Journal of the Korean Institute
of Industrial Engineers
Vol. 16, No. 1, June, 1990

An Optimal Boundary Shape for Class-Based Storage Assignment Policy in Automated Storage/Retrieval Systems

Hark Hwang* and Jae-Won Ha*

자동 입출고 시스템에서 계급 할당규칙에 대한 최적 경계모양

황 학* · 하재원*

Abstract

With two-class-based storage assignment policy and dual command cycle in Automated Storage/Retrieval Systems(AS/RS), the problem of determining the region dedicated for class-one item is considered. First, the expected travel time of the S/R machine is derived when the boundary of the class-one region is square. Secondly, a heuristic procedure is proposed which determines sequentially the class-one region in a discrete rack. An application of the procedure generates leaf shape region which confirms that the L-shape partition is not necessarily optimal.

1. Introduction

There are three kinds of storage assignment policies in Automated Storage/Retrieval Systems(AS/RS): random storage assignment, turnover-based assignment, and class-based turnover assignment. In K calss-based turnover assignment rule, the racks and pallets are partitioned into K classes based on one-way travel times of the S/R machine and turnover and within any given class, pallets are assigned to locations randomly. Earlier work related to class-based rule was done by Hausman et al.(1976), Graves et al.(1977), Schwarz et al.(1978) and Hwang et al.(1988). Hausman et

* Department of Industrial Engineering, Korea Advanced Institute of Science and Technology, P.O. Box 150, Chungryang, Seoul, Korea

al.(1976) compared the operating performance of storage assignment rules under single command cycle, which was extended by Graves et al.(1977) to include dual command cycle. By means of a computer simulation, Schwarz et al.(1978) supported these previous analytical work on the scheduling of stacker cranes. Relaxing the assumption that rack is square in time, Hwang et al.(1988) derived the expected travel time for single command cycle under two and three-class systems, from which an optimal class partition can be determined.

All these studies are based on L shaped classes in the rack with square outer boundaries. Under class-based storage assignment policy, it is known that the optimal boundary shape with single command is square-L shape. This statement is not necessarily true with dual command since the rack partitioned by a concentric square should give the minimum expected interleave time. So, the objectives of this study has two folds. First, we evaluate the expected round-trip time for the case when the center of the square-shaped region for class one items is located at a point of the line connecting the I/O point and the center of the rack. Secondly, for the case with no restriction on the boundary shape, we develop a heuristic procedure to obtain an optimal partition of discrete rack.

2. Model Development for Continuous Rack

2.1. Assumptions

1) Each pallet holds only one item.

2) The S/R machine travels in Chebyshev type.

3) The system analyzed consists of a single S/R machine serving a single one-sided aisle.

4) The I/O point is located at the lower left-hand corner.

5) The rack length and height is known.

6) The trunover characteristic of pallets is known and represented by the skewness of ABC-curve, s.

7) Actual time for the S/R machine to load or unload a pallet at the I/O point or a storage location is ignored.

8) Horizontal and vertical speed of the S/R machine are known and are such that the system is "square" in time. The S/R machine travel measure will be time rather than distance. Time is normalized so that the maximum one-way travel time for a store or retrieve is 1 time(Fig. 1).

9) The systems adopts two-clas based turnover assignment rule and operates under dual command cycle. The items and rack locations are ranked according to turnover and distance(in travel time) from

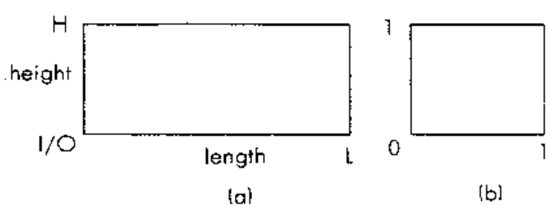

Fig. 1. Rack Normalization.

the I/O point, respectively. These ranked lists are then partitioned into two of matched classes such that the class of items with the highest turnover is assigned randomly within the class of loactions closest to the I/O point, etc.

We want to find the expected travel time in the rack partitioned as shown in Fig. 2. For the sake of the convenience of the evaluation, the region for class II items is partitioned into 4 sub-regions, i.e., II_i, i=1, 2, 3, 4. Let L denote the location of the left corner of the square region dedicated for class I item and R one side length of the region. The expected round-trip time, ERT, is obtained as

$$ERT = 2 \times E[\text{one-way travel time}] + E[\text{interleave time}]. \quad \cdots\cdots\cdots\cdots (1)$$

Since within any given class, pallets are assigned to locations randomly, it is

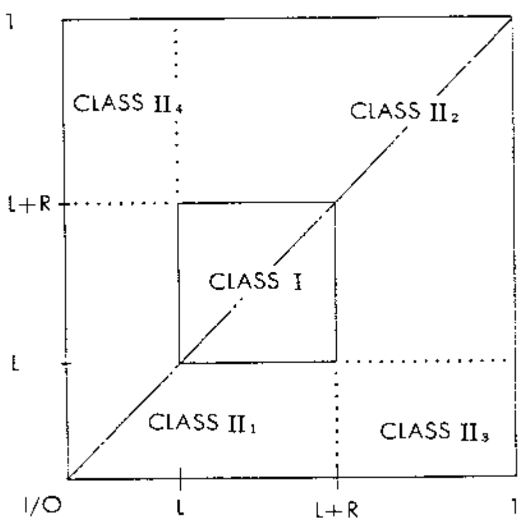

Fig. 2. Square-shape Boundary for Class-one Item.

assumed that the storage location of a pallet within each class is uniformly distributed. Thus with (x, y) being the storage location, the coordinates can be assumed to be independently generated.

2.2. Inventory Turnover Distribution

To consider the turnover of pallets, we introduce a concept of the inventory turnover distribution. A continuous ABC curve can be expressed as

$$G(i) = i^s, \ 0 < s \leq 1, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where G(i) is cumulative percentage of demand in pallet loads, i is the number of pallets in percentage, and s is the skewness of ABC curve. Assuming the basic EOQ model and the ABC phenomenon for inventories, Hausman et al.(1976) derived the turnover of the 100jth pallet, $\lambda(j)$, and

$$\lambda(j) = (2s/K)^{1/2} \cdot j^{(s-1)/(s-1)}, \ 0 \leq j \leq 1, \quad \cdots\cdots (3)$$

where K is the ratio of order cost to holding cost.

2.3. Expected One-Way Travel Time

The expected one-way travel time, EOT, is

$$EOT = p(I) \cdot OT(I) + p(II) \cdot OT(II), \quad \cdots\cdots\cdots (4)$$

where p(i) is the probability that a pallet to be stored or retrieved belongs to class i and OT(i) is the expected one-way travel time to a point in class i for i=1, 2.

EOT can be expressed as the following:

Since $p(\mathrm{I}) = \int_0^{R^2} \lambda(j)\,dj / \int_0^1 \lambda(j)\,dj = R^{2z}$,

$p(\mathrm{II}) = 1 - R^{2z}$,

$OT(\mathrm{I}) = L + (2/3)\cdot R$, and

$OT(\mathrm{II}) = [2/3 - R^2L - (2/3)\cdot R^2]/(1-R^2)$,

where $z = 2s/(1+s)$,

$$EOT = R^{2z}\cdot(L + \frac{2}{3}R) + (1-R^{2z})\cdot \left[\frac{2}{3}\cdot\frac{1-R^3}{1-R^2} - \frac{R^2L}{1-R^2}\right]. \quad \cdots\cdots (5)$$

2.4. Expected Interleave Time

Let p(i, j) be the probability that an interleave occurs from a location in class i to a location in class j and L(i, j) the associated expected interleave time. The expected interleave time, EIT, can be expressed as

$$EIT = p(\mathrm{I}, \mathrm{I})\cdot L(\mathrm{I}, \mathrm{I}) + p(\mathrm{II}, \mathrm{II})\cdot L(\mathrm{II}, \mathrm{II}) + 2\cdot p(\mathrm{I}, \mathrm{II})\cdot L(\mathrm{I}, \mathrm{II}). \quad \cdots\cdots (6)$$

Note that $p(i, j) = p(i)\cdot p(j)$. Graves et al. (1977) gave the expected interleave time within class I and

$$L(\mathrm{I}, \mathrm{I}) = (7/15)\cdot R. \quad \cdots\cdots (7)$$

L(I, II) can be otatined as

$$L(\mathrm{I}, \mathrm{II}) = q_1\cdot L(\mathrm{I}, \mathrm{II}_1) + q_2\cdot L(\mathrm{I}, \mathrm{II}_2) + q_3\cdot L(\mathrm{I}, \mathrm{II}_3) + q_4\cdot L(\mathrm{I}, \mathrm{II}_4), \quad \cdots\cdots (8)$$

where q_i, i = 1, 2, 3, 4, is the proportion of the area of class II_i to class II. $L(\mathrm{I}, \mathrm{II}_1)$ and $L(\mathrm{I}, \mathrm{II}_2)$ become

$$L(\mathrm{I}, \mathrm{II}_1) = \frac{1}{60R^2(L+2R)}\left[L^4 - 5RL^3 + 50R^2L^2 + 80R^3L + 70R^4\right], \quad \cdots (9)$$

if L<R.

$$= \frac{1}{60L(L+2R)}\left[40L^3 + 90RL^2 + 65R^2 + R^3\right], \quad \cdots\cdots (10)$$

if L>R.

$$L(\mathrm{I}, \mathrm{II}_2) = \frac{1}{60[(1-L)^2 - R^2]}\left[-40L^3 + 120L^2 - 30RL^2 - 120L + 60RL - 5R^2L + 40 - 30R + 5R^2 - 14R^3\right], \quad \cdots (11)$$

if L<1−2R.

$$= \frac{1}{60R^2(1-L+R)}\left[L^4 - 4L^3 + 9RL^3 + 6L^2 - 27RL^2 + 71R^2L - 4L + 27RL - 142R^2L + 39R^3L + 1 - 9R + 71R^2 - 39R^3 + 46R^4\right], \quad \cdots\cdots (12)$$

if L>1−2R.

Since the locations of clas II_3 and II_4 are symmetric about class I, $L(\mathrm{I}, \mathrm{II}_4)$ is the same as $L(\mathrm{I}, \mathrm{II}_3)$. Noting that $L(\mathrm{I}, \mathrm{II}_3)$ depends on the relative values of R and L, we obtain the following results for $L(\mathrm{I}, \mathrm{II}_3)$.

i) $$L(\mathrm{I}, \mathrm{II}_3) = \frac{1}{120R^2[1-(L+R)]}\left[-L^4 + 5RL^3 + 70R^2L^2 - 120R^2L + 70R^3L + 60R^2 - 60R^3 + 5R^4\right], \quad \cdots\cdots (13)$$

for either $(0 < R < \frac{1}{4},\ 0 < L < R)$

or $(\frac{1}{4} < R < \frac{1}{2},\ 0 < L < \frac{1-2R}{2})$

ii) $$L(\mathrm{I}, \mathrm{II}_3) = \frac{1}{120L[1-(L+R)]}\left[80L^3 - 120L^2 + 60RL^2 + 60L - 60RL + 10R^2L - R^3\right], \quad \cdots\cdots (14)$$

for $0<R<\frac{1}{4},\ R<L<\frac{1-2R}{2}$

iii) $$L(I,\ II_3)=\frac{1}{120LR^2[1-(L+R)]}\Big[-32L^5+80L^4-160RL^4-80L^3+320RL^3-240R^2L^3+40L^2-240RL^2+360R^2L^2-260R^3L^2-10L+80RL-180R^2+260R^3L-150R^4L+1-10R+40R^2-80R^3+80R^4-33R^5\Big], \quad \cdots\cdots (15)$$

for either $(0<R<\frac{1}{4},\ \frac{1-2R}{2}<L<\frac{1-R}{2})$

or $(\frac{1}{4}<R<\frac{1}{3},\ R<L<\frac{1-R}{2})$

iv) $$L(I,\ II_3)=\frac{1}{120LR^2[1-(L+R)]}\Big[-33L^5+80L^4-155RL^4-80L^3+320RL^3-250R^2L^3+40L^2-240RL^2+360R^2L^2-250R^3L^2-10L+80RL-180R^2L+260R^3L-155R^4L+1-10R+40R^2-80R^3+80R^4-32R^5\Big], \quad \cdots\cdots (16)$$

for either $(\frac{1}{4}<R<\frac{1}{3},\ \frac{1-2R}{2}<L<R)$

or $(\frac{1}{3}<R<\frac{1}{2},\ \frac{1-2R}{2}<L<1-2R)$

v) $$L(I,\ II_3)=\frac{1}{120R^2[1-(L+R)]}\Big[-32L^4+75L^3-145RL^3-70L^2+280RL^2-210R^2L^2+30L-180RL+240R^2L-170R^3L-5+40R-60R^2+100R^3-75R^4\Big], \quad \cdots\cdots (17)$$

for either $(\frac{1}{3}<R<\frac{1}{2},\ 1-2R<L<\frac{1-R}{2})$

or $(\frac{1}{2}<R<1,\ 0<L<\frac{1-R}{2})$

The explicit expression of L(I, II) is possible once R and L are known and in general, we have

$$L(I,\ II)=\frac{(L+R)^2-R^2}{1-R^2}\cdot L(I,\ II_1)+\frac{[1-(L+R)]^2-R^2}{1-R^2}\cdot L(I,\ II_2)+2\cdot\frac{L\cdot[1-(L+R)]}{1-R^2}\cdot L(I,\ II_3). \quad (18)$$

In order to find the expected interleave time within class II, we utilize the expected interleave time of 7/15 given by Graves et al.(1977). Since $p(I,\ I)=(R^2)^2$, $p(II,\ II)=(1-R^2)^2$, and $p(I,\ II)=R^2\cdot(1-R)^2$ with random storage assignment,

$$7/15=p(I,\ I)\cdot L(I,\ I)+2\cdot p(I,\ II)\cdot L(I,\ II)+p(II,\ II)\cdot L(II,\ II). \quad \cdots\cdots (19)$$

Consequently,

$$L(II,\ II)=\frac{\frac{7}{5}-R^4\cdot L(I)-R^2\cdot(1-R^2)\cdot L(I,\ II)}{(1-R^2)^2} \quad (20)$$

Since the expected interleave time is quire complicated in its form, the expected round-trip time is calculated utilizing micro-computer. With the skewness of ABC-curve, s, being given, L and R values to minimize the expected round-trip time can be determined by grid search method.

2.5. Evaluation

With four different values of s, 0.065, 0.139, 0.222, and 0.318, the optimal value of L and R are calculated. Contrary to our expectation, the optimal L is zero for all cases(see Fig. 3), leading to the conclusion

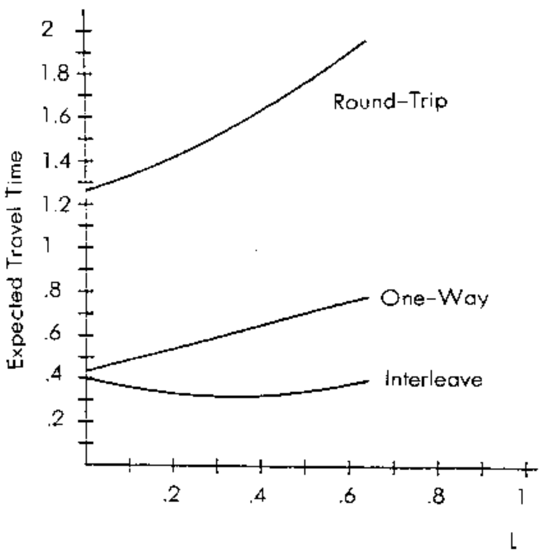

Fig. 3. Expected Travel Time(s=0.139)

that the optimal boundary has still square-L shape. In other words, the decreasing rate of the expected interleave time as L moves to the center of the rack is less than the increasing rate of twice the expected one-way travel time. These observations are coming from the fact that the boundary shape is restricted to a square.

3. Heuristic Procedure for Discrete Rack

In this section, a procedure to partition a discrete rack into two regions is presented, one for class I item and the other for class II item. The procedure is heuristic in nature and based on the concept of potential determined on each rack opening of class II region. Let POT(i, j) be the potential of the rack opening located in i-th column and j-th row, and defined as,

POT(i, j)=[one-way travel time from the I/O point to (i, j)]+[sum of weighted average interleave time between (i, j) and other openings, and one-way travel time to the I/O point from the opening interleave has occurred].

Initially, assuming that all rack openings of the discrete rack belong to class II region, a rack opening is determined which has the smallest value in POT(i, j). Then the rack opening chosen is assigned to class I region and then determine POT(i, j) again for all the remaining rack openings belonging to class II region. We repeat the above steps until further increase in the area of class I is undesirable. In other words, the area of class I region increases like crystal growth until the point is reached where a further increase only contributes to an increase in the expected round-trip time. The flow diagram of the heuristic procedure is represented in Fig. 4.

3.1. Experimentation

The procedure is applied to 50×50 rack and the results are depicted in Fig. 5 for the case with s=0.139. It is observed that the area for class I item increases as s increases and resembles the shape of "leaf". Note that this boundary shape is more efficient in terms of the travel time compared with the square-L shape. Table 1 shows the extent of improvements, even though not substantial, obtained by the heuristic procedure over the square-L shape boundary. Through this

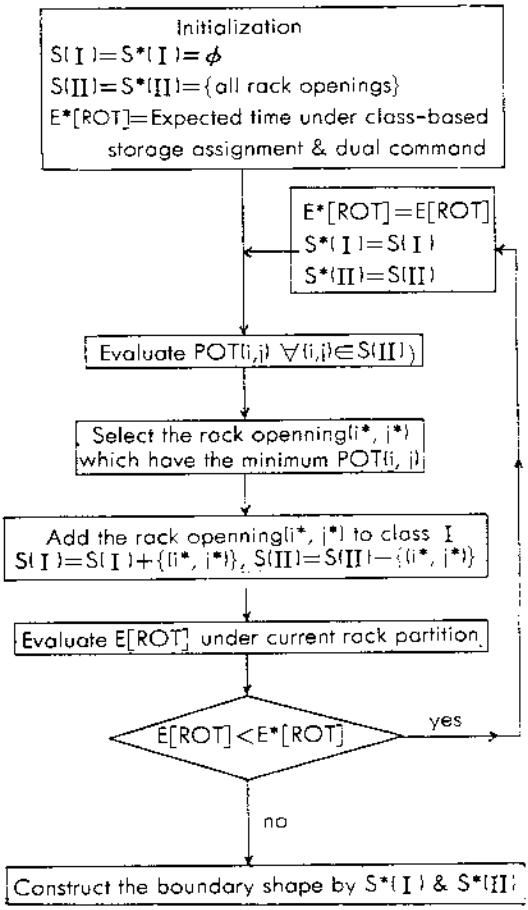

Fig. 4. Flow Diagram of Heuristic Procedure.

example, we can confirm that the square-L shape which is used in the previous research works for partitioning the rack is not

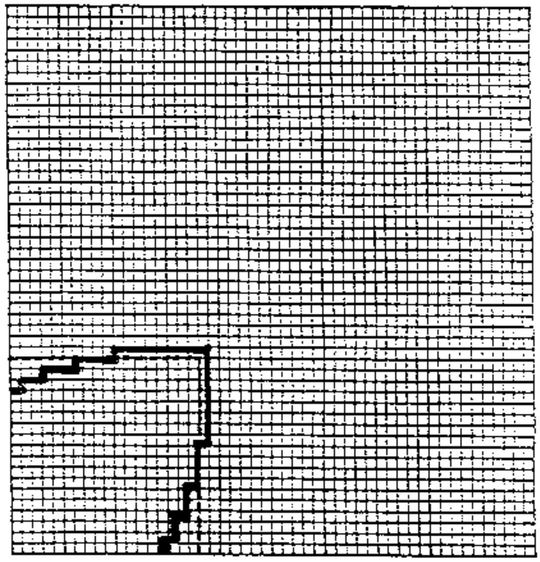

Fig. 5. Boundary Shape Obtained with the Heuristic Procedure(s=0.139)

optimal.

4. Conclusion

With 2 class-based turnover assignment policy, we examined square-shape boundary of the region dedicated for class one item. Contrary to our expectation, L-shape boundary turns out to be better than square -shape boundary. Utilizing the procedure

Table 1. Comparison of New Shape with Square-L Shape

Skewness of ABC curve (s)	Expected Round-Trip Time		Improvement (%)
	Square-L shape	Leaf shape	
0.065 (20/90)	0.972495	0.9715362	0.07
0.139 (20/80)	1.261872	1.2599920	0.15
0.222 (20/70)	1.425647	1.4231030	0.18
0.318 (20/60)	1.537694	1.5352810	0.16

developed to partition a discrete rack into two regions, we obtained leaf shape region for class one even though the improvement over L-shape is not substantial in terms of the expected round-trip time of the S/R machine.

5. References

[1] Bozer, Y.A., White, J.A., "Travel-Time Model for Automated Storage/Retrieval Systems," *IIE Transactions*, Vol. 16, No. 4, pp.329-338, 1984.

[2] Graves, S.C., Hausman, W.H., and Schwarz, L.B., "Storage-Retrieval Interleaving in Automatic Warehousing Systems," *Management Science*, Vol. 23, No. 9, pp.935-945, 1977.

[3] Ha, J.W., "An Optimal Boundary Shape for Class-based Storage Assignment Policy in Automated Storage/Retrieval Systems," Master Thesis, *Korea Advanced Institute of Science and Technology*, 1989.

[4] Hausman, W.H., Schwarz, L.B., and Graves, S.C., "Optimal Storage Assignment in Automatic Warehousing Systems," *Management Science*, Vol. 22, No. 6, pp.629-638, 1976.

[5] Hwang, H., Koh, S.G., and Jang, G.S., "A Study on Class-based Storage Assignment in Automated Storage/Retrieval Systems with Continuous Rectangular Rack Face in Time," *Journal of the Korean Institute of Industrial Engineers*, Vol. 14, No. 1, pp.31-42, 1988.

[6] Schwarz, L.B., Graves, S.C., and Hausman, W.H., "Scheduling Policies for Automatic Warehousing Systems: Simulation Results," *AIIE Transactions*, Vol. 10, No. 3, pp.260-270, 1987.