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Production and Inventory Management Using Multiple Objective Decision Making

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Abstract

Up to the present, the evaluation measures in the production and inventory management have been studied under the pre-condition that the costs for major factors (e.g., cost of carrying inventory, cost of demand shortage) are given easily, although in practice, it is difficult. The case in which multiple participants have a different viewpoints in production and inventory management has not been studied, in spite of its frequent occurrence.

This study suggests a production and inventory model with multiple objectives corresponding to major factors and the related interactive algorithm based on the preference structures of participants. The problem can be solved through a weighting vector generated by an interaction with participants. The concept of equity is also used in order to guarantee the reasonable distribution of group utility in determining the individual relative weights of participants. This study includes the reality of the model and the decision process in the production and inventory management.

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1. Introduction

A general production planning problem (PPP) is to determine the required size of the Work-force, rate of production, and inventory level for each period given a product demand forecast. Many production planning algorithms have been developed under the pre-condition that the following major factors, i.e., cost of payroll (regular and overtime), cost of carrying inventory, and cost of demand shortage, etc. are given. However, it is practically very difficult to assess the costs of carrying inventory and demand shortage. It is desirable to adopt practical measures, instead of costs, in order to appreciate the alternatives (solutions) of PPP. For example, the factor related to inventory can be easily measured by the amount of inventory instead of the cost of carrying inventory. By treating each factor as an objective, PPP can be converted into one of the multiple objective decision making (MODM) problems.

Recently, MODM techniques have been used in PPP. Lawerence and Burbridge (1976), Lockett and Muhlemann (1978, Gonzalez and Reeves (1983), Rakes et al. (1984), and Michalowski and Zolkiewski (1983) used Goal Programming (GP) solving PPP with multiple objectives. Masud and Hwang (1980) solved multiobjective PPP using three MODM methods, e.g., GP, Step Method, and Sequential Multiple Objective Problem Solving. Multiobjective production planning based on the participants' different conflicting preference structures has not been considered, since there is not a MODM technique suitable for the case in which participants' different conflicting preference frequently occurs in PPP.

The decision process of PPP with participants' conflicting viewpoints is very complex since the supplier is interested in selling as many products as possible and wants the stockist to carry a high level of inventory in order to minimize lost sales. At the same time, the stockist has to bear the cost of the carrying inventory and is seeking to maintain a balance between inventory carrying costs and sales revenue.

In earlier studies, the conflict among the objectives which are the factors for evaluation of each PPP alternative reflecting the conflicting viewpoints of the participants have not been considered. In this paper, the interactive algorithm to be extended is based on the weighting method using a weighting vector. The weighting vector can be interpreted as the synthesis of participants' preference structures and is a key to finding the best compromise solution. The weighting vector is generated iteratively by the combination of the previous gradient (i.e., the weighting vector of the previous iteration) and the desirable gradient of group (consisting of stockist and supplier) utility. The desirable direction gradient in which the group utility increases most rapidly is produced by using the values of marginal rate of substitution (MRS) assessed from both stockist and supplier. In determining the desirable gradient, the concept of equity (Keeney and Kirkwood 1975, Brock 1980) is used in order to guarantee the fair and reasonable distribution of group utility.

2. Definition of the problem

The situation of interest is the multi-period, multi-product PPP with multiple objectives, which are : to minimize production cost (f_1), to maintain the balanced labor force (f_2), to minimize the amount of overtime production (f_3), and to maintain the proper amount of inventory (f_4). Both stockist and supplier take part in the decision process of production planning.

The stockist and the supplier express their opinions to determine, for each product and each of the planning periods, the best production level inventory level, and amount of overtime production. They also offer their views for seeking the adequate work-force level for each period. Therefore, they agree on some aspects, i.e., maintaining a balanced work-force level, minimizing production cost, and minimizing the amount of overtime production. But with respect to the best inventory level, there is a conflict between stockist and supplier. The stockist intends to minimize the inventory level because of the cost of the carrying inventory. The supplier, on the other hand, intends to maximize the inventory level in order to minimize lost sales. Accordingly, the different viewpoints between them must be considered simultaneously in PPP. If we let x be the decision variable(s) and X be a feasible region of production planning, then the model from each viewpoint can be represented as follows :

Stockist's Viewpoint ; $\min_{x \in X} \ (f_1(x), f_2(x), f_3(x), f_4(x))$

Supplier's Viewpoint ; $\min_{x \in X} \ (f_1(x), f_2(x), f_3(x))$

$\max_{x \in X} \ f_4(x)$

There are two possible ways to obtain nondominated solutions of the above problem with different viewpoints. First, after solving the stockist's and supplier's models, the nondominated solutions of the problem are represented by the intersection of nondominated solutions for each model. Second, after eliminating the objectives which cause a complete conflict in each model, a new monolithic formulation is made by the objectives which remain in the model. The new monolithic model is as follows :

$$\min_{x \in X} \ (f_1(x), f_2(x), f_3(x))$$

In the second case, the nondominated solutions of the problem are simply represented by the nondominated solutions of the new monolithic model. However, it is doubtful whether the intersection of nondominated solutions in the first case is not always empty and is always identical with the nondominated solutions in the second case. In the suggested problem, the objective f_4 generates a complete conflict. Let $\nabla f_4^t, \nabla f_4^u$ be the gradient vectors of f_4 in stockist's and supplier's viewpoints and $\nabla f_1, \nabla f_2, \nabla f_3$ be the gradient vectors of f_1, f_2, f_3, respectively. Let C_t and C_u be the polar cones generated by ($\nabla f_1, \nabla f_2, \nabla f_3, \nabla f_4^t$) and ($(\nabla f_1,$

∇f_2, ∇f_3, ∇f_4^u), respectively. And let C_G be a polar cone by (∇f_1, ∇f_2, ∇f_3). In particular, C_t and C_u have a property such that $(C_t \cap C_u)$ is not a empty set.

Theorem 1. Let X be a convex and nonempty set and let D_t, D_u, D_G be the nondominated solution sets generated by each polar cone C_t, C_u, C_G, where D_t, D_u, D_G, are the subsetst of X. Then $D_t \cap D_u = D_G$ and $D_G \neq \phi$.

Proof. $D = \{x \mid X \cap C(x) = \phi,\ x \in X\}$. C(X) means the remainder except for x when x is a vertex of cone C. Since the set X is not empty and convex, there exists more than a vector in X, which satisfies $X \cap C(x) = \phi$. Therefore, $D_G \neq \phi$.

If we let $D_t = \{X \mid X \cap C_t(x) = \phi,\ x \in X\}$, $D_u = \{x \mid X \cap C_u(x) = \phi,\ x \in X\}$, $D_t \cap D_u$ is defined as follows:

$$
\begin{aligned}
D_t \cap D_u &= \{x \mid X \cap C_t(x) = \phi \text{ and } X \cap C_u(x) = \phi,\ x \in X\} \\
&= \{x \mid (X \cap C_t(x)) \cup (X \cap C_u(x)) = \phi,\ x \in X\} \\
&= \{x \mid X \cap (C_t(x) \cup C_u(x)) = \phi,\ x \in X\}.
\end{aligned}
$$

Let the cone C^+ be a sum of two polar cones C_t and C_u. $C_t \cup C_u$ is identical to C^+ when $C_t \cap C_u \neq \phi$ and C^+ is identical to C_G. Therefore,

$$
\begin{aligned}
D_t \cap D_u &= \{x \mid X \cap (C_t(x) \cup C_u(x)) = \phi,\ x \in X\} \\
&= \{x \mid X \cap C^+(x) = \phi,\ x \in X\} \\
&= \{x \mid X \cap C_G(x) = \phi,\ x \in X\}.
\end{aligned}
$$

Accordingly, $D_t \cap D_u = D_G$ since $D_G = \{x \mid X \cap C_G(x) = \phi,\ x \in X\}$. Q.E.D.

Using theorem 1, the nondominated solutions of the problem are identical in both cases. Hence, we will solve the new monolithic model for obtaining nondominated solutions of the problem instead of solving the stockist's and supplier's models separately.

3. The model

The notations in the multiobjective model for solving the problem are as follows : Variables :

H_t : worker hired in period t (man-day).

I_{it} : inventory of product i at the end of period t (units).

L_t : worker lay-off in period t (man-day).

P_{it} : regular time production of product i in period t (units).

W_t : work-force level in period t (man-day).

Y_{it} : overtime production of product i in period t (units).

Parameters and constants :

D_{it} : demand for product i in period t (units).

a_i : labor time for product i (man-hour/unit).

b_i : machine time for product i (machine-hour/unit).

c_{pi} : production cost (other than labor cost) for product i (\$/unit).

c_{lt} : labor cost in period t (\$/man-day).

M_t : *regular time machining capacity in period* t (machine-hour).

$M_{t\ min}$: lower bound on the utilization of machine capacity in period t (machine-hour).

$W_{t\ max}$: maximum work-force available in period t (man-day).

α_t : fraction of regular machine capacity available for use in overtime in period t.

β_t : fraction of regular work-force capacity available for overtime use in period t.

δ : regular time per worker (man-hour/man-day).

T : *time horizon*.

N : total products.

For each period, the constraints are as follows :

$$W_t = W_{t-1} + H_t - L_t, \qquad t = 1,\ldots,T, \tag{1}$$

$$W_t \leq W_{t\,max}, \qquad t = 1,\ldots,T, \tag{2}$$

$$\sum_{i=1}^{N} a_i P_{it} \leq \delta W_t, \qquad t = 1,\ldots,T, \tag{3}$$

$$\sum_{i=1}^{N} a_i Y_{it} \leq \delta \beta_t W_t, \qquad t = 1,\ldots,T, \tag{4}$$

$$I_{it} = I_{it-1} + P_{it} + Y_{it} - D_{it}, \quad t = 1,\ldots,T, \quad i = 1,\ldots,N, \tag{5}$$

$$\sum_{i=1}^{N} b_i P_{it} \leq M_t, \qquad t = 1,\ldots,T, \tag{6}$$

$$\sum_{i=1}^{N} b_i Y_{it} \leq \alpha_i M_t, \qquad t = 1,\ldots,T, \tag{7}$$

$$\sum_{i=1}^{N} b_i P_{it} \geq M_{t\,min}, \qquad t = 1,\ldots,T. \tag{8}$$

Eq. (1) shows that the available labor-force in any period equals labor-force in the previous period plus labor-force change in the current period. The limit of maximum available labor-force in any period can be ensured in Eq. (2). This maximum would come from labor market or available plant capacity. Total regular time and overtime production in each period are limited by the available production capacity, shown in Eq. (6) and (7) respectively. The regular time and overtime production to the available labor are limited by Eq. (3) and (4), respectively.

Eq. (5) ensures that the demand of each product in a period plus the inventory at the end of the period equals the total supply consisting of inventory from the previous period plus the regular and overtime production in the current period. Eq. (8) ensures that the utilization of production capacity will be at least up to a minimum level. Depending on the actual problem, one can add other resource balance constraints.

The objectives which are the same according to the viewpoints of stockist and supplier are as follows : the objectives for the cost of production and labor, the changes in the work-force level, and the amount of overtime production.

$$\text{Min } f_1 = \sum_{t=1}^{T} \sum_{i=1}^{N} \{ c_{pi}(P_{it} + Y_{it})\} + \sum_{t=1}^{T} c_{lt} W_t$$

$$\text{Min } f_2 = \sum_{t=1}^{T} (H_t + L_t)$$

$$\text{Min } f_3 = \sum_{t=5}^{T} \sum_{i=1}^{N} Y_{it}$$

On the other hand, the inventory level can create a conflict between stockist and supplier. The objective for inventory level is as follows, but this is not included in the monolithic model.

$$\text{Min } f_4 = \sum_{t=1}^{T} \sum_{i=1}^{N} I_{it}$$

4. The methodology

A multiobjective optimization problem for the new monolithic model can be represented as follows :

$$\min_{x \in X} U_G(f), \qquad f = (f_1(x), f_2(x), f_3(x)), \quad f_R(x) = f_4(x) \tag{9}$$

where $U_G(\cdot)$ is group utility consist of stockist's and supplier's utility. If $U_G(\cdot)$ is explicitly known, (9) could be solved by any appropriate scalar technique. An important requisite for solving (9) by scalar optimization techniques is a feasible direction method. It is based on the gradient of (9) at f, and the gradient of (9) is as follows :

$$\nabla U_G(f) = \sum_{i=1}^{3} \frac{\partial U_G(f)}{\partial f_i} \nabla f_i(x) \tag{10}$$

Where $\nabla f_i(x)$ is the gradient of $f_i(x)$ at x and $\partial U_G / \partial f_i$ is the partial derivative of $U_G(\cdot)$ with respect to $f_i(\cdot)$.

However, it is impossible to know $U_G(\cdot)$ explicitly, and it becomes evident that an interaction with stockist and supplier is nessary.

Hence, we will propose some assumptions.

Assumption 1. Each utility function (or preference function) of the stockist and the supplier exists and is known only implicitly to them, which means they cannot specify each functional form, but they can answer simple choice questions comparing two prospects. Moreover, each of the utility functions is a strictly decreasing and continuously differentiable function.

Assumption 2. A group utility function exists but we cannot specify its functional form. Instead, the group utility function is assumed to be represented by aggregating the individual utility functions. Moreover, the grorp utility function by the linear aggregation rule can be assumed as an additive form of the individual utility functions.

Assumption 3. The stockist and the supplier are rational decision makers. And they have a co-operative attitude for the improvement of group preference.

Let $m^t_{iR}(f)$, $m^u_{iR}(f)$ be the marginal rate of substitution (MRS) of stockist and supplier, and its meaning is, at any f, the amount of inventory that each of them is willing to sacrifice to acquire an additional unit of other objective f_i. Let $U_t(f)$, $U_u(f)$ be the utility functions of them.

Theorem 2. The gradient of group utility is

$$\nabla U_G(f) \cong \sum_{i=1}^{3} [\lambda_t \cdot m_{iR}^{t}(f) + \lambda_u \cdot m_{iR}^{u}(f)] \cdot \nabla f_i(x)$$

$$\text{where } m_{iR}^{t}(f) = \frac{\partial U_t(f)/\partial f_i}{\partial U_t(f)/\partial f_R}, \quad m_{iR}^{u}(f) = \frac{\partial U_u(f)/\partial f_i}{\partial U_u(f)/\partial f_R}$$

and where f_R is chosen as a reference criterion $(\partial U_t/\partial f_R \neq 0, \partial U_u/\partial f_R \neq 0, i \neq R)$

Proof. By the assumption 2, $U_G(f) = \lambda_t \cdot U_t(f) + \lambda_u \cdot U_u(f)$, where λ_t, λ_u is the stockist's and supplier's relative weight. Then,

$$\frac{\partial U_G(f)}{\partial f_i} = \lambda_t \cdot \frac{\partial U_t(f)}{\partial f_i} + \lambda_u \cdot \frac{\partial U_u(f)}{\partial f_i} \tag{11}$$

By Eq. (11), Eq. (10) can be represented as follows :

$$\nabla U_G(f) = \sum_{i=1}^{3} \left[\lambda_t \cdot \frac{\partial U_t(f)}{\partial f_i} + \lambda_u \cdot \frac{\partial U_u(f)}{\partial f_i} \right] \nabla f_i(x) \tag{12}$$

By $m_{iR}^{t}(f)$, and $m_{iR}^{u}(f)$, Eq. (12) can be represented as follows :

$$\nabla U_G(f) = \sum_{i=1}^{3} \left[\lambda_t \cdot \frac{\partial U_t(f)}{\partial f_i} + \lambda_u \cdot \frac{\partial U_u(f)}{\partial f_i} \right] \cdot \nabla f_i(x)$$

$$\cong \sum_{i=1}^{3} [\lambda_t \cdot m_{iR}^{t}(f) + \lambda_u \cdot m_{iR}^{u}(f)] \cdot \nabla f_i(x)$$

$$\text{where } m_{iR}^{t}(f) = \frac{\partial U_t(f)/\partial f_i}{\partial U_t(f)/\partial f_R}, \quad m_{iR}^{u}(f) = \frac{\partial U_u(f)/\partial f_i}{\partial U_u(f)/\partial f_R}.$$

f_R can be chosen as a reference criterion based on the inventory level. Q.E.D.

In theorem 2, we know that $\partial U_t/\partial f_i$, $\partial U_u/\partial f_i$ are estimated by stockist and supplier respectively by the MRS, i.e., $m_{iR}^{t}(f)$, $m_{iR}^{u}(f)$. Consequently, the gradient of group utilify, when it is not known explicitly, is generated by an interaction with the decision makers. The gradient $\nabla U_G(\cdot)$ in decision space can be converted into the desirable gradient in objective space. Its meaning in the objective space is a weighting vector between the objectives at some f. Hence, the proposed model (9) can be converted as follows in the objective space :

$$\underset{f \in F}{\text{Min}}\ V = \alpha \cdot f$$

where $f \in R^N = \{f_1(x), f_2(x), \cdots, f_N(x)\}$, $F = \{f_i(x) \mid x \quad X, i = 1, 2, \ldots, N\}$, $\alpha \in R^N = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$. Let f^1 be a solution for V with a given weighting vector $\alpha = \alpha^1$. Let $\nabla V(f^1)$ be the gradient of V at f^1. On the other hand, let $\nabla U_G(f^1)$ be the desirble gradient of objectives at f^1, by an assumption accurately estimated by stockist and supplier.

Theorem 3. There is a weighting vector $\alpha^2 = (1-\beta) \cdot \nabla V(f^1) + \beta \cdot \nabla U_G(f^1) \in R^N, 0 \le \beta \le 1$, such that the new solution f^2 of V, with $\alpha = \alpha^2$, will give $U_G(f^2) \ge U_G(f^1)$.

Proof. As f^1 and f^2 are nondominated solutions of V, they are at the efficient frontier of F, and the vector $f^2 - f^1$ defines a feasible direction at f^1 due to the convexity of F, then

$$\nabla V(f^2) \cdot (f^2 - f^1) = \alpha^2 \cdot (f^2 - f^1) \ge 0$$
$$\nabla V(f^1) \cdot (f^2 - f^1) = \alpha^1 \cdot (f^2 - f^1) \le 0$$

From the equation for α^2, we have

$$\alpha^2 \cdot (f^2 - f^1) = (1-\beta) \cdot \nabla V(f^1) \cdot (f^2 - f^1) + \beta \cdot \nabla U_G(f^1) \cdot (f^2 - f^1)$$
$$\alpha^2 \cdot (f^2 - f^1) = (1-\beta) \cdot \alpha^1 \cdot (f^2 - f^1) + \beta \cdot \nabla U_G(f^1) \cdot (f^2 - f^1)$$

Therefore,

$$\nabla U_G(f^1) \cdot (f^2 - f^1) = \frac{1}{\beta}\, \alpha^2 \cdot (f^2 - f^1) - \frac{(1-\beta)}{\beta} \cdot \alpha^1 \cdot (f^2 - f^1).$$

In the above equation, we prove in the case of $0 < \beta \le 1$, since α^2 is equal to α^1

where $\beta=0$. By the given results (i.e., $\alpha^2 \cdot (f^2-f^1) \geq 0$), $\alpha^1 \cdot (f^2-f^1) \leq 0$), we see that $\nabla U_G(f^1) \cdot (f^2-f^1) \geq 0$. Consequently, the new weighting vector α^2 will give $U_G(f^2) \geq U_G(f^1)$. Q.E.D.

In theorem 3, the choice of β is an important thing. Hence, we introduce a reasonable method for choosing of β by the local proxy preference function (Oppenheimer, 1978). The local proxy preference function is a local approximation of utility function using the deterministic additive independence condition and assumption about a MRS variation. It is estimated by assessing the MRSs and its functional forms are sum-of-exponentials, sum-of-powers, and sum-of-logarithms. Let $P_t(f)$, $P_u(f$ be the proxy functions of stockist and supplier and let $P_G(f)$ be the grorp proxy function. the proxy value about new weight α^2 is

$$\begin{aligned} P_G(\alpha^2 f) &= P_G[(1-\beta^1)\cdot \nabla V(f^1) f] + P_G[\beta^1 \cdot \nabla U_G(f^1) f] \quad , P_G(f) = \lambda_t \cdot P_t(f) + \lambda_u \cdot P_u(f) \\ &= P_G[\nabla V(f^1) f] + P_G[\beta^1 \{\nabla U_G(f^1) - \nabla V(f^1\} f] \quad , 0 \leq \beta^1 \leq 1 \end{aligned}$$

Hence, β^1 is selected as follows :

$$\text{Maximize } P_G(\alpha^2 f) = P_G[\beta^1 \{\nabla U_G(f^1) - \nabla V(f^1\} f]$$

The fixed small change $\triangle\beta^1$ is suggested to them and they determine the β^1 (i.e., $0 \leq \triangle\beta^1 \leq 2\triangle\beta^1 \leq ; \cdots\cdots \leq 1$). That is, β is determined under maximizing the group preference.

The above theorems and results allow the representation of an interactive algorithm. The overview of this algorithm is represented in Figure 1 and let the model V be :

$$\begin{aligned} &\underset{x \in X}{\text{Min}}\ V = \alpha \cdot f, \qquad \text{where } f = (f_1(x), f_2(x), f_3(x)), \quad f_R(x) = f_4(x) \\ &\text{s/t} \ \sum_{i=1}^{3} \alpha_i = 1, \qquad \alpha_i > 0 \quad i = 1, 2, 3. \end{aligned}$$

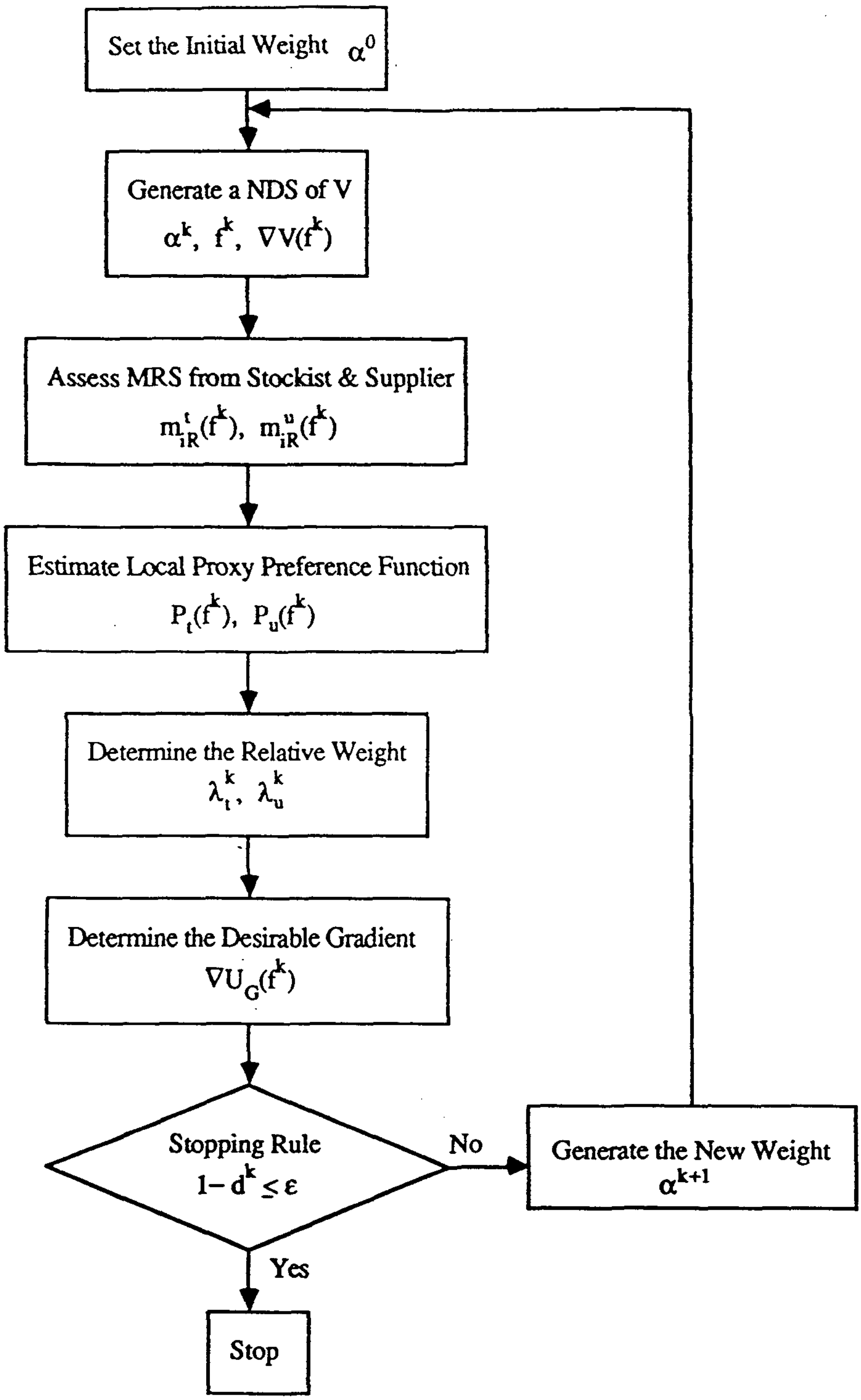

Figure1. Overview of the algorithm

The procedure of this algorithm is as follows;

(Step 1) The initial weighting vector $\alpha(\alpha_i>0)$ may be selected arbitrarily (for example, $\alpha_1=\alpha_2=\alpha_3=1/3$). The model V with the initial weighting vector α^k, k= 1, generates a nondominated solution using the weighting method and we obtain the vector solution f^k and its gradient $\nabla V(f^k)$ of the model V

(Step 2) After the stockist and supplier are showed the nondominated solution and the vector solution f^k, they assess the values of MRS based inventory level.

$$m^l_{iR}(f^k) = \frac{\partial U_l(f^k) / \partial f^k_i}{\partial U_l(f^k) / \partial f^k_R} = -\frac{df^k_R}{df^k_i}\Big|_{dU=0\ df^k_r=0\ ,r\neq i,j,\ l=t,u.}$$

(Step 3) Using the values of MRS of stockist and supplier, each of their local proxy preference funtions is estimated. If we let the type of $P_t(f)$, $P_u(f)$ be the sum-of-logarithms, its its parameters, i.e., τ^l_{ik}, are estimated as follows :

$$m^l_{iR}(f^k) = \frac{\partial P_l(f^k) / \partial f^k_i}{\partial P_l(f^k) / \partial f^k_R} = \frac{\tau^l_{ik}\cdot(M^l_R - f^k_R)}{\tau^l_{Rk}\cdot(M^l_i - f^k_i)}, \quad i = 1, 2, 3,\ \ R = 4,$$

$$\text{where } P_l(f^k) = \Sigma\, \tau^l_{ik}\cdot \ln(M^l_i - f^k_i), \quad l = t, u.$$

(Step 4) The relative individual weights of stockist and supplier, λ^k_t and λ^k_u, are estimated by the equation, i.e., $\lambda^k_t \cdot P_t(f^k) = \lambda^k_u \cdot P_u(f^k)$ based on the concept of equity. At f^k, λ^k_t and λ^k_u are produced by the ratio of their proxy value and scaling factor :

$$P_t(f^k)/P_u(f^k) = \lambda^k_u / \lambda^k_t = \eta \quad \text{and} \quad \lambda^k_t + \lambda^k_u = 1$$

(Step 5) By theorem 2, the gradient of group utility is as follows :

$$\nabla U_G(f^k) = \sum_{i=1}^{3} [\lambda^k_t \cdot m^t_{iR}(f^k) + \lambda^k_u \cdot m^u_{iR}(f^k)]\cdot \nabla f_i(x)$$

$$\text{where } m^t_{iR}(f^k) = \frac{\partial U_t(f^k)/\partial f^k_i}{\partial U_t(f^k)/\partial f^k_R},\quad m^u_{iR}(f^k) = \frac{\partial U_u(f^k)/\partial f^k_i}{\partial U_u(f^k)/\partial f^k_R}$$

f_R can be chosen as a reference criterion based on inventory level. According to the above results, we determine the desirable gradient $U_G(f^k)$ at some f^k.

(Step 6) In order to find when this algorithm has to stop, we use the discrepancy index d^k between the disposable gradient $V(f^k)$ and the desirable gradient $U_G(f^k)$ at f^k. The discrepancy index d^k means the difference of the direction gradients between the previous iteration and current one. This can be obtained by their normalized scalar product, i.e.,

$$d^k = \frac{\nabla V(f^k) \cdot \nabla U_G(f^k)}{\|\nabla V(f^k)\| \cdot \|\nabla U_G(f^k)\|}$$

If the discrepancy vanishes, i.e., if $\nabla V(f^k)$ is as near as colinear with $\nabla U_G(f^k)$, this algorithm stops. That is, if $1-d^k \le \varepsilon$, stop, since the solution is the preferred one which is accepted by both stockist and supplier. Otherwise, go to step 7.

There is a question which necessitates a decidion making skill. It is the selection of ε value. If the ε value decreases, then the quality of solutions increases, on the other hand, the computational efforts (the number of iterations) increase. That is, the tradeoffs exist between the computational efforts and the quality of solutions. accordingly, the decision maker selects a proper ε value case by case.

(Step 7) The new weighting vector is generated by $\nabla V(f^k)$, $\nabla U_G(f^k)$ and reasonable β^k.

$$\alpha^{k+1} = (1-\beta^k)\cdot \nabla V(f^k) + \beta^k \cdot \nabla U_G(f^k), \qquad 0 \le \beta^k \le 1$$

The β^k is selected to maximize $P_G(\alpha^{k+1} f)$. By the new weighting vector generated in this step, the weighting vector of Model V in step 1 is updated, go to step 1 and repeat.

5. Example

The hypothetical example shows the procedure of this algorithm on a two-product, three-period production planning problem. Numerical data, parameters and constants are presented Table 1,2,3 and 4, respectively.

Table 1 Demand, work-force and machine capacity data

Period	1	2	3
D_{1t}(unit)	8000	14500	15000
D_{2t}(unit)	4500	12500	6500
M_t (machine-hour)	32000	28400	29600
$M_{t\ min}$(machine-hour)	5300	4000	4500
$W_{t\ max}$(man-day)	24000	24000	24000

Table 2 Miscellaneous data $\delta = 8$(man-hour/man-day)

Period	1	2	3
α_t	0.5	0.6	0.5
β_t	0.3	0.3	0.3

Table 3 Operating and cost data $c_{1t} = 64$(\$/unit)

Product	Production cost c_{pi} (\$/unit)	Labour time a_i(hour/unit)	Machine time b_i(hour/unit)
1	15	2	1.5
2	20	3	2.0

Table 4 Initial data

Initial inventory of product 1	$I_{10} = 500$
Initial inventory of product 2	$I_{20} = 500$
Initial work-force level	$W_0 = 3500$

(1st Iteration)

(Step 1) We select the initial weighting vector, $\alpha_1^1=0.4$, $\alpha_2^1=0.3$, $\alpha_3^1=0.3$. Then the vector solution f^1 of the model V and its gradient $\nabla V(f^1)$ is generated. $f^1=(f_1^1, f_2^1, f_3^1)=(758000, 3158.624, 3300)$, $f_R^1=1080$, $\nabla V(f^1)=(0.4, 0.3, 0.3)$.

(Sep 2) The stockist and the supplier assess the values of MRS, i.e., $m_{iR}^t(f^1)$, $m_{iR}^u(f^1)$, i=1,2,3, respectively, as follows :

$$m_{1R}^t(f^1) = 4.5\ ,\ m_{2R}^t(f^1) = 4.2\ ,\ m_{3R}^t(f^1) = 3.5$$
$$m_{1R}^u(f^1) = 1.1\ ,\ m_{2R}^u(f^1) = 1.3\ ,\ m_{3R}^u(f^1) = 1.5$$

(Step 3) Using the values of MRS of stockist and supplier, each local proxy preference function of them can be estimated. Suppose that each function is the form of sum-of-logarithms for the reduction of computational efforts to estimate the parameters. The values of the individual proxy function are $P_t(f^1)=12.718$, $P_u(f^1)=13.045$.

$$P_l(f^1) = \sum_{i=1}^{4} a_{il}^l \cdot \ln(M_i - f_i^1) \qquad l = t, u.$$

$$M_1 = 1000000,\ M_2 = 10000,\ M_3 = 5000,\ M_4 = 20000$$

(Step 4) The relative individual weight of stockist and supplier is $\lambda_t^1=0.506$, $\lambda_u^1=0.494$.

$$P_t(f^1)/P_u(f^1) = \lambda_u^1/\lambda_t^1 = 12.718/13.045 \text{ and } \lambda_t^1 + \lambda_u^1 = 1$$

(Step 5) The desirable gradient of group utility, i.e., $\nabla U_G(f^1)$, which is organized by the preferences of stockist and supplier. The desirable gradient $\nabla U_G(f^1)$ translated from the decision space to the objective space and scaled, is $\nabla U_G(f^1)=[0.35, 0.35, 0.31]$

(Step 6) Let the value of ε be 0.0005, arbitrarily. The discrepancy index is $d^1=0.9938$. Hence, $1\text{-}d^1=0.0062$.

(Step 7) The new weighting vector is represented as $\alpha^2=(1-\beta^1)\cdot\nabla U_G(f^1)$, $0\le\beta$

$^{1}\leq 1$. the β^1 which maximize $P_G[\beta^1\{\nabla U_G(f^1)\}-V(f')\text{-}DV(f')f]$ is selested. The increments of local proxy values by the fixed change are shown as follows. The *optimal value of* β^1 *is* 1.0 *and* $\alpha^2=[0.35, 0.34, 0.31]$ because the maximum value of group proxy is 13.027 when β^1 is 1.0.

Table 5 Additional Proxy value

β^1	Incremental of Proxy Value
0.0	0
0.2	0.080
0.4	0.146
0.6	0.202
0.8	0.272
1.0	0.328

(2nd Iteration)

(Step 1) By $\alpha^2(\alpha_1^2=0.35, \alpha_2^2=0.34, \alpha_3^2=0.31)$, the vector solution f^2 and its gradient $\nabla V(f^2)$ is generated. $f^2=(f_1^2, f_2^2, f_3^2)=663250, 3579.807, 3410)$, $f_R^2=10800$, $\nabla V(f^2)=0.35, 0.34, 0.31)$.

(Step 2) The stockist and the supplier assess the MRSs.

$$m^t_{1R}(f^2) = 3.3\ ,\ m^t_{2R}(f^2) = 2.7\ ,\ m^t_{3R}(f^2) = 2.2$$
$$m^u_{1R}(f^2) = 2.1\ ,\ m^u_{2R}(f^2) = 2.3\ ,\ m^u_{3R}(f^2) = 2.5$$

(Step 3) Using the values of MRS, each proxy value is $P_t(f^2)=12.945$, $P_u(f^2)=13.049$.

(Step 4) The relative individual weight of stockist and supplier is $\lambda_t^2=0.502$, $\lambda_u^2=0.498$.

$$P_t(f^2)/P_u(f^2) = \lambda_u^2/\lambda_t^2 = 12.945/13.049 \text{ and } \lambda_t^2 + \lambda_u^2 = 1$$

(Step 5) The desirable gradient of group utility is $U_G(f^2)=[0.36, 0.33, 0.31]$.

(Step 6) The discrepancy index is $d^2=0.9997$. Hence, $1\text{-}d^2=0.0003\leq\varepsilon(=0.0005)$.

Accordingly, the algorithm is stopped and the best compromising solution is in

Table 6. Best compromise solution

Total cost (f_1)	Change in work-force level (f_2)	Overtime production (f_3)	Inventory level (f_4)
663250 ($)	3580 (man-day)	3410 (unit)	10800 (unit)

6. Conclusion

This study developed an interactive algorithm for production planning with two conflicting participants, i.e., stockist and supplier. This interactive approach in the production planning has some advantages. First, the estimation of the intangible cost is not required because the model considers several objectives simultaneous to appreciate the alternatives. Second, the preferences of two conflicting participants, i.e., stockist and supplier, concerned in the decision process of prlduction planning are considered in this study. Therefore, the solution of the suggested interactive algorithm is one which can be agreed upon by both participants and guarantees the concept of equity, and includes the reality of the decision process. In addition, the assessment of each preference pattern for stockist and supplier and the determination of relative individual weight for stockist and supplier are performed by this interactive method.

This algorithm can be extended to the cases of more than two conflicting participants or more than two conflicting objectives.

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