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市場造成危險과 株式價格의 形成

〈국문요약〉

張 大 洪*.

국내외에서 주식가격의 과잉변동현상에 대한 실증적 연구결과가 보고되어 왔다. 이에 대한 이론적인 설명은 대체로 두가지 접근방법에 의해 시도되어 왔다. 그 하나는 거품현상으로 이해하고자 하는 것이며, 다른 하나는 자본시장의 미시적 구조를 규명함으로써 전통적인 왈라스 균형가격 형성과정을 수정하고자 하는 것이다. Kraus 와 Smith[1989]는 시장상태에 대한 불완전정보와 이의 해결과정이 균형주식가격의 과잉변동과 위험을 초래할 수 있음을 보여주고 있다. 그러나, 그들의 논문에서 지적되고 있는 시장정보 즉, 위험중립적 및 위험회피적 투자자집단의 증권보유상태에 대한 불완전한 정보는 그 속성상 지속적으로 보기 어렵고, 베이지안 과정(Baeysian Process)에 의해 해소될 가능성을 가지고 있다.

이 논문에서는 이러한 단점을 보완하여, 시장의 거래행위자체가 시장상태에 대한 불완전정보를 영속적으로 발생시킬 수 있으며, 이에 따라 형성된 주식가격이 과잉변동할 수 있음을 보여준다. 논문에서 사용된 모형은 평균-분산 구조하의 복수거래 모형으로서, 위험증권의 수요 및 가격을 유도하고, 이를 구성하는 모수 즉 총괄적 효용특성이 거래행위에 따라 변동함을 보여준다. 이와같은 총괄적 모수에 대한 불완전정보는 거래행위를 반복함으로써도 해소될 수 없으며, 주식수익율 과잉변동의 원인이 될 수 있음을 입증해준다. 또한, 불완전시장정보와 완전시장정보하에서의 주식수익율에 대한 상대적 분산을 비교하여 구한 과잉변동계수를 비교해 보면, 과잉변동이 자기실현적(Self-justifying)이 될 수 있음을 알 수 있다.

* 翰林大學校 經營學科 副敎授

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Market Created Risk and the Formation of Stock Price

Jaang, Dae hong*

〈Contents〉

Abstract

This paper developes a multiperiod trading model of securities price formation which extends the notion of market created risk introduced by Kraus and Smith [1989]. It is shown that stock price volalitility can depend on combinations of market parameters known to the market participants only imperfectly. Resulting portfolio rebalancing equilibria generate self-justifying price movements while market fundamental remain unchanged.

I. Introduction

Economists have tried to rationalize the volatility of security price movements for long.[1] Early theoretical discussion had addressed the issue around the existence of

* Associate Professor of Finance, Hallym University

1) Keynsian claim for the desirability of active government intervention in the fiscal or the monetary policy is baesd, in part, on the observation that asset prices seemed to be unduely volatile.

speculative 'bubble' in dynamic asset market. In dynamic rational expectations economy, the existence of multiple equilibria is not uncommon and may lead to highly unstable price patterns or bubble. The latter arises essentially due to the lack of perfect arbitrage because of the mismatch between the life spans of the market and market participants. In the sunspot literature prices might move only because the participants believe they will. The source of such a belief, say sunspots, market psychology, animal spirits etc., has been termed extrinsic uncertainty, contrary to that on the market fundamentals which is called intrinsic uncertainty.[2)]

In financial economics, the issue has been closely related with the asset market's informational efficiency. While the dissatisfaction with the idea has long been around, especially among the practitioners, a serious question was frist raised by Shiller[1983] which argued that the stock price movements has been far too volatile to be justifed by the subsequent dividend payments which is thought to represent the relative stability of the underlying economy or market fundamentals. Though many of his claims have been critically reexamined since then, the very notion of excessive stock price movements has been firmly placed as a unresolved problem. The idea of excessive volatility seemed to have gained ground by the world wide stock market crashes happened around 1986. In Korea, it has been confirmed that stock prices have been overeacting to "randomly" arriving new information.[3)]

On applications level, the volatility can be treated within a model as a trading noise as, for example, in Black[1986] or Amihud and Mendelson[1987]. However, for an analytical purpose, not to mention of getting normative solution for the problem, the nature of noise needs certainly to be analyzed.

One way to do this is to study the market microstructures, namely to analyze the institutional structure that affects the price movements.[4)] The other is to incorporate the trading mechanism that potentially create volatility in the equilibrium process of price formation, as is done by Kraus and Smith[1989]. They have shown that beliefs about market states, in particular, the distribution of securities endowment, known only imperfectly, may lead to a pooling equilibrium price that will eventually be bro-

2) The terms originated from Cass and Shell[1983].

3) See Kim et al [1988].

4) See, for example, Scholes[1973] or Amihud and Mendelson[1987].

ken to a different price on a later trading in which the uncertainty about the endowment are resolved. Within this framework securities price can change without corresponding changes in the market fundamentals.

While successful in demonstrating the possibility of excessive volatility in an intuitively appealing way, their analysis seems to have a few shortcomings. Most of all, the role of market parameters, intrinsic as well as extrinsic, in forming stock prices in each trading periods is unclear. Instead, they seem to suggest the inabilities of investors to tell the exact distribution of securities endowments causing the uncertainty about market state. However, the distribution itself should be a result of market equilibrium, under the given set the market parameters, which investors need not know.[5] This is inevitable with the simple dichotomized market structure used in their model, where there are only two classes of utilities, one with the risk neutral and the other with the log utility. Another consequence of their market structure is that it can not explain why a market state or any market parameter that might have resulted the particular state should remain unknown. Thus it does not follow from their analysis that the market persistently generate volatile price movements.

This paper builds on the notion of market created risk by Kraus and Smith[1989] and attempts to incorporate explicitly the parameters of beliefs as well as 'market' fundamentals in a familiar framework of mean-variance model. In addition to extending their results for a more familiar market setting, this paper brings forth a few important insights on the formation of stock price, market risk premium and the issue of why the stock price should exhibit a persistent volatility.

The contents of the paper is organized in the following order. Section I describes the market setting. In section II portfolio rebalancing equilibria are derived and characterized. Section III explicitly derives the risky asset prices under portfolio rebalancing equilibria and examines their properties. Section IV contains concluding remarks.

5) In fact, Kraus and Smith[1989] refers vaguely to markert states, which they actually identifies with the distribution of the securities endowments.

II. Multiperiod Model of Securities Trading

There are three dates, t=0, 1 and 2. The final date, t=2 is when each security makes its final payment and dissolves. No interim payoff occurs. There are two securities, one riskfree, the other risky security. The latter may be considered as a market portfolio.[6] Riskfree security pays r, which is known in each period, units of numeraire good at t=2 per each unit purchased.[7] One share of risky stock pays R units of the good at t=2 with a given probability distribution, its mean and variance denoted by μ and σ^2 respectively. Each market participant or investor, indicated by superscript h=1, 2, ⋯, starts period 0 with endowments z_0^h, z_1^h, shares of the riskfree and the risky securities respectively. The states of nature affecting the payoff of risky security, are assumed a common knowledge, though itself may be imperfectly known.[8]

Each investor trades x_k^h, k=0, 1, shares of each securities at t=0 to maximize the expected value of the derived utility as of date 1. In the following the superscript will be suppressed unless required for clarity. Date 2 can be considered as an artificial date at which all the uncertainties, including those of market states, are resolved. On the other hand, t=1 is considered to be the final trading date at which remaining doubts about market states, if any, becomes irrelevant. Uncertanities about market include the distribution of investors' utility parameters, the security endowments and even the choice of (or beliefs about) parameters[9], μ, σ^2. For convenience, it is assumed that dates t=0 and t=1 are close enough that no intertemporal arbitrage between the two dates is meaningful.

As usual it is easier to describe the optimization process backward at t=1, at which the market reopens after the initial trading at t=0 and y_k^h, k=0, h = 1, 2, ⋯, shares are traded. An investor's portfolio problem at t=0 is described by

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6) The analysis can be easily extended to many risky assets with results similar to that follows.

7) Its price is therefore known in each period and can be taken as one.

8) For example, the choice of distribution parameters μ, σ^2, may be a common knowledge.

9) Note that while the true parameters remain unchanged investors may have only its probability distribution (or belief about) at t=0.

$$\max_{y_0,\, y_1} E[\alpha\omega - (\beta/2)\omega^2] \tag{1}$$

with $\omega \equiv y_0 r + y_1 R, \quad \alpha > 0, \ \beta > 0$

subject to $y_0 + y_1 s = x_0 + x_1 s,$

where s is the market price of one share of risky security and E denotes the expectation about R. In keeping with the focus here E is homogenuous across investors as well as across time, that is $E^h_t = E$, $t = 1, 2$, $h=1, 2, \cdots$. The uncertainty about the market state at $t=0$ is introduced by probability π_m, $m = 1, 2, \cdots M$. Each market state corresponds to a set of values for utilities, α_m and β_m, endowments, z_{0m}, z_{1m}, x_{0m}, x_{1m}, and μ_m, σ^2_m. The portfolio problem at $t = 0$ can be written as

$$\max_{x_0,\, x_1} \Sigma_m \ \pi_m E_0[E_1\alpha\,\omega^*_m - E_1(\beta/2)\omega^{*2}_m] \tag{2}$$

subject to $x_0 + x_1 p = z_1 + z_1 p$

where p is the market price of the risky share at $t = 0$ and the terms in the bracket indicates the derived utility as of date 2, with ω^* being the wealth for y^*_0, y^*_1, the solution to problem (1)[10]. However, y^*_0, y^*_1, in problem (2) are now market-state dependent, namely y^*_{0m}, y^*_{1m}, respectively,to be precise.

III. Portfolio Rebalancing Equilibrium

The solution to (1) can be given as

$$y^h_1 = [c^h - (x^h_0 r + x^h_1 sr)] \ (\mu - sr)/[(\mu^2 + \sigma^2 - 2rs + r^2 s^2)] \tag{3}$$

where $c^h \equiv \alpha^h/\beta^h$. Those investors for whom $c^h < [\mu - rs + \sigma^2/(\mu - rs)]y_1 + (x_0 + x_1 s)r = x^h_0 r + x^h_1 sr$ for all $y_1 > 0$ would hold only riskfree assets. Aggregating (3) over the

10) See equation (5) below.

investors who hold nonzero amounts of the risky security and assuming that aggregate number of shares are 0 and 1 respectively for the riskfree and the risky securities, we can derive the market price as[11)]

$$s = (\mu^2+\sigma^2-c\mu)/[(\mu-c)r], \quad c \equiv \Sigma_{h \in H}\ \alpha^h/b^h$$

$$\text{with } H = \{h/y_1^h \neq 0\} \tag{4}$$

Set H in (4) contains only the investors who chooses to hold nonzero amounts of the risky share. For later purpose, we note that

$$\partial s/\partial r > 0,\ \partial s/\partial \mu > 0,\ (\text{provided that } (u-c)^2 > \sigma^2),\ \partial s/\partial \sigma^2 > 0$$
$$\partial s/\partial c > 0,\ \partial^2 s/\partial c^2 > 0$$

The aggregate utility parameter, c in (4), is partially determined by the market process while c^h themselves are exogenuously given, if only known subjectly to the investor. Therefore, the market price itself does not reveal the true distribution of the parameters without a knowledge of the distribution of the security holdings at equilibrium. It should be also noted that the distribution of security endowments, x_k^h, k=0, 1, h = 0, 1, ···, does not appear in (3) and (4) beacause they are assumed out as a common knowledge throughout the different trading dates. This is reasonable since only aggregate number of shares is necessary information for trading purposes[12)] which is often one of the most easily accesible and stable data for the market.

Substituting (3) for y_0^*, y_1^*, ω^* in problem (2) can be rewritten as

$$\omega_m^{h*} = \{r[(s_m-p)x_{1m}^h + z_0^h + z_1^h p](R-rs_m)\ (\mu_m-rs_m)+c_m^h(R-rs_m)\}/D_m,$$
$$D_m \equiv \mu_m^2+\sigma_m^2-2\mu_m rs_m+r^2 s_m^2 \tag{5}$$

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11) The expression (4) may appear to be counter intuitive in that the higher variance implies the higher price. This is so because the variance contains the level of price itself. Letting $\mu^*=E(R/s)$ and $\sigma^{*2} = var(s)$, equation (4) can be rewritten as $s = c(\mu^*-r)/(\mu^{*2}+\sigma^{*2}-\mu^* r)$

12) This can be easily from equation (3) where each investor's trading does only on his or her own number of shares and share price. The reason why aggregate number of shares does not appear in (4) is because of our simplifying assumption that $\Sigma_h x_k^h=0$ and 1 respectively for k = 0, 1. Otherwise they should be present in (4).

It can be easily checked, using (1), (3) and (5), that $E_0\omega^*$ reduces to $E_1\omega$ in problem (1) if $p = s_m$ for some market state m,in which case the two problems become indistinguishable.[13] If $p \neq s_m$ for at least one m, problem (2) with (5) can be solved to yield

$$x_1^h = \frac{\Sigma_m\ \pi_m[c_m^h(s_m-p)-r(z_0^h+z_1^h p)]\ \sigma_m^2/D_m}{\Sigma_m\ \pi_m\ r(s_m-p)\sigma_m^2/D_m} \tag{6}$$

By construction, equation (3) and (5) impliy that a portfolio rebalancing must occur at $t = 1$. Aggregating (6) yields

$$p=\Sigma_m\ \sigma_m s_m,$$
$$\sigma_m \equiv [\pi_m(c_m-rs_m)\ \sigma_m^2/D_m]/[\Sigma_m\ \pi_m(c_m-rs_m)^2\ \sigma_m^2/D_m] \tag{7}$$

Equations (3), (4) and (6), (7) constitute portfolio rebalancing equilibrium except for some trivial cases where p in (7) become identical to s in (4) for some market state. Comparative statics results with ex post share prices s_m hold exactly the same as before. In addition, ex post prices are independent of probability belief π_m, while the effect of belief on ex ante share price is ambiguous.[14] The latter can be illustrated in relation with the familiar notion of market risk premium which can be written as

$$mrp_m \equiv (\mu_m-rs_m)/s_m-r = \mu_m(\mu_m-c_m)/(\mu_m^2+\sigma_m^2-c_m\mu_m)-r$$

It can be easily shown that the ex post market risk premium is strictly decreasing in utility parameter c_m. Ex ante market risk premium is similarly defined as

$$mrp^p \equiv \Sigma_m\ \mu_\mu/p-r=\Sigma_m\gamma_m mrp_m$$
$$\gamma_m \equiv [\pi_m s_m(c_m-rs_m)/D_m]/[\Sigma_m\pi_m s_m(c_m-rs_m)/D_m]$$

The weghts for the ex post market risk premiums can be shown to be strictly increa-

13) This is essentially the case of separating equilibrium discussed in Kraus and Smith[1989].
14) Note that
$\partial s_m/\partial\pi_m=0,\ \partial\Sigma_m(\pi_m s_m)/\partial\pi_m=s_m+\Sigma_{k\neq m}(\partial\pi_k/\partial\pi_m)s_k$

sing in c_m. Thus market risk premium, in the ex ante sense, does not necessarily decrease with the risk aversion parameter.

While the portfolio rebalancing prices above may appear to depend on the artificial division of trading dates, it can be easily shown that, should there be more trading dates before $t=1$, the equations for security holdings and for the price remain to be of the same general form as those in (6) and (7) except for the fact that the interim price p would replace s in (7). In this sense, rebalancing may persist relative to interim price price p which by itself is the pooling price in the sense of Kraus and Smith [1989]. Such a rebalancing may continue as long as the price reveal unambiguously the distribution of utility parameters.

As is clear from the configuration of equilibrium prices,sources of such a rebalancing colud be any set of imperfectly known market parameters of $\{\alpha, \mu, \sigma^2\}$. Furthermore, aggregate number of shares could easily be incorporated as such a parameter within the present model. It is unlikely, however, that the distribution or the aggregate number of shares, given the accesibility of the data,would cause a market uncertainty.[15)]

Ⅳ. The Volatility of Stock Price

Given the portfolio rebalancing equilibria derived above, share prices are likely to be volatile simply because there are likely to be more trading as long as the 'true' state of nature remain unknown. As observed earlier, this is likely so for the market state associated with the aggregate utility parameters even if parameters of the states of nature, or those of exogenous market parameters for that matter, remaining unchanged. Since more trading does not necessarily mean more volatility, a deeper question

15) This is in contrast to Kraus and Smith[1989] where such a data is the source of a pooling equilibrium price like (7). With a log utility case, as used in their results, utility parameters plays little role but the distribution of shares can not be aggregated out. This is why the distribution of shares is the source of the volatility in Kraus and Smith[1989]. They also had to introduce risk neutral investor to circumvent the problem caused by a signgle utility parameter of log utility.

to be answered must be whether equilibria associated with market created risk leads to a higher volatility than with no such a risk. To study the nature and the extent of such a volatility in a tractable way, we will abstract from other sources of volatility but that of utility parameters. Utility parameters are often the least known data of the market but also is closely related with what might be called an 'investor psychology'. Assuming that μ and σ^2 are a common knowledge and there are M distinct values for the equilibrium utility parameter, c_m, $m=1, \cdots, M$, each corresponding to a market state, the euqations for the price in each date can be written as,

$$s_m = (\mu^2+\sigma^2-c_m\mu)/[(\mu-c_m)r] \qquad (8)$$

$$p=\Sigma_m\delta^*_m s_m, \ \delta^*_m \equiv [\pi_m(c_m-rs_m)/D_m]/[\Sigma_m\pi_m(c_m-rs_m)/D_m] \qquad (9)$$

In (8) and (9), a market state refers to that for c_m only. Note that $t=0$ price is a weghted average of s_m where weights can be considered to be market-state-adjusted probabilities for different utility parameters.[16)]

To study the volatility of stock return under different prices, define a relative measure of volatility as,

$$v=[\mathrm{var}(R/p)/\mathrm{var}(R/p_n)]^{1/2} = p_n/p \qquad (10)$$

If the relative volatility measure is greater (less) than one, then the return variance under price p is greater (less) than that under a benchmark price p_n. If the conditional price s_m and the (portfolio) rebalancing equilibrium price p are used for p_n, v measures the volatility of stock price under the rebalancing equilibrium relative to that with no market created risk. Since δ^*_m in (9) does not depend on it follows that

Proposition 1. The relative volatility for (p, s_m) is independent of the risk free rate.

16) To be considered as probabilty weights, $(c_m-rs_m)/D_m$ must be nonnegative for all m. It is assumed that other market parameters are such that the nonnegativity holds.

However, the volatility measure is not independent of other parameters.

Since s_m is conditional on a market state, it is reasonable to think of an expected measure of relative volatility defined as

$$v^p = [\mathrm{var}(R/p)/\mathrm{var}(R/\Sigma_m \pi_m s_m)]^{1/2} = \Sigma_m \pi_m s_m / p \qquad (11)$$

The following two results show the cases where the rebalancing equilibria with the market created risk generate more volatility in the stock return in the sense of (11).

Proposition 2. The expected volatility, v^p, is greater than (less than, equal) if and only if

$\Sigma^k_m \pi_m \leq (\geq, =) \ \Sigma^k_m \pi_m (c_m - rs_m)/D_m]/[\Sigma^M_m \pi_m (c_m - rs_m)/D_m]$ for all $k \leq M$.

Proposition 3. Suppose $\Delta c_m = c$ for all m. The expected volatility, v^p, is greater than (less than, equal) if and only if

$\Sigma^n_{k=1} \Sigma^k_m \pi_m \leq (\leq, =) \ \Sigma^n_{k=1} \Sigma^k_m \pi_m (c_m - rs_m)/D_m]/[\Sigma^M_m \pi_m (c_m - rs_m)/D_m]$
for all $n \leq M$.

The proofs for the two positions above, making use of a straightforward application of the first and the second order stochastic dominance for the two probability distributions π and δ with the convex price function $s = s(c)$, are omitted.

An intuitively more appealing result for the volatility can be obtained by restricting the probability belief on market states as following

Proposition 4. Suppose

$\Sigma^n_m [(c_m - rs_m)/D_m]/[\Sigma^k_m \pi_m (c_m - rs_m)/D_m] \geq n/k$ and $\Sigma^k_m \pi_m (c_m - rs_m)/D_m] \geq k$ for all $k \leq M$. If the common knowledge belief, π_m, is such that the higher share price, in ex post sense, is more likely, that is, π_m increasing with s_m, than the volatilty increases in the sense that $v^p > 1$.

(proof) By a direct differentiation of $(c_m - rs_m)/D_m$ with respect to c_m, it can be shown that it is monotonically increasing in c_m. Since s_m is also a monotonic increasing function of c_m, it follows that π_m is increasing in s_m if and only if it is increasing in $(c_m - rs_m)/D_m$. Let $q_m \equiv [(c_m - rs_m)/D_m]/[\Sigma_j^k \pi_j (c_j - rs_j)/D_j]$ and $A_k \equiv \Sigma_m^k q_m$. By construction, q_m/A_k is a probability measure. Since

$$\Sigma_{j=1}^n q_j(k/A_k)/k \le n/k = \Sigma_{j=1}^n (1/k) \text{ for all } n \le k$$

and $1/k$ is also a probability measure for each k, it follows that q_m/A_k stochastically dominates $1/k$, in the first order sense. Since π is strictly increasing in q, it follows that

$$\Sigma_{j=1}^k q_j \pi_j \ge \Sigma_{j,=1}^k q_j(k/A_k)\pi_j \ge \Sigma_{j=1}^k \pi_j \text{ for all } k \le M.$$

The assertion holds by proposition 2 above since p is increasing with $(c_m - rs_m)/D_m$.

QED

V. Conclusion

The existence of self-fulfilling stock market equilibrium with price volatility demonstrated in this paper is perhaps not surprising. Our contribution lies mostly in the expositional strength in that the set of parameters for generaring the volatility is much more diverse and their relation much more complicated than the first impression on the sources of volatility. Furthermore, the utility parameters and the beliefs on them are shown to be important in creating the volatility. These factors are undoubtedly what people have in mind, at least partially, when they say stock prices are sensitive to market psychology[17]. Recall that the volatility obtained here is essentially of the nature that imperfect knowledge of market parameters persist. In that sense stock prices tend to be volatile with true parameters, so called market fundamentals remaining unchanged. However, the true states of market parameters may never be known but only the beliefs about them are realized in the subsequent trading periods. In that

17) See Shiller[1983].

cases the portfolio rebalancing equilibrium persist on the realized beliefs.

On the methodological level, it may be argued that the simple market setting used in the paper needs to be generallized, in particular that of quadratic utility. Its use may be defended,however, on the ground that it generates intuitively pleasing results in a tractable way and yet they do not depend on the objectionable property of the function, namely increasing relative risk aversion.

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