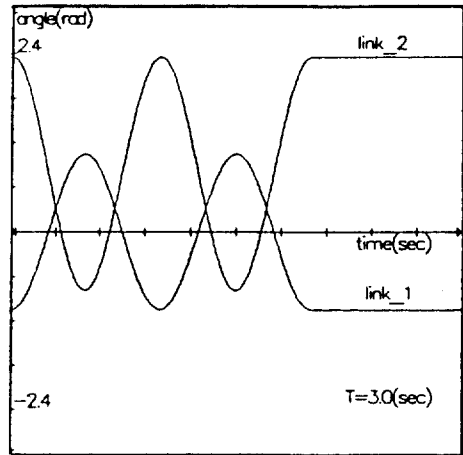
**Fig. 11** Trajectories of the system after the 70th iteration with payload change during on-line operation



**Fig. 13** Trajectories of the system after the 10th iteration with bounded disturbances



**Fig. 12** Trajectories of the system at the first iteration with bounded disturbances



**Fig. 14** Trajectories of the system after the 70th iteration with bounded disturbances

the same system converges to the desired ones as the learning proceeds.

Second, In order to observe the behaviour of the learning controller when there exists a step change in the system parameters due to increase of payload during the on-line tracking operation, a payload (5.0Kg) was picked up at  $t=4.0(\text{sec})$ . This gives rise to a time-varying mode to the parameter values (variation of the center of mass etc.) so that the usual adaptive control methods as well as classical controllers can not keep system

trajectory deviation from increasing significantly. Figure (9)-(11) show that the actual trajectories of the system as the learning control mechanism proceeds. In this case, convergence occurs more slowly due to step change in parameter values due to payload pick-up.

Third, we simulated the case when there exist substantial disturbances ( $T_1 \neq 0$ ) in the system dynamics, so that we may check disturbance rejection capability of the learning controller. In Figures (12)-(14), one may observe that the learning

control action achieves disturbance rejection even though it is not known.

### 5. Concluding Remarks

A new iterative learning control scheme for a class of uncertain robotic systems is presented in this paper. The main feature of the control scheme is that exact-tracking and feedforward input learning can be achieved simultaneously, so that an inverse dynamics control can be achieved after perfect learning. It is also expected that estimation of parameters of time-varying as well as time invariant uncertain systems will converge with only short duration system trajectories, provided that the trajectories are persistently exciting in the iteration sequence domain. Moreover, the acceleration terms are not used in the proposed control scheme, which makes it more feasible to practical applications.

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