論 文
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Mixture of Cumulants Approximation법에 의한 발전 시뮬레이션에 관한 연구

A Study on the Probabilistic Production Cost Simulation by the Mixture of Cumulants Approximation

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요 약

전력계통에 있어서 발전시뮬레이션은 계통확충계획 및 운용계획에 있어서 중요한 역할을 하고 있다. 더욱이 계통의 규모가 성장하고, 구성이 복잡해짐에 따라 발전비용 및 여러가지 신뢰도지수를 제공하는 발전시뮬레이션에 있어서 계산의 신속성 및 결과의 정확성이 더욱 요구되어 진다. 본 논문에서는 Approximate법의 대표적인 방법인 Cumulant법이 계산소요시간은 빠르나, 계통의 상태에 따라 부정확한 결과가 얻어지므로 이러한 단점을 개선하기 위해 계통부하의 multi-modal 특성을 고려하고, 발전기 사고용량확률분포를 발전기 고장대수에 따른 Subest로 구성하여 이들을 각각 상승적 분합으로써 계통의 등가부하지속곡선을 구하는 Mixture of Cumulants Approximation방법을 제안하였으며, 이를 IEEE RTS, EPRI-D 및 TEXAS계통에 적용하여 그 결과를 Booth-Baleriaux법 및 Cumulant법과 비교하여 본 논문에서 제안한 Miture of Cumulants Approximation법의 유용성을 검증하였다.

Abstract- This paper describes a new method of calculating expected energy generation and loss of load probability (L.O.L.P) for electric power system operation and expansion planning. The method represents an equivalent load duration curve (E.L.D.C) as a mixture of comulants approximation (M.O.C.A), which is the general case of mixture of normals approximation (M.O.N.A). By regarding a load distribution as many normal distributions-rather than one normal distribution-and representing each of them in terms of Gram-Charlier expansion, we could improve the accuracy of results. We developed an algorithm which automatically determines the number of distribution and demarcation points. In modeling of a supply system, we made subsets of generators according to the number of generator outage: since the calculation of each subset's moment needs to be processed rapidly, we further developed specific recursive formulae. The method is applied to the test systems and the results are compared with those of cumulant, M.O.N.A. and Booth-Baleriaux method. It is verified that the M.O.C.A method is faster and more accurate than any other method.

1. Introduction

The more complex power systems grow, the more important an accurate representation of probabilistic production cost simulation. Since the production is performed many times in capacity expansion planning and screening studies in order to determine the best scenario, the computing time and accuracy are major concerns. Many research works have been done for these purposes, representing the E.L.D.C and convolution procedures in various ways. They can be categorized by exact and approximate methods as follows.

Exact method
- Booth-Baleriaux method

Approximate method
- Piece-wise linear approximation method
- Fourier Series approach
- Fourier, Hartley, Z transformation method
- Segmentation algorithm
- Equivalent Energy Function algorithm
- Monte-Carlo simulation method
- Cumulant-Based technique
- Mixture of Normals Approximation method

Exact method represents E.L.D.C on a uniformly spaced grid. As generators are convolved, E.L.D.C is evaluated successively at each grid point. We can get accurate results. However, the computing time in convolution process increases exponentially as the power system grows[1].

Approximate methods represent E.L.D.C analytically by using various transformation techniques or series approaches[2]. Among approximate methods, the cumulant method is widely used because of its speed and reasonable accuracy [3, 4, 5].

But, this method has some drawbacks [2, 6]; (a) the expansion are liable to yield inaccurate results when the system is small and/or the forced outage rates(F.O.R) of an individual unit has small values close to zero; (b) negative probability estimates are sometimes obtained; (c) an addition of successive higher order cumulant terms in the expansion does not necessarily improve the accuracy of the expansion. These are due to the following two properties[7]: (1) Edgeworth/Gram-Charlier expansion is useful especially when the basic r.v. is close to normal. (2) The central limit theorem is a large sample properties.

To solve these problems, we propose the M.O.C.A method. In this method, the system load is modelled as r.v. which can be interpreted in terms of partitioning the load into various categories[8, 9]. For each load category, system load r.v is approximated by a mixture of cumulants. We can consider the load shape of multi-modal characteristics with this method. The demarcation point of each load category is calculated automatically by using least square method which minimizes the square of the errors between the model and the actual load data[10].

Each generating unit of a supply system is modelled as r.v of generation outage capacity according to the number of generator outage, as generators are convolved to the system. For each generator's outage subset, r.v is also approximated by a mixture of cumulants. And the r.v of each load category and generator outage subsets are combined individually to get the cumulants of mixed-E.L.D.C.

This method has been applied to, and validated for, the test systems such as IEEE Reliability Test System(IEEE R.T.S)[11], EPRI Scaled-down Synthetic Utility D System[12] and Texas Electric System[6].

2. Formulation

In the probabilistic production simulation, the system load of the study period is represented as r. v-L. And its cumuative probability distribution function is represented as Eq.(2.1)

$$F_L(x)=1-G(x) \tag{2.1}$$

where,

$G(x)$; Inverted load duration curve.
$=\text{Prob}\{L>x\}$

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Generators are also modelled as r.v of outage capacity. Effective load L_k is defined as Eq.(2.2).

$$L_k = L + \sum_{i=1}^{k}(C_i - A_i) \tag{2.2}$$

where,

C_i ; Capacity of i-th generator in the merit order.

A_i ; Available capacity of i-th generator in the merit order.

The. E.L.D.C$\mathcal{L}_k(x)$ is derived as Eq.(2.3).

$$\mathcal{L}_k(X) = \text{Prob}\{L_k > x\} \tag{2.3}$$

The expected energy served of k-th generator(E.E.S $_k$) can be derived by Eq. (2.4). From $k_N(x)$, which represents the E.L.D.C after all the generators are convolved, expected unserved energy(E.U.E) and L.O.L.P can be expressed as Eq.(2.5) and Eq.(2.6), respectively.

$$\text{E.E.}S_k = (1-q_k)\, T \int_{Ck-1}^{Ck} \mathcal{L}_{k-1}(x)dx \tag{2.4}$$

$$\text{E.U.E} = T \int_{CN}^{\infty} \mathcal{L}_N(x)dx \tag{2.5}$$

$$\text{L.O.L.P} = \text{Prob}\{\mathcal{L}_N(x) > C_N\} \tag{2.6}$$

where,

$C_k = \sum_{i=1}^{k} C_i$

T ; Total load duration hour during the study period.

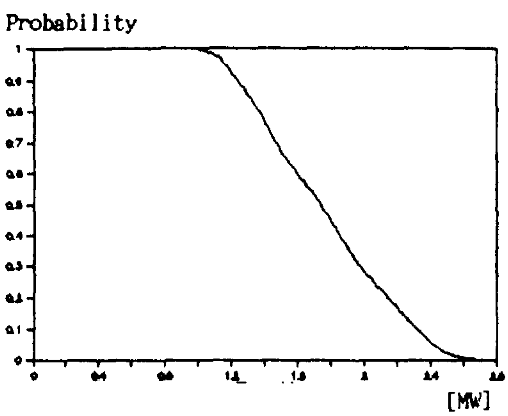
N ; Total number of generator in the system.

3. Mixture of Cumulants Approximation Method

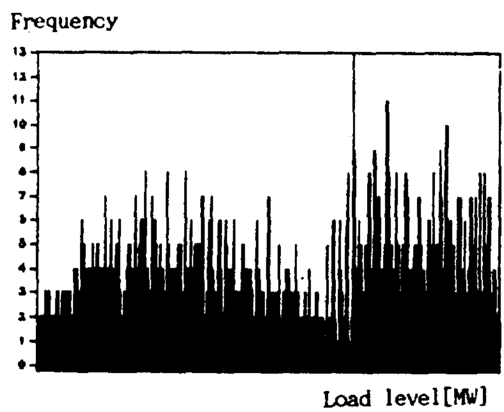
The mixture of cumulants approximation method consists of three steps : first, system load r.v is approximated by a mixture of cumulants ; second, the r.v of generation outage capacity is also approximated by a mixture of cumulants ; finally the mixture of two cumulants are combined to get the M.O.C.A of E.L.D.C. Here, we use the Gram-Charlier A-type expansion using 4-cumulants in the representation of E.L.D.C. The algorithm for determining the optimal demarcation points of load categories is proposed by using least square method. And the number of generator outage subset is decided automatically according to the individual unit's F.O.R value. Two-state generator model is used for simplicity of calculation.

3.1 System Load Pepresentation

The load duration curve(L.D.C) is obtained by rearranging the hourly load decreasing order of magnitude. Then, the load is categorized by peak, intermediate, base load populations and so forth. Each categorized load populations can be regarded as a uni-modal distribution. The system L.D.C is considered to be a multi-modal distribution. By modelling the system L.D.C as a multi-modal distribution, we can obtain better accuracy on probability than any other approximate methods, expecially in the tail of E.L.D.C. The various



(a) L.D.C of R.T.S.



(b) Frequency histogram of load by load level.

Fig. 1 The concept of multi-model distribution.

reliability indices are calculated exactly. Fig. 1(a) shows the L.D.C of IEEE R.T.S. And Fig. 1(b) illustrates the concept of multi-modal distribution.

The system load is approximated by Eq.(3.1)

$$\mathcal{L}^{a}(x)=\sum_{k=1}^{K}\alpha_k[1-\text{G.C.E}] \quad (3.1)$$

where, K : The number of load population.

α_k : Weighting factor of k-th load population.

G.C.E = Gram-Charlier A-Type Expansion.

$N(Z_k)=\exp(-Z_k^2/2)/\sqrt{(2\pi)}$

$Z_k=(x-\mu_k)/\sigma_k$

μ_k : Mean of k-th load population.

σ_k^2 : Variance of k-th load population.

G_{1k} : Standardized Skewness of k-th load population.

G_{2k} : Standardized Kurtosis of k-th load population.

$N^{(r)}$: r-th derivatives of standard normal distribution.

Eq. (3.1) expresses L.D.C as a mixture of many distributions. The necessary steps for calculating the equation can be shown as follows:

Step. 1 Categorize chronological load into K load populations. Here, the demarcation point of individual load category is calculatd automatically as follows.

(1) Devide the system load into segments with same MW-increment. And calculate the first moments for each segment load duration hour and load magnitude as Eq. (3.2)

$$m_1^k=\frac{\text{Sum of load duration hour about } K\text{-th segment}}{\text{Sum of load number about } K\text{-th segment}}$$

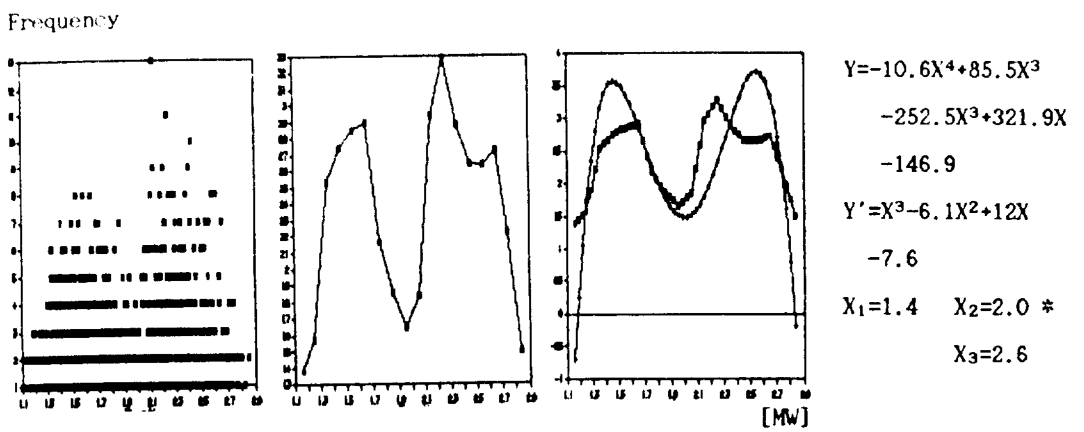
$$m_2{}^k=\frac{\text{Sum of load magnitude about } K\text{-th segment}}{\text{Sum of load number about K-th segment}}$$

$$(K=1,\ 2,\cdots NS) \quad (3.2)$$

where, NS ; Total number of segment.

(2) Based on a calculated set of data $\{m_1^K, m_2^K\}$, we construct a functional relationship between the dependent variable (m_2^K) and the independent variable(m_1^K) with least square method. In order to keep the original load shape, interpolation is performed between the data. In this paper, we use the chebyshev-polynomial curve fitting method with Lagrangian interpolation in regression analysis, because normal equation method become unsatisfactory when the resultant normal equations are ill-conditioned.

(3) Differentiate the least-squared polynomial in previous calculation and solve the equation. We solve the algebraic equation by Bairstow method. (The extremal values are : $X_1=1.4$, $X_2=2.0$, $X_3=2.6$). Demarcation points are decided among these solutions. In the least squared polynomial, the coefficient of the highest order is negative signed ; and the power of the highest order term is even-numbered. Thus, the demarcation points are the minimum values(minimum value is $X_2=2.0^*$).



(a) Original load (b) Modified load with segment (c) Interpolation & approximation (d) Decision of demarcation point

Fig. 2 Calculation procedure of demarcation point.

Fig. 2 shows the example of the above mentioned procedures about IEEE R.T.S.

Step. 2 Calculate load cumulant for each load population by Eq. (3.3).

$$
\begin{aligned}
LK_1^k &= LM_1^k \\
LK_2^k &= LM_2^k - (LM_1^k)^2 \\
LK_3^k &= LM_3^k - 3LM_2^k LM_1^k + 2(LM_1^k)^3 \\
LK_4^k &= LM_4^k + 6LM_2^k (LM_1^k)^2 - 4LM_3^k LM_1^k \\
&\quad - 3(LM_1^k)^4 - 3(LK_2^k)^2
\end{aligned} \tag{3.3}
$$

where,

$$LM_i^k = (1/T^k) \sum_{t=1}^{Nk} (L_t^k)^i$$

N^k ; Number of load of k-th load population.

T^k ; Total duration hour of k-th load population.

L_t^k ; Hourly load of k-th load population at time t.

Step. 3 Evaluate α_k in Eq. (3.4). Here, α_k is weighting factor of k-th load population. α_k means the fraction of the period for which the loads belong to k-th load population.

$$\alpha_k = \frac{\text{Load duration hour of k-th load population}}{\text{Total load duration hour of study period}} \tag{3.4}$$

where, $\sum_{k=1}^{k} \alpha_k = 1,\ \alpha_k \geq 0$

3.2 Supply System Representation

Whenever the generator is convolved to the system, we make the subset of generator according to the number of generator outage and calculate the generator cumulant for each subset. The weighting factor for each subset $\varphi_{n,j}$ is calculated by Eq. (3.5). Here, $\varphi_{n,j}$ means the probability that j generators are down-state when n generators are convolved to the system.

$$
\begin{aligned}
\varphi_{n,0} &= \varphi_{n-1,0} \times p_n \\
\varphi_{n,j} &= \varphi_{n-1,j} \times p_n + \varphi_{n-1,j-1} \times q_n;\ 0 < j < r \\
\varphi_{n,r} &= \varphi_{n-1,r-1} \times q_n
\end{aligned} \tag{3.5}
$$

where, p_n ; Availability of n-th generator.

q_n ; F.O.R of n-th generator.

$r = n + 1$.

In Eq. (3.5) r means that r or more units have failed out of a group of n units. As units are convolved, the probability that many units are simultaneously down-state becomes very small. In this paper, we aggregate the subsets of these r or more units, which are down-state to one subset for the purpose of reducing calculation burden.

Next, we calculate the moment for each subset. When n generators are convolved to the system, the number of events in subset $(j+1)$ is $_nC_j$. The calculation burden for each subset increases exponentially. In order to overcome these problems, we derived formulae from the first to fourth moments for each subset. The moments for each subset are calculated as Eq. (3.6).

$j=0$ (3.6)

$$
\begin{aligned}
\mu_{n+1,0} &= 0 \\
\sigma^2_{n+1,0} &= 0 \\
\nu_{n+1,0} &= 0 \\
x_{n+1,0} &= 0
\end{aligned}
$$

$0<j<r$

$$
\begin{aligned}
\mu_{n+1,j} &= \varphi_{n,j} p_{n+1} \mu_{n,j} + \varphi_{n,j-1} q_{n+1} (\mu_{n,j-1} + C_{n+1}) / \varphi_{n+1,j} \\
\sigma_{2n-1,j} &= \varphi_{n,j} p_{n+1} (\sigma^2_{n,j} + \mu^2_{n,j}) + \varphi_{n,j-1} q_{n+1} [\sigma^2_{n,j-1} + \\
&\quad (\mu_{n,j-1} + C_{n+1})^2] / \varphi_{n+1,j} - \mu^2_{n+1,j} \\
\nu_{n+1,j} &= \varphi_{n,j} p_{n+1} (\nu_{n,j} + 3\sigma^2_{n,j} + \mu^3_{n,j}) + \varphi_{n,j-1} q_{n+1} \\
&\quad [\nu_{n,j-1} + 3\sigma^2_{n,j-1} (\mu_{n,j-1} + C_{n+1}) \\
&\quad + (\mu_{n,j-1} + C_{n+1})^3] / \varphi_{n+1,j} - 3\sigma^2_{n+1,j} \mu_{n+1,j} \\
&\quad - \mu^3_{n+1,j} \\
x_{n+1,j} &= \varphi_{n,j} p_{n+1} (x_{n,j} + 4\nu_{n,j} \mu_{n,j} + 6\sigma^2_{n,j} \mu^2_{n,j} + \mu^4_{n,j}) \\
&\quad + \varphi_{n,j-1} q_{n+1} [x_{n,j-1} + 4\nu_{n,j-1} (\mu_{n,j-1} + C_{n+1}) \\
&\quad + 6\sigma^2_{n,j-1} (\mu_{n,j-1} + C_{n+1})^2 + (\mu_{n,j-1} + C_{n+1})^4] / \varphi_{n+1,j} \\
&\quad - 4\nu_{n+1,j} \mu_{n+1,j} - 6\sigma^2_{n+1,j} \mu^2_{n+1,j} - \mu^4_{n+1,j}
\end{aligned}
$$

$j=r$

$$
\begin{aligned}
\mu_{n+1,r} &= [\varphi_{n,r} (\mu_{n,r} + q_{n+1} C_{n+1}) + \varphi_{n,r-1} q_{n+1} \\
&\quad (\mu_{n,r-1} + C_{n+1})] / \varphi_{n+1,r} \\
\sigma^2_{n+1,r} &= [\varphi_{n,r} [\sigma^2_{n,r} + p_{n+1} \mu^2_{n,r} + q_{n+1} (\mu_{n,r} + C_{n+1})^2] \\
&\quad + \varphi_{n,r-1} q_{n+1} [\sigma^2_{n,r-1} + (\mu_{n,r-1} + C_{n+1})^2]] \\
&\quad / \varphi_{n+1,r} - \mu^2_{n+1,r} \\
\nu_{n+1,r} &= [\varphi_{n,r} [\nu_{n,r} + p_{n+1} (3\sigma^2_{n,r} \mu_{n,r} + \mu^3_{n,r}) \\
&\quad + q_{n+1} \{3\sigma^2_{n,r} (\mu_{n,r} + C_{n+1}) + (\mu_{n,r} + C_{n+1})^3\}] \\
&\quad + \varphi_{n,r-1} q_{n+1} [\nu_{n,r} + 3\sigma^2_{n,r-1} (\mu_{n,r-1} + C_{n+1}) \\
&\quad + (\mu_{n,r-1} + C_{n+1})^3] / \varphi_{n-1,r} - 3(\sigma^2_{n+1,r} + \mu^2_{n+1,r}) \\
&\quad \mu_{n+1,r} + 2\mu^3_{n+1,r}
\end{aligned}
$$

$$x_{n+1,r}=[\varphi_{n,r}[x_{n,r}+p_{n+1}(4\nu_{n,r}\mu_{n,r}+6\sigma^2_{n,r}\mu^2_{n,r}+\mu^4_{n,r})+q_{n+1}\{4\nu_{n,r}(\mu_{n,r}+C_{n+1})+6\sigma^2_{n,r}(\mu_{n,r}+C_{n+1})^2+(\mu_{n,r}+C_{n+1})^4\}]+\varphi_{n,r-1}q_{n+1}\{x_{n,r-1}+4\nu_{n,r-1}(\mu_{n,r-1}+C_{n+1})+6\sigma^2_{n,r-1}(\mu_{n,r-1}+C_{n+1})^2+(\mu_{n,r-1}+C_{n+1})^4\}]/\varphi_{n+1,r}-4\nu_{n+1,r}\mu_{n+1,r}-6\sigma^2_{n+1,r}\mu^2_{n+1,r}-\mu^4_{n+1,r}$$

Then the cumulant for each subset are calculated by Eq. (3.7).

$$GC1_{n,j}=\mu_{n,j}$$
$$GC2_{n,j}=\sigma^2_{n,j}-(\mu_{n,j})^2$$
$$GC3_{n,j}=\nu_{n,j}-3\sigma^2_{n,j}\mu_{n,j}+2(\mu_{n,j})^3$$
$$GC4_{n,j}=x_{n,j}+6\sigma^2_{n,j}(\mu_{n,j})^2-4\nu_{n,j}-3(\mu_{n,j})^4-3(GC2_{n,j})^2 \quad (3.7)$$

where,

$GCi_{n,j}$; i-th cumulant of subset j when n generators are convolved to the system.

3.3 Representation of E.L.D.C and Expected Energy, Reliability Index Calculation

E.L.D.E is obtained by combining the r.v of each load category and the generator outage subset as Eq. (3.8)

$$\text{E.L.D.C}(n)=1-\sum_{k=1}^{K}\sum_{j=0}^{r}\alpha_k\varphi_{n,j}[\int_{-\infty}^{Zs}N(Zs)dZs+(G1_{n,j}^{k}/3!)N^{(2)}(Zs)-(G2_{n,j}^{k}/4!)N^{(3)}(Zs)-(10/6!)(G1_{n,j}^{k})^2N^{(5)}(Zs)] \quad (3.8)$$

where,

E.L.D.C(n) ; E.L.D.C when n generators are convolved to the system.

$$Z^k=\frac{X-\mu_{n,j}^{k}}{\sigma_{n,j}^{k}}=Zs$$

$N(Z)=(1/\sqrt{2\pi})\times\exp(-Z^2/2)$

$N^{(N)}(z)=N$-th derivative of the Normal.

The procedure of obtaining E.L.D.C are summarized in the following:

Step. 1 Calculate system equivalent load cumulant by adding load cumulant and generator cumulant.

$$ELKi_{n,i}^{k}=LK^ki+GCi_{n,j} \quad (3.9)$$

where, $i=1,2,3,4$

$j=0,1,2,\cdots,r$

$k=1,2,3,\cdots,K$

Step. 2 Calculate coefficients of Gram-Charlier expansion.

$\mu_{n,j}^{k}=ELK1_{n,j}^{k}$; Mean for each subset about k-th load population.

$\sigma^2{}_{n,j}^{k}=ELK2_{n,j}^{k}$; Variance for each subset about k-th load population.

$G1_{n,j}^{k}=ELK3_{n,j}^{k}/(\sigma_{n,j}^{k})^3$; Standardized skewness for each subset about k-th load population.

$G2_{n,j}^{k}=ELK4_{n,j}/(ELK2_{n,j}^{k})^2$; Standardized kurtosis for each subset about k-th load population.

Step. 3 Represent E.L.D.C as Eq. (3.8).

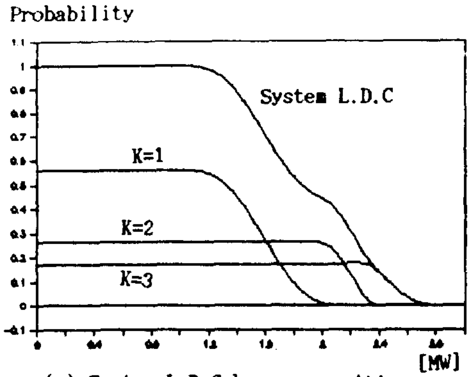
The expected energy of each generator and system reliability indices are calculated by Eq. (2.4), Eq. (2.5) and Eq. (2.6).

4. Case Study

The proposed M.O.C.A method is applied to the EPRI scaled-down synthetic utility D system, IEEE R.T.S and Texas electric system. Table 1. presents the basic characteristics of these three test systems. The test results are compared with

Table 1 IEEE R.T.S, EPRI-D and Texas Electric System characteristics.

	IEEE R.T.S	ERPI-D	TEXAS
Installed Cap.	3405MW	11420MW	4466MW
Peak Load	2850MW	8740MW	3350MW
No. of Units	32	60	27



(a) System L.D.C by superposition of each load population.

K	Demarcation point	Weighting (a_k)
1	1102～2000	0.5618
2	2001～2348	0.2624
3	2350～2850	0.1758

(b) Location of Demarcation point for each load population.

Fig.3 The concept of L.D.C by M.O.C.A.

Table 2 The effects of the number of load category and demarcation point for the IEEE R.T.S system.

Case	K	Weighting factor	Initial Engergy	$err_{\max}$	err_{mean}	$err_{s\text{-mean}}$
①	2	0.56, 0.44	4163695	−.01608	−.00543	.00061
②		0.82, 0.18	4163862	−.01418	−.00544	.00062
③	3	0.34, 0.28, 0.38	4163696	−.01523	−.00543	.00061
④	4	0.22, 0.24, 0.16, 0.38	4163698	−.01510	−.00543	.00061
⑤		0.56, 0.19, 0.13, 0.12	4163704	−.01667	−.00544	.00062
⑥*	3	0.56, 0.26, 0.18	4163709	−.01665	−.00544	.00060

Booth-Baleriaux, Cumulant and M.O.N.A method. Fig.3 shaws configulation of L.D.C about IEEE R.T.S. The number of load population and demartation points are shown in Fig. 3(b)

As shown in Fig. 3(a), the system L.D.C by M.O.C.A is represented by superposition of individual load population with weighting a_k. Because the individual load population is decided in a way similiar to normal distribution, the L.D.C by M.O.C.A. method is well approximated to the actual L.D.C.

In order to examine the validity of the proposed procedure for the decision of number of load category and demarcation points, we compare the various indices-initial unserved energy, maximum error (err_{max}), average error (err_{mean}) and square average error ($err_{s\text{-mean}}$)-by changing the number of load category and demarcation points. Table 2. shows the results for the IEEE R.T.S. In table 2, case 1, 2, 3, 4 and 5 show the result of calculation with the number of load population and demarcation points on off-line basis. In the other hand, case 6 shows the result of the method proposed in this paper.

In table 2, $err_{\max}$, err_{mean}, $err_{s\text{-mean}}$ is defined as Eq.(4.1), (4.2) and (4.3).

$$err_i = err(x_i), i = 1, 2 \cdots I$$
$$err(x) = L(x) - L^b(x)$$

where,

I ; The number of data point.

$\mathscr{L}(x)$; Approximated value of L.D.C or E.L.D.C at the point of x.

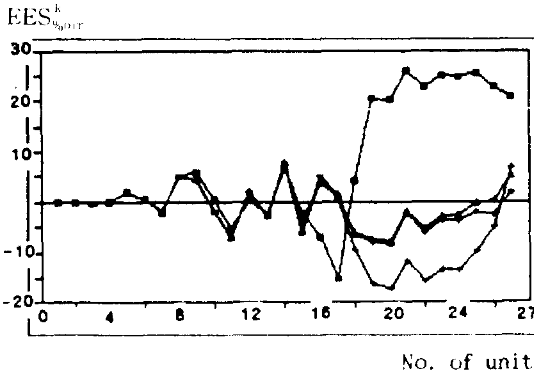
$\mathscr{L}^b(x)$; Benchmark value of L.D.C or E.L.D.C at the point of x

$$err_{\max} \cong \{\text{sign} \, err_i^{\max}\} \hat{err}_{\max} \quad (4.1)$$

where,

$$\hat{err}_{\max} = \text{Max}\{Abs(err_i) ; 1 \le i \le I\}$$

Table 3 The effect of number of generator outage subset.



No. of subset	E.U.E	Normalized time
2 : □	885	1.0
3 : +	766	1.3
4 : ◇	779	1.6
5 : △	778	2.0

Table 4 The expected unserved energy and reliability index for the test systems.

METHOD	INDICES	IEEE R.T.S	EPRI-D	TEXAS	Time
Booth	EUE LOLP	831 0.002894	742 0.0000043	3999 0.002099	5.2
Cumulant	EUE LOLP	678 0.002361	948 0.0000140	129243 0.005119	1.0
M.O.N.A	EUE LOLP	1035 0.002607	6766 0.0000012	4567 0.002021	1.2
M.O.C.A	EUE LOLP	778 0.002884	748 0.0000038	3597 0.002087	1.3

i_{max} ; The value of the index i for which err_i is maximum.

$$err_{mean} \cong \frac{1}{I}\sum_{i=1}^{I-1}\frac{err_i + err_{i+1}}{2} \tag{4.2}$$

(uniformly spaced grid case)

$$err_{s-mean} \cong \frac{1}{I}\sum_{i=1}^{I-1}\frac{err_i^2 + err_{i+1}^2}{2} \tag{4.3}$$

(uniformly spaced grid case)

Table 2. makes it clear that values of indices in the number of load population and demarcation points determined on off-line basis and the result of the proposed method do not reveal any significant disparity between them. Consequently, this fact testifies the validity of the proposed algorithm which determines the number of load population and demarcation points.

Table 3. illustrates the % error of expected energy served (EES_{err}^k) for each generator. % error is calculated as Eq. (4.4). We use the expected energy of Booth-Baleriaux method (EES_{booth}^k) as a benchmark value.

$$EES\%_{err}^k = (EES^k - EES_{booth}^k) / EES_{booth} \times 100 \tag{4.4}$$

In table 3, there is no significant difference in the E.U.E values. The more number of subset, the less % error of each units. But the computer time increases linearly according to the number of generator outage subset. The number of generator outage subset is decided by the weighting factor of the subset. The weighting factor is calculated by the individual unit's F.O.R value as Eq.(3.5). From Table 3, we decided the number of generator outage subset so that the weighting factor is less than, or equal to, 10^{-5}(In this case, the number of subset is 5). Finally, we compare the expected unserved energy, reliability index and computer time. Table 4. shows the results.

In table 4, the point of reference is Booth-Baleriaux method. When applied to different test systems, the E.U.E and L.O.L.P values of M.O.N.A and Cumulant methods expose obvious inaccuracy compared with the values of Booth-Baleriaux method. Meanwhile, the result of M.O.C.A method corresponds to that of Booth-Baleriaux method. In terms of computer time, M.O.C.A method is a little bit slower than cumulant method, but it is four times as fast as Booth-Baleriaux method. The testing results fully demonstrates the M.O.C.A method's robustness and reliability. It's performance is good both in pathological and well-behaved systems.

5. Conclusion

This paper has described a newly developed production costing simulation method by using the mixture of cumulants approximation. The main results are summarized as follows.

(1) In this study, mixtures of cumlant are used to represent L.D.C and E.L.D.C ; in order to do that, the formulae for the moment of generator's outage subset are derived. By applying these formulae to the moment calculation of generator outage subset, we can reduce computational burden, Also, the algorithm for the decision of number of load category and demarcation points is proposed by using least square approximation method.

(2) We evaluate the proposed method by applying it to the various test systems. The test results show that the M.O.C.A method is considerablely reliable and stable both pathological and well-behaved system.

(3) M.O.C.A method is characterized by a combination of high accuracy and considerably reduced computing time.

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