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均衡퓨처價格(equilibrium futures prices)을 예측하기 위한 財務省 長期債券(Treasury bond)의 퓨처옵션價格(futures option prices)에 대한 研究

〈국문요약〉

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주식옵션(stock options)에 대한 연구에 비교하여 상품 및 퓨처 옵션(commodity & futures options)에 대한 연구는 선진국에서도 지금 한창 연구를 하고 있는 단계에 있다. 우리나라에서도 이 분야에 대한 이론을 바탕으로 하는 제도를 곧 도입하려는 준비를 하고 있다.

본 연구는 블랙의 "블랙의 컴모디티 옵션의 가격모형(Black commodity option pricing model)"을 이용하여 재무성 장기채권의 퓨처의 균형가격을 예측하는데 있다. 이 블랙모형의 적용가능성을 검증해 본 것이다. 실제퓨처가격(observed futures prices)과는 달리 재무성 장기채권 퓨처 옵션에서의 묵시적 퓨처가격(futures prices implicit)은 시장효율성(market efficiencies)의 전제하에 성립되거나, 아니면 옵션가격모형을 사용하여서는 아니되거나 둘 중의 하나이거나 둘 다 섞이거나 일 것이다. 본 실증적인 연구, 즉 묵시적인 표준편차(implied standard deviations)를 사이멀테니어스(simultaneously)하게 계산한 묵시적인 퓨처가격(implied futures prices)을 사용한 실증적인 연구는 옵션모델에 의하여 퓨처가격을 계산하는 데에 문제가 있음을 발견하였다. 그 이유는 옵션가격결정모형을 이용하여 계산한 재무성 장기채권의 퓨쳐가격은 재무성 장기채권의 미래가격변동의 방향을 제시하는 지표로써 사용할 수 없기 때문일 것이다.

우리나라에서도 이 분야에 대한 이론과 제도를 곧 도입하는 입장에서 선행되는 문헌이 될 것이다.

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Treasury Bond Futures Option Prices as Predictors of Equilibrium Futures Prices

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Ⅰ. Introduction

In recent years, there has been a rejuvenation of options on futures and commodities. The existence of commodity and futures options can be traced back to the Civil war era, although their trading history has been tainted with lawsuits, scandal and skepticism. The prospects for commodity options trading remained bleak until 1981, when the Commodity Futures Trading Commission allowed options to be traded on organized exchanges for an initial three year period. Among the options granted approval under this pilot program, the market for options on Treasury bond futures has grown into the biggest market on any of the nation's eleven commodity exchanges. The growth of these markets is perhaps best exemplified by a recent excerpt from the financial press [Wall Street Journal, 30 April 1984, p.48], which stated that"... after trading only 19 months, Treasury bond futures options this month passed the 107

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year old corn futures market in open interest. Currently, Treasury bond futures options and the underlying futures together often make up more than 40% of the monthly trading volume on the Chicago Board of Trade."

Academic research in the valuation of commodity and futures options is not as exhaustive as research on stock options, primarily because of the sporadic history of commodity options trading. Black(1976) has developed a futures option pricing model, similar in spirit to the widelt known and used Black and Scholes(1973) stock option pricing model. Limited applications of the Black model have shown some degree of success regarding the applicability of the model in pricing Treasury bond futures call options. The purpose of this paper is to provide further evidence on the validity of the Black commodity pricing model as an empirical paradigm by assessing the efficacy of the model in predicting equilibrium futures prices. Futures prices implicit in call option premia different from observed futures prices would represent the existence of market in efficiencies and/or a mis specification of the option pricing model. In order to further this end, the Black option pricing model is discussed in the next section. The methodology is presented in Section III while the results of the tests are discussed in Section IV. A brief summary and conclusions are contained in the final section.

II. Pricing of Options on Treasury Bond Futures

Since options on futures are a relatively recent financial innovation, the literature in the valuation of thesse contingent claims is sparse compared to the work in stock options. An option on a futures contract is similar in concept to stock options. A call option on a futures contract gives the buyer the right to establish a long position in a futures contrct of a specified maturity whereas a put option gives the buyer the right to establish a short position in a futures contract of a specified maturity.

The earliest investigation of the valuation of futures options was conducted by Black (1976). Using logic similar in spirit to that used in deriving the familiar Black and Scholes(1973) stock call option valuation model, Black showed that the value of futures call option (C) could be modelled as a function of the futures price (F), exercise price of the option (X), the risk free rate (r), the time to expiration (T) and the variation of the underlying futures contract (V^2). Using the continuous dividend assumption developed by Merton (1973), this model has been extended by Merville and Overdahl (1985) to value Treasury bond futures call options. Merville and Overdahl (1985) argue that the continuous payment assumption can be satisfied by the daily marking to market feature of Treasury bonds futures. Moreover, since the underlying asset for the Treasury bond futures contract is Treasury bonds which earn continuously at the risk free rate, the futures option can be treated for analytical purposes as an option on either a zero coupon cash Treasury bond or a cash Treasury bond which pays interest continuously. Under these assumptions, the model for valuing call options on Treasury bond futures may be stated as :

$$C=e^{-rT}[FN(d_1)-XN(d_2)] \qquad (1)$$

where

N(.) = cumulative normal density function ;

$d_1=[\ln(F/X + (V^2/2T]/V\sqrt{T}$ and $d_2=d_1 - V\sqrt{T}$

Empirical tess of this model to the pricing of Treasury bond call options have met with varied degrees of success. Belongia and Gregory (1984) have shown that the market for Treasury bond options is characterized by a high degree of efficiency. This conclusion was strengthened by the absence of any arbitrage profit opportunities in instances where the model estimates were different from observed market values. Merserschmidt (1984) used the Black model to assess the efficacy of Treasury bond options and futures in hedging mortgage risk. However, in a recent study of the efficacy of the valuation model, Merville and Overdahl (1985) have shown that the Black model tends to underprice in the money options and overprice at and out of the money options. These latter results cast doubts on the validity of the Black model in valuing Treasury bond futures call options. This conflicting evidence is disturbing in view of the observation by Wolf (1984) that institutional traders use the Black model for evaluating option premium as well as the fact that several exchanges [Chicago Board of Trade, Commodity Exchange and the New York Futures Exchange] use the model to calculate margin requirements for floor traders. We offer additional evidence on the efficiency issue, albeit in an indirect fashion, on the validity of the model and informational content of call option prices by assessing the degree of simslarity between implied (option model) futures prices and actual futures prices. If Treasury bond call options are actually priced according to the Black model, then the implied futures prices should not deviate significantly from the actual futures prices.

III. Data and Methodology

While several studies have examined the information content of stock option prices, the literature on this subject in the futures area is scant. Stock option prices have been used to investigate expectations regarding stock volatility [e.g., Schmallensee and Trippi (1978) and Chiras and Manaster(1978)], investors' ex ante reaction to the quality of economic events [e.g., Patell and Wolfson (1981) and Whaley and Cheung (1982)] and the degree of arbitrage possibilities indicated by the deviation of actual security prices from implied prices [e.g.,Manaster and Rendleman (1982) and Peterson and Tucker (1985)]. It is only recently that researchers [e.g., Park and Sears (1985)] have begun exploring similar issues in the futures markets using options prices.

Daily closing Treasury bond call option and the underlying futures price data for a period ranginr from June 1983 to June 1985 were used in the study.[1] While options on Treasury bond futures started trading in October 1982, the initial trading history was excluded from the analysis. The data was excluded in order to base the analysis upon market history indicative of "steady stste" conditions on account of the relative newness of such options as well as the relative inexperience of traders and investors alike in using them for risk transfer and speculation. Options on Treasury bond futures are currently traded in the expiration cycle March, June, September and Decem-

1) The options and futures data was obtained from the Chicago Board of Trade OHLC/VS/OI data tapes. In instances where the closing prices were reported as a range, the mean of the range was used in the empirics.

ber. However, the options expire on the Saturday following the Friday preceding by at least five business days the first notice day of the Treasury bond futures contract. Therefore, since the options do not expire around the same time as the underlying futures contract, the results of the study are expected to be free from the effects of any abnormal valatility in the price bshavior of the underlying futures contract which occur at expiration. The data for the risk free rate was collected daily from the Wall Street Journal. This rate was calculated as the average of the bid and ask discount rate, converted to the bond equivalent yield for U.S. Treasury bills having the maturity closest to th expiration of the call option.

The Black commodity option pricing model is strictly applicable for European options, which cannot be exercised prior to maturity. Recently, several authors, particularly Brenner, Courtadon and Subrahamanyam (1985) and Ramaswamy and Sundaresan (1985) have also shown that despite the fact premature exercise may be optimal for American optons on futures contracts, the value of this feature is negligible and the Black formula for valuing these options appears to be a reasonable approximation. Nontheless, in view of these considerations, options which violated the lower boundary condition, $C \leq F - Xe^{-rT}$, and hence may be prime candidates for early exercise were deleted from the analysis.

In empirical applications of the Black futures options pricing model, the only unobservable factor is the instantaneous variance of the underlying futures contract. Much has been written in the options literature as to the correct method for estimatingthe volatility of the underlying security. Techniques ranging from simple historical methods, implied methods based on various weighting schemes, Bayesian methods where more recent observations are given more weight and methods which incorporate high, low, open and closing prices have been proposed in the literature.[2] Since this debate has not clearly identified the "best" estimator of varince, the implied Treasury bond futures prices were estimated simultaneously with the implied standard deviation using data from several options at the same time. A similar procedure was used by Manaster and Rendleman (1982) in assessing the informational content of implied stock prices in an option valuation framework.

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2) See Cox and Rubinstein (1985) and Jarrow and Rudd (1983) for a description of the various methods for estimating variance in an option pricing context.

Mathematically, this procedure is stated as :

$$\text{Minimize } Z\,[F_{jt},\ V_{jt}] = \sum_{i=1}^{N_{jt}} [C_i - C_i(F_{jt},\ V_{jt})]^2 \tag{2}$$

where C_i is the market price of the call option, $C_i(F_{jt}, V_{jt})$ is the model price determined from the Black valuation model, as specified in Equation (1) and N is the number of options on futures contract j at time t. Computationally, the algorithm simultaneously determines the implied futures price (F^*_{jt}) and the implied variance (V^*_{jt}) by minimizing the sum of squared deviations between C_i and $C_i(F_{jt}, V_{jt})$ using a non linear search procedure.[3)]

Ⅳ. Results

Using equation (2) and all available options quotes which met the selection criteria above, numbering over 11000 observations, were used to compute the implied futures price (F^*_{jt}) on a daily basis. Upon computation of the implied Treasury bond futures prices, percentage deviation errors (E_{jt}) for futures contract j on day t, defined as :

$$E_{jt} = [(F^*_{jt} - F_{jt})/(F_{jt})] \times 100 \tag{3}$$

were computed for the futures contract maturing in three months, designated as the

3) Several non-linear optimization techniques such as the Newton method, the Marquardt method and the method of Steepest Descent are available to solve equation (2). In this study, the solution to the equation was determined using the the sub-routine PROC NLIN of the Statistical Analysis System (SAS). While all three methods are available in SAS, the Marquardt method was used in the study. The choice of this alternative was based on the observations by Bazaraa and Shetty (1979, pp.289-297) regarding the pitfalls of the other methods. The Newton method may fail to converge if the starting point is not sufficiently close to the optimal point. The Steepest Descent method may run into the problem of "zigzagging" as the optimal solution is approached. In other words, the algorithm may take only small orthogonal steps which may be computationally expensive. Complete details of the algorithm are available to interested readers upon request.

"nearby" contract and the futures contract maturing in three to six months, designated as the "deferred" contract. The frequency distributions for these percentage deviations are presented in Table 1. An analysis of this table reveals thst for the Treasury bond contract of the most immediate maturity about 78 percent of the observations while about 55 percent of the observations for the contract with the maturity in three to six months are within 1 percent of observed futures prices. Additionally, for the deferred contractabout 88 percent of the observations are within 2 percent of the observed prices, although there is a positive bias in the percentage deviations for this conrtact. These results suggests that the Black model tends to overprice Treasury bond cal options which have a longer time to maturity.

In order to assess the informational value of the difference between implied and actual Treasury bond futures prices, the analysis was conducted in a market efficency context. As noted by Manaster and Rendleman (1982), the implied price and the observed futures price are estimates of the unobservable equilibrium price. This suggests that movements in implied and observed futures prices should be highly correlated if there is informational value in implied prices. A practical implication of this contention is that high (low) proportional deviations (E_{jt}) should be associated with high (low) price changes in observed prices. Therefore, the finding that Treasury bond option prices contain information which is not reflected in observed Treasury bond futures prices would violate the concept of market efficiency. Recall that the notion of market efficiency as propounded by Fama (1970) indicates that security prices instantaneously reflect all available information.

Table I

Frequency Distribution of Percentage Deviations(E_{jt})

Nearby Contract

Range of E_{jt}	Number of Observations	Percentage of Observations
〈 -2	16	2.95%
-2 to -1	54	11.23%
-1 to 0	193	40.12%
0 to 1	183	38.05%
to 2	35	7.28%
Total	481	99.65% *

Deferred Contract

Range of E_{jt}	Number of Observations	Percentage of Observations
< -1	9	1.50%
-1 to 0	63	13.10%
0 to 1	204	42.41%
1 to 2	157	32.64%
2 to 3	20	4.16%
3 to 4	18	3.74%
> 4	10	2.08%
Total	481	99.63% *

* less than 100% due to rounding errors

Previous studies of the informational content off implied (option model) security prices [e.g., Manaster and Rendleman (1982) and Peterson and Tucker (1985)] have investigated the relationship between security returns and proportional deviations. However, since the initial investment in futures contracts represents a "good faith" deposit which may be posted in the form of interest bearing escurities, futures contracts do not generate "returns" in the sense commonly accepted and used in the finance literature. In view of these considerations, the relationship between price changes and proportional percentage changes between implied and observed prices is investigated instead. The use of price changes instead of returns in the analysis is tantamount to assuming that futures prices follow a martingle process and is based on the premise that futures prices, as noted by Telser (1958), do not contain a risk premium. This assumption is fairly standard in futures market research and has been employed extensively [e.g., Figlewski (1981) and Bortz (1984)], although some authors [e.g., Danthine (1977)] have questioned the validity of this assumption.

The informational content hypothesis was investigated by regressing observed closing percentage price changes in the Treasury bond futures market (z_{jt}) on lagged oercentage deviations (E_{jt}). These percentage price are defined as :

$$z_{jt} = [(F_{jt} - F_{jt-1})/F_{jt-1}] \times 100 \qquad (4)$$

If implied futures prices do not contain any information beyond that already present in observed Treasury bond futures prices, then tere should be no relationship between the E_{jt}'s and future price changes in the Treasury bond futures market. The equation for these tests is stated as :

$$z_{jt} = a_j + b_j \ E_{jt-1} + u_{jt} \qquad (5)$$

where a_j and b_j are parameters and u_{jt} is the error term. In eqation (5), if implied Treasury bond futures prices do not contain any information about futures prices, then the coefficient b_j is expected to be zero. These regressions were performed individually for the different classification of the percentage deviation errors (E_{jt}) specified in Table for each of the futures contracts under consideration. The rationale for conducting this segmented analysis is based on the assumption that if implied futures prices contain additional information about Treasury bond futures prices beyond that found in observed prices, then hihger values of the E_{jt} are expected to be associated with greater futures price changes.

Table II

Relationship Between Proportional Deviations (E_{jt})

and Treasury Bond Futures Price Changes

Near Contract

Range of E_{jt}	a	b	F-value	R-square	DW Statistic
〈 -2	0.9291 (0.609)	-0.1257 (-0.063)	0.045	0.0032	2.01
-2 to -1	-0.1267 (-0.160)	-0.2829 (-0.911)	0.831	0.0157	1.83
-1 to 0	-0.0621 (-0.625)	-0.5397 (-0.624)	1.883	0.0350	1.83
0 to 1	0.1715 (0.103)	-0.0625 (-0.747)	1.032	0.0243	1.84
1 to 2	0.2452 (0.566)	-0.0751 (-1.394)	1.518	0.0258	2.06

(t-statistics in parentheses)

Table III

Relationship Between Proportional Deviations (E_{jt})

and Treasury Bond Futures Price Changes

Deferred Contract

Range of E_{jt}	a	b	F-value	R-square	DW Statistic
〈 -1	-0.2091 (-0.495)	-0.2352 (-0.216)	0.792	0.0562	2.42
-1 to 0	0.3139 (0.939)	0.2706 (0.812)	0.842	0.0176	1.59
0 to 1	0.1417 (0.456)	-0.0338 (-0.159)	0.025	0.0001	1.74
1 to 2	0.4308 (0.103)	-0.4964 (-0.747)	1.558	0.0137	1.86
2 to 3	-0.6057 (-0.135)	0.7657 (0.919)	0.913	0.0483	1.21
3 to 4	-0.5982 (-0.966)	0.6973 (0.919)	0.844	0.0501	2.29
〉 4	0.3873 (0.840)	-0.6247 (-0.859)	0.754	0.0493	2.32

(t-statistics in parentheses)

The results of these tests are presented in Table for the near contract and in Table for the deferred contract. An analysis of these results reveals that in all cases the regression relationship between the E_j and futures Treasury bond prices changes is not statistically significant, as indicated by the F-statistics.

Moreover, the slope coefficient b_j which measures the differential response of price changes to the E_{jt} is not statistically different from zero, regardless of the level of the E_{jt} and the maturity of the futures contract. These results sugesst that while observed futures prices may be different from implied (option model) prices, the informatonal content of these implied pricces is negligible indicating that the Treasury bond futures and options markets are characterized by a high degree of price efficiency.

V. Concluding Comments

While the results of this research have cast doubts on the ability of the Black commodity option model to price options with a longer time to maturity, it appears that the implied Treasury bond futures prices determined from the option pricing model cannot serve as early warning indicators of the directon of future price movements. However, a caveat is in order. As noted by Bookstaber(1981), a problem with using summary information such as closing prices for the options and the underlying security is the non-simultaneity of quotations of the options and the underlying security. For instance, the futures prices that is used to detect deviations from implied prices may not be the same price that existed in the market when the last option quoted. This non-simultaneity may cause inaccuracies in the empirical results, especially in the pricing of options. In the present context, whether the results of this study would be improved using simultaneous quotes on Treasury bond futures and options data is a question, which we hope to address in futures research.

However, the conduct of such studies is severely limited by the availability of simultaneous data on options and futures quotations. While simultaneous quotes, time stamped to the second are available for stock options from the Berkeley Options Data Base, there is no comparable data base for futures market transactions. An inexpensive source of data which appears promising for research which avoids any biases created by the non-simultaneity of options and futures quotations is the Time and Sales File compiled by the Chicago Board of Trade. However, the limitation of this data base is that it records transactions only at price changes. Therefore, the results of tests conducted using this data may be limited in that any option mispricing when prices remain unchanged would not be included in the scope of the analysis.

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