論 文
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새로운 슬라이딩 라인을 갖는 가변구조 방식에 의한 직류 모터의 위치 제어

DC Motor Position Control Using Variable Structure Systems with a New Sliding Surface

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요 약

가변구조제어 시스템에 있어서, 상태 궤적이 슬라이딩 모우드 내에 존재하게 되면, 시스템은 매개변수나 부하의 변동에 강인한 특성을 가지게 된다. 기존의 가변구조제어 시스템은 슬라이딩 라인이 상태 변수들의 선형 조합으로 이루어지므로, 상태 궤적이 슬라이딩 라인에 도달하는 리칭 페이즈가 존재한다. 이 영역에서는 슬라이딩 모우드가 일어나지 않으므로 매개변수나 부하의 변동에 대한 둔감성을 보장할 수 없게 된다. 본 논문에서는 새롭고 간단한 비선형 슬라이딩 라인을 제안하여 이 단점을 보완하였다. 제안된 가변구조제어 시스템을 직류 모터 위치 제어에 적용하였고, 기존의 가변구조제어 시스템과의 성능을 비교 검토하였다.

Abstract- In the VSS control, the trajectories with a conventional sliding surface have a reaching phase which is an interval from the initial state to the first touching of the sliding surface. Since the sliding mode control can not be realized in a reaching phase, the trajectories may be sensitive to the disturbances and parameter variations. A simple nonlinear sliding surface is proposed to improve the robustness in a reaching phase. The position control of a PM DC servo motor using a new sliding surface is carried out and is compared to the one using the conventional surface. The sliding mode occurs in entire trajectories with the proposed new sliding surface and the improved robustness is obtained.

1. Introduction

DC machines have been used extensively in many applications, such as servo systems, cutting-wheel systems, robot manipulators, and so on. In the motor applications requiring accurate control of speed, torque and/or position, the DC motor is most popular because of its easy use and high efficiency in spite of its relatively high cost and maintenance requirements. Most of these applications require precise and robust control characteristics. But several nonlinear effects are involved in a DC motor, for examples, friction as a function of speed, torque ripple caused by nonsinusoidal rotor flux, and load disturbances, etc.

The high-gain feedback method and compensation technique have been used to cope with the above nonlinear effects[3]-[4]. Although the high-gain feedback method can reduce the effects of parameter variations and load disturbances, it may be unstable due to the high-gain effect. The compensation technique may be easily applied to a specific system, but it may be sensitive in the presence of parameter variations and load disturbances.

The theory of variable structure systems (VSS) with a sliding mode control has been studied in great detail in Soviet literatures[2], [9]-[10]. The VSS are a special calss of nonlinear systems characterized by a discontinuous control action which changes the control structure under the condition of a sliding surface $s(x)=0$. If a system is forced to constrain its evolution on a predetermined sliding surface, it results in a new dynamical behavior that is largely determined by the design parameters defining the sliding surface. Consequently, new properties which are not present in the original system can be obtained for the controlled motions. Therefore the system is robust and insensitive to the disturbances and parameter variations. The representative point of the state trajectories undergoes high frequency chattering along the surface with small amplitude known as the sliding mode or the sliding regime. The designed surface is then referred to as the sliding manifold or the sliding surface.

In the previous methods, the linear sliding surface is used[2], [5], [8], [10]-[12] and the analysis for a linear time varying sliding surface is given by Slotine and Sastry[8]. However, VSS with a linear sliding surface always have a "reaching phase" in which the trajectories starting from a given initial condition tend towards the sliding surface. The trajectories in this phase are sensitive to the parameter variations and load disturbances [8]. Thus the advantages of a sliding mode control in a reaching phase can not be realized. The alleviation of these problem by the use of high-gain feedback to reduce the time in a reaching phase is suggested by Young[7]. This has the drawbacks related to high-gain feedback which gives extreme sensitivity to the unmodeled dynamics and actuator saturation[8]. This problem is solved by Harashima et al.[6]. The sliding surface of this method consists of four segments such as maximum acceleration, maximum speed, and maximum deceleration within certain system limits. Not only robust but time suboptimal performances can be obtained. However, this method can not be used for the higher order systems and also the sliding surface is segmented discontinuously so that the determination of the coefficients for each sliding surface segment is much more complicated.

In this paper, a simple continuous nonlinear sliding surface is proposed and the robustness in the entire trajectories can be obtained. The position control of a permanent magnet (PM) DC servo motor with the proposed sliding surface is investigated and the experimental results show good agreements with the computer simulations.

2. Overall System Descriptions and Modelling

The schematic diagram representation of the overall control system is shown in Fig. 1. It consists of a DC servo motor, an H-type PWM amplifier, an incremental encoder, a counter, a main computer (IBM PC/AT Compatable) with a 12 bit 712 interface card, and a dynamometer. The servo motor used in this analysis is a PM DC motor driven by the current controlled MOS-FET PWM amplifier. The current of a PM DC motor must be limited because of the possible damage to the permanent magnet caused by over-current. The current controller controls the armature current to

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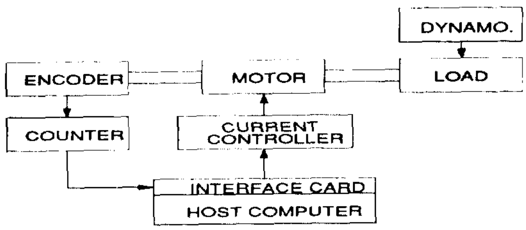


Fig. 1 Schematic diagram of overall systems

follow the current command given by the host computer. It can be assumed that these values are equal in steady state. Furthermore, the electrical dynamics of the motor are much faster than the mechanical ones so that electrical dynamics can be neglected in the procedures of modelling, simulation and controller design. The incremental encoder and counter provide the information about shaft position. The 712 interface card supports digital input/output (DIO), analog-to-digital conversion (ADC), and digital-to-analog conversion (DAC). By using the 712 interface card, the main computer can carry out the position control of a PM DC servo motor based on the control algorithm. The dynamometer is used to electrically change the load as a load disturbance.

As explained previously, the electrical dynamics can be neglected and only the mechanical dynamics are considered. Written in state space form with $x_1=\theta$, $x_2=\dot{\theta}=\omega$, the following equation can be obtained:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -B/J\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ k_t/J\end{bmatrix}i_a - \begin{bmatrix}0\\ 1/J\end{bmatrix}T_f sgn(x_2) \qquad (1)$$

where

$$J=J_m+J_L$$
$$B=B_m+B_L$$
$$T_f=T_{fm}+T_{fl}\,.$$

3. VSS with Linear Sliding Surface

1) Basic VSS Theory

Consider a single-input single-output system of canonical form as

$$\dot{x}=Ax+Bu+F \qquad (2)$$

where

$$A=\begin{bmatrix}0 & 1 & 0 & \cdot\cdot & 0\\ 0 & 0 & 1 & \cdot\cdot & 0\\ \cdot & \cdot & \cdot & \cdot\cdot & \cdot\\ a_1 & a_2 & a_3 & \cdot\cdot & a_n\end{bmatrix}, B=\begin{bmatrix}0\\ 0\\ \cdot\\ 1\end{bmatrix}$$

$$F=\begin{bmatrix}0\\ 0\\ \cdot\\ f\end{bmatrix}$$

x is the state, f is a disturbance and $a_i's$ are constants or time-varying parameters. Furthermore $a_i's$ and f may be unknown but the boundary values are assumed to be known. The VSS are characterized as the following discontinuous control,

$$u=\begin{cases}u^+(x,t) & \text{for } s(x)>0\\ u^-(x,t) & \text{for } s(x)<0\end{cases} \qquad (3)$$

The above system with a discontinuous control is termed as the VSS since the feedback control structure is switched alternatively according to the state of the predetermined sliding surface in the state space. Then the controlled system will be forced to slide along or nearby the vicinity of the switching surface defined as follows:

$$s(x)=C^TX=\sum_{i=1}^{n}c_ix_i,$$
$$c_i=const.,\ c_n=1. \qquad (4).$$

The sliding mode occurs on a switching surface, $s(x)$, when trajectories of the state are not away from the switching surface so that the trajectory of the system slides and remains on the predetermined switching surface. The equation $s(x)$ is solved to prove the invariancy of the sliding mode with respect to the system parameters $a_i's$ and the disturbance f. From (4), the resultant dynamics of the system in the sliding mode are written as follows:

$$\dot{x}_i=x_{i+1},\ i=1,2,\cdots,n-1$$
$$\dot{x}_{n-1}=-\sum_{i=1}^{n-1}c_ix_i. \qquad (5)$$

The above equations are dependent on only parameters $c_i's$ given by the designer. Therefore, the system in sliding mode is completely robust

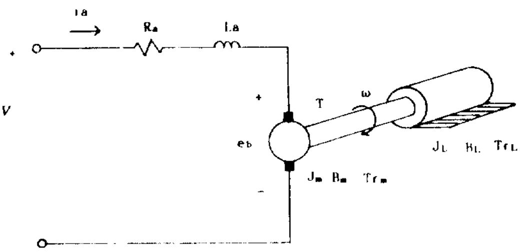


Fig. 2 Simplified motor and load

and insensitive to the disturbances and parameter variations[2], [8]-[11]. The sufficient condition for the existence of the sliding mode is

$$\lim_{s(x)\to 0} s(x)\dot{s}(x) < 0. \quad (6)$$

In the sliding mode, the folowing equations are satisfied

$$s(x)=0 \text{ and } \dot{s}(x)=0. \quad (7)$$

It is noted that the parameter c_i's of a desired sliding surface are determined to satisfy the Hurwitz criterion for asymptotic stability and a discontinuous control input is determined by the existence condition (6) of the sliding mode.

The advantages of VSS with a linear sliding surface are robustness in sliding mode, asymptotic stability, predetermination of dynamics, and no overshoot[9]-[10]. However, the disadvantages are chattering on sliding surface and no guarantee of robustness in a reaching phase[9]-[10]. Thus the purpose of this paper is to improve the robustness in a reaching phase using a simple continuous nonlinear sliding surface.

2) New Sliding Surface Design

Before designing the new sliding surface, some requirements which the new sliding surface must satisfy are considered. One requirement is that the value of a sliding surface function must be zero at the origin in state space, which implies that the steady state error must be zero. It can be written in a mathematical form as follows:

$$\lim_{x\to 0} s(x) = 0. \quad (8)$$

To satisfy the above condition, the sliding surface has to be only a function of the system states. Note that this condition is also satisfied in the conventional sliding surface. The other requirement is that the value of a sliding surface function must be zero at initial states to make the system slide from the beginning and it can be stated as

$$s(x_0)=0. \quad (9)$$

It is now assumed that the initial states are known and let the initial states x_0 be as follows:

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} x_{1o} \\ 0 \end{bmatrix} \quad (10)$$

Since the initial states except x_1 are allowed to be zeros, the initial states can always be on the horizontal axis in a second order state space as shown in Fig. 3.

The conventional linear sliding surface of a second order system can be represented as follows:

$$s(x)_L = c_1 x_1 + x_2,\ c_1 > 0 \quad (11)$$

Let a new sliding surface be

$$s(x) = s(x)_L + s(x)_{NL} \quad (12)$$

where $s(x)_{NL}$ represents a nonlinear function of system states. Since this new sliding surface must satisfy the conditions (10) and (11), $s(x)_{NL}$ term should satisfy the following equations,

$$\lim_{x\to 0} s(x)_{NL} = 0 \quad (13)$$

$$s(x_0)_{NL} = -s(x_0)_L. \quad (14)$$

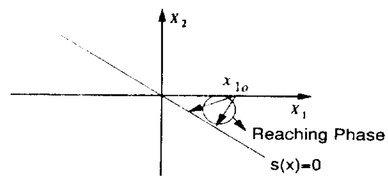


Fig. 3 Conventional sliding surface in phase plane

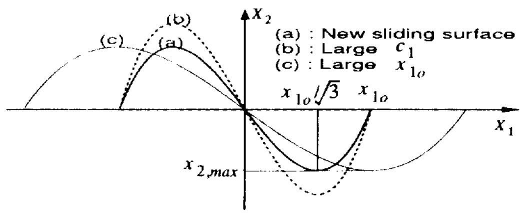


Fig. 4 New sliding surfaces in phase plane

Therefore, the initial value of $s(x)_{NL}$ can be obtained as

$$s(x_0)_{NL} = -c_1 x_{1_o}. \tag{15}$$

This means that a polynomial of x_1 may be chosen for $s(x)_{NL}$ mathematically. Choosing ax_1^3 in view of the practical reasons such as calculation time, simple form, easy control, and resultant performance, a new sliding surface becomes

$$s(x) = s(x)_L + ax_1^3,\ a = -\frac{c_1}{x_{1_o}^2}$$
$$= c_1\left(1 - \frac{x_1^2}{x_{1_o}^2}\right)x_1 + x_2. \tag{16}$$

In the sliding mode, $s(x)=0$, x_2 satisfies the following equation:

$$x_2 = \dot{x}_1 = -c_1\left(1 - \frac{x_1^2}{x_{1_o}^2}\right)x_1. \tag{17}$$

Equation (17) is a first order nonlinear differential equation and it can be solved using a numerical method. Since $\dot{x}_1(0) = x_2(0) = 0$, x_1 does not decrease at initial time. In other words, x_1 needs to be activated by the control input at initial time. Since the behavior of x_1 is only dependent on parameter c_1, it can be said that the closed loop system using the sliding mode control is robust. The maximum desired operating value of x_2, $x_{2,max}$ can be given by the designer and equals to $\frac{2c_1x_{1_o}}{3\sqrt{3}}$. The design parameter c_1 of the surface (a) in Fig. 4 can be easily obtained for a given intial condition. The case for the same initial position but different $x_{2,max}$ is shown in surface (b) of Fig. 4. The surface (c) shows the case of the same $x_{2,max}$ but different initial position.

4. Simulations and Expermental Results

In the digital control, the selection of the sampling time is known to be important in the system performance, especially for the VSS control with a sliding mode. The sampling time is chosen as 2 [*msec*] in simulations and also in experiments [13]. The control algorithms are implemented using C-language which has a merit of access to the hardware directly. The position information is obtained using the counter which counts two phase signals from the encoder with a precision of 2500 [*pulse/rev*]. The motor speed is calculated by using euler's method as

$$\omega(k) = \frac{\theta(k) - \theta(k-1)}{T}. \tag{18}$$

Thus speed informations are quantized by 500 [*pulse/sec*].

The simulations and experiments are carried out with or without a load disturbance of $T_f = 36.8$[kg-cm] which is 40 [%] of the full load torque. The control input is taken as the following form,

$$u = (k_1|x_1| + k_2|x_2| + k_3)sgn(s) \tag{19}$$

where, $sgn(s) = \begin{cases} 1 & \text{for } s>0 \\ 0 & \text{for } s=0. \\ -1 & \text{for } s>0 \end{cases}$

The gains in (19) selected for a conventional sliding surface are $k_1 = 0.388$, $k_2 = 0.277$, and $k_3 =$

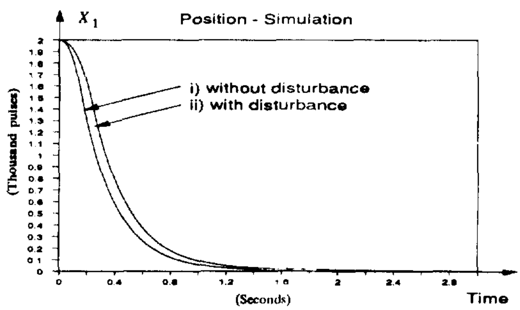


Fig. 5 Position comparison for the conventional sliding surface

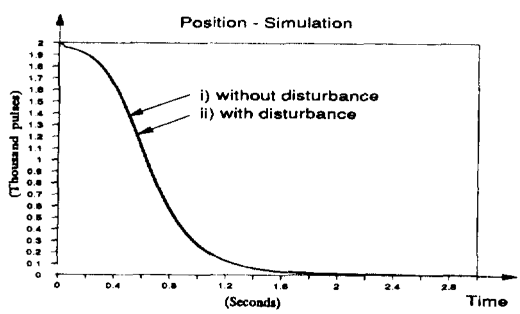
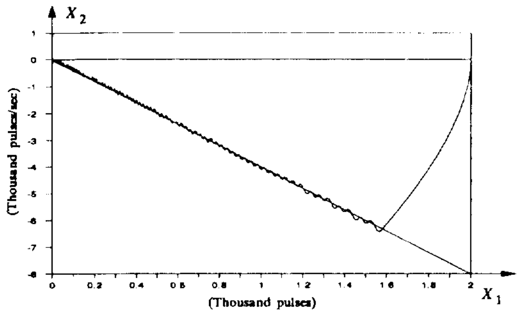
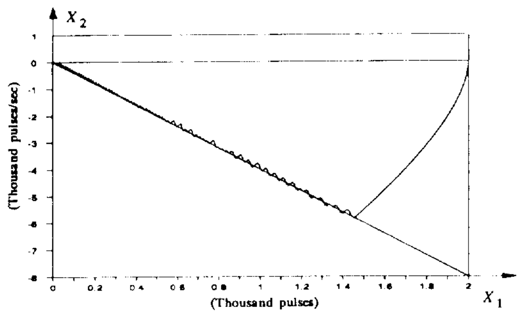


Fig. 6 Position comparison for the new sliding surface

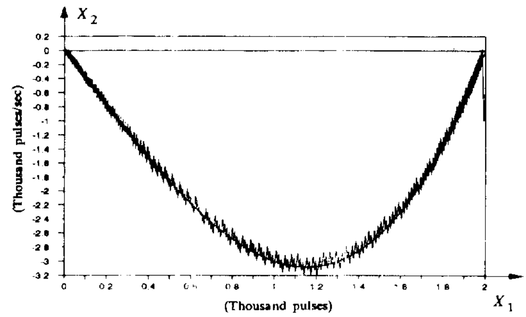


(a) without disturbance

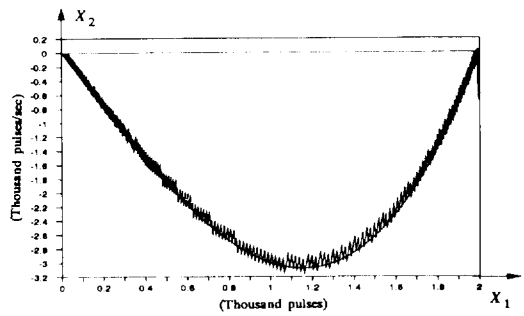


(b) with disturbance

Fig. 7 Phase trajectiories for the conventional surface in phase plane



(a) without disturbance



(b) with disturbance

Fig. 8 Phase trajectories for the new surface in phase plane

3.68 based on the existence of a sliding mode (6). However, for a new sliding surface these become $k_1=0.776$, $k_2=1.3855$, and $k_3=3.68$. The position command is 2000 [*count*] (=288 [*degree*]). The modelled DC motor dynamics (1) is solved by the use of Runge-Kutt 4th order method. The parameters used in this analysis are $B/J=0.32$ [1/sec], $k_t/J=19.616$ [$rad/sec^2/A$], and $T_f/J=36.31$ [rad/sec^2]. The position responses with or without the load disturbances are given in Fig. 5 for the conventional sliding surface. Notice that there is a discrepancy between two output. Therefore the control with the conventional sliding surface may be sensitive to the load disturbances and parameter variations. This problem is solved by the use of a simple nonlinear continuous sliding surface(16). The position outputs for the new sliding surface are shown in Fig. 6 where it can be seen that there is no reaching phase with this

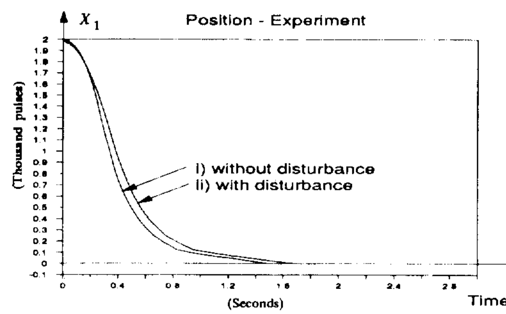


Fig. 9 Position comparison for the conventional sliding surface

sliding surface. As can be seen in Feg. 6, there is a little discrepancy between the two outputs. This comes from the bandwidth existing in a sliding surface due to the finite sampling frequency, namely a quasi-sliding mode. It is, however, considered that the robust control with a new

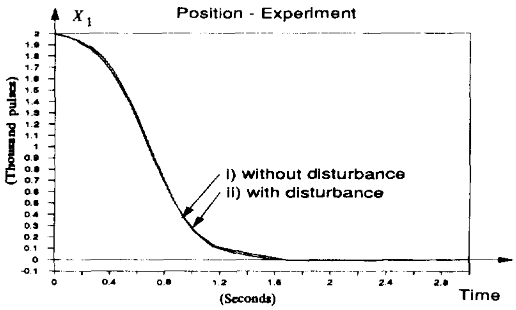
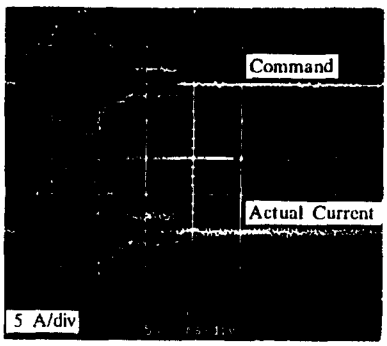
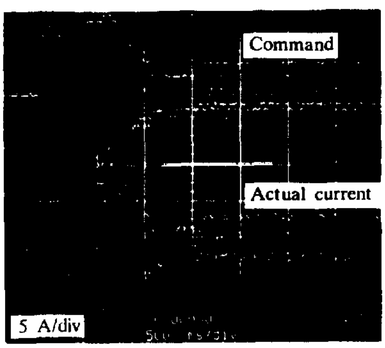


Fig. 10(a) Position comparison for the new sliding surface



(a) without disturbance



(b) with disturbance

Fig. 10(b) Current commands and actual currents for the new sliding surface

sliding surface can be realized in all state space. In Fig. 7 and Fig. 8, the phase trajectories for the conventional sliding surface as well as for the new sliding surface with or without disturbances are depicted. It is noted that the reaching phase where the trajectories are senstive to disturbances in Fig. 7 is removed in Fig. 8.

The experimental results are shown in Fig. 9 and Fig. 10. The position outputs for the conventional sliding surface are shown in Fig. 9 and those for the new sliding surface are shown in Fig.10(a). By comparison, the new sliding surface gives robust output than the conventional one as expected through simulations. Fig. 10(b) shows the current commands and the actual currents of the motor for the new sliding surface.

Using the proposed scheme, it has been shown that the better performances can be obtained. Therefore, it is considered that the robust control with a new sliding surface can be realized in all state space.

5. Conclusions

A new simple nonlinear sliding surface is proposed to make the system more robust with respect to the disturbanced and parameter variations. The conventional VSS have always a reaching phase which is an interval from the initial state to the first touching of the sliding surface. Since the sliding mode does not occur in a reaching phase, the trajectories may be sensitive to the load disturbances and parameter variations. Thus the robustness of the system is not guaranteed in all state space. Using the proposed scheme, the robustness to the external disturbances and parameter variations especially during the reaching phase can be improved.

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