A Simple Efficient Rank-Order Contract Under Moral Hazard And Adverse Selection

Jung-Yoll Yun *

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* Ewha Woman's University
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I . Introduction

With the growing interest in so-called rank-order contract, its efficiency aspect under asymmetric information between employer and employees is being analyzed. The main concern of the related literature is to identify the cases where the rank-order contract is more efficient than individual piece-rate contracts in controlling the moral hazard problems.

One of the favorable circumstances for the rank-order contracts would be where the observed outputs of agents are positively correlated, as has been pointed out by Stiglitz & Nalebuff (1983) and Green & Stokey (1983). Another important rationale for the rank-order contracts is the possibility than the employer has an incentive to cheat his agent under a piece-rate contract when the absolute performance level of an agent is unverifiable, as has been argued by Bhattacharya (1984), Bhattacharya & Guasch (1988), Malcomson (1984, 1986), and Lazear & Rosen (1981). this argument is can be proven as valid in many cases, particularly when the employer hires more than one agent and when production process is not sufficiently separable for an unbiased estimate of each employee's own contribution to output to be made verifiable.

In this paper, it is assumed that for the above-mentioed reasons, emplyers offer contracts that are based upon the rankings of performances of agents. In particular, the model focuses on a set of simple rank-order contracts that pay two different wage levels to the agents in the same contest, since this type of simple contract represents a promotion scheme we can often observe in reality. This paper will then analyze the efficiency of a certain type of simple rank-order contract under asymmetric information about the agents' effort levels (moral hazard) and under asymmetric information about their ability levels (adverse selection).

Lazear & Rosen (1981) have shown that a rank-order contract they have designed cannot yield the first-best outcome under both moral hazard and adverse selection even when agents are resk-best outcome under both moral hazard and adverse selection even when agents are risk-neutral. And Bhattacharya & Guasch (1988) suggested a particular form of rank-order contract under which different types of agents play against the standard set by the performance of the agent of the lowest ability. This, however, is hardly observable in reality.

Despite their excellent analysis of some important aspects of rank-order contracts, the contest they designed is of a special type, in which there are only two agents and one of them is always penalized (or given a prize). then one may ask how the nature of a rank-order contract would change as the fraction of the contestants being penalized varies.

This paper deals with this very issue, and shows that as the fraction of the penalized decreases (together with an appropriate increase in the size of penalty), the first-best efficiency is more likely to be achieved under both moral hazard and adverse selection. The two first-best results under the two informational asymmetries are established : i) when agents are risk-neutral, in which case, there exists a first-best rank-order contract that imposes a certain level of penalty upon a certain fraction of the contestants ; ii) when agents are risk-averse, where the first-best outcome can be approximated arbitrarialy as the fraction of the penalized gets smaller (and as the penalty gets larger appropriately).

This first-best result may advocate the promotion structures we can observe in some countries such as Korea and Japan, Where the fraction of the demoted in the group of a given rank is relatively lower than that of the United States.[1] M. Aoki(1988) argued that in Japan, the demoted often gets separated through the pressure from his supervisor and peer group, and that the financial penalty for the midcareer separation is substantially high due to the disadvantageous treatement in the separation payment, the stigma effect that follows the few demoted, and few job vacancies for the midcareer agents.[2)]

Another important implication can be drawn from the efficiency of the rank-order contracts that penaliges smaller fraction of the contestants heavily ; a manager who hires a great unmber of workers could use a set of the efficient rank-order contracts to comply with the union's demand for pay equality for job equality without affecting adversely the incentives of agents under the two informational asymmetries. This is because then different types of agents self-select by choosing different rank-order contracts, and because only a small portion of the contestants under each contract is paid lower wages. In other words, the demand for equal pay for equal job may not be incompatible with the incentive pay structure under moral hazard and adverse selection on the part of workers. This may shed new light on the relationship between pay equality and incentives.

In the next section a basic model is outlined and a particular type of rank-order contract is suggested. In sections 3 and 4, this type of contract is shown to resolve the two informational problems efficiently when agents are risk-neutral and when agents are risk-averse, respectively. And this first-best result is extended in section 5 to the case where there is a privately informed common shock among the agents in the same contest, which is followed by some concluding remarks.

II. Model

Consider a competitive employer that hires an agent who can produce output Q in the following way.

$$Q = V\mu,$$

where μ is the level of effort chosen by the agent. The utility function of an agent of ability z is assumed to be additive in income x and dffort μ :

$$U(x, \mu ; z) = u(x) - C(\mu ; z),$$

where $C_{\mu} > 0$, $C_z < 0$, $C_{\mu\mu} > 0$ and $C_{\mu z} < 0$. (1)

The utility of income u(X) can be linear in x or concave in x. Note that, as in Lazear & Rosen (1981) and others, different types of agents are characterized by the different dffort cost functions $C(\mu : z)$, not by the different production functions. The type of an agent is distributed within an interval $[z_1, z_2]$, where $z_1 < z_2$. Then the first-best outcome under this circumstance would be the competitive outcome under perfect information in which eadh agent of ability z chooses his first-best level of effort μ_z such that

$$Vu'(V\hat{\mu}_z) = C'(\hat{\mu}_z ; z)$$

and in which each type of agent gets his first-best utility $\hat{\mu}(z)$ $(=u(V\hat{\mu}_z) - C(\hat{\mu}_z ; z))$ given the zero profit for the employer.

An employer, however, cannot observe the output Q. He also can neither identify the abilith of its agent (ddverse selection) not abserve the effort level that its agent chooses (moral hazard). He can, however, observe the effort level that his agent exerts in the following way :

$$y = \mu + \varepsilon,$$

where y is the employer's observation of the agent's true effort level μ, and where the observational error ε is a random variable distributed with mean zero given μ. Let us denote the distribution function of ε and its density function by H( ) and h( ), respectively, and make a following standard assumption on h( ).

Al : $h(\varepsilon)$ is differentialble, symmetric and unimodal at $\varepsilon = 0$.

We will also assume, for the purpose of simplicity, that ε is identically and independently distributed across the agents, and that the support of the distribution of ε is infinite.

A contract in this model should be based on the observations y's of performances of agents. One of the well-known contract is a piece-rate contract under which pay for an agent depends on the observation of his performance y only. Another type of contract is a rank-order contract under which pay for an agent is based on the ranking of his observed performance y among the performances y's of the agents in the same contest.

When agents are risk-neutral, there exists a certain type of piece-rate contract that can support the first-best outcome despite these informational asymmetries.[3] Under the following circumstances, however, an employer may be constrained to offer the first-best piece-rate contracts, and may choose to offer rank-order contracts. First, it may be more costly to measure the absolute performances levels for individual agents rather than just to identify the rankings of performances among the agents. More importantly, the effort levels (or the output levels) which agents choose are only privately observed by their employer.

This is especially true for a large organization in which the performance of agent is subjectively evaluated by the employer. In such case, the employer may have incentive to cheat his agents under a piece-rate contract. This sould bring both employer and agents to prefer a rank-order contract to deter the employer's adverse incentive.[4] Another circumstance under which an employer may want to offer a rank-order contract is where there is a privately-informed common shock in the effort cost functions of the agents in the same contest. As we shall see in the section 5, a rank-order contract can solve efficiently the two informational asymmetry problems in this case while some notable piece-rate contracts cannot.

Taking into consideration the above-mentioned circumstances, we will focus on the set of rank-order contracts that are based upon the rankings of

y_i 's among agents. In particular, we will be dealing with the efficiency of simple rank-order contracts that pay two different wage levels to the agents in the same contest, because this type of rank-order contract would repre -sent one of the most popular contracts that are used by firms - a promotion scheme. So we will examine how we should design a contract that sets the levels of high and low wages to be assigned to different rankings of performances of agents in order to achieve the efficiency when confronted with moral hazard and adverse selection problems simultaneously.

To see the problem more formally, let us suppose that an employer offers two wage levels W_H and W_L based on the relative performances of agents. Then the expected utility of an i-th agent of ability z under a contract R is going to be

$$(1-f(\mu^i, \mu^{-i}; R))u(W_H) + f(\mu^i, \mu^{-i}; R)u(W_L) - C(\mu^i; z)$$
$$= F - f(\mu^i, \mu^{-i}; R)D - C(\mu^i; z),$$

where F(fixed payoff) $= u(W_H)$, D(penalty payoff) $= u(W_H) - u(W_L)$ and $f(\mu^i, \mu^{-i}; R)$ is the probability that the i-th agent is penalized under R when he chooses μ^i and all the others choose μ^{-i} $(= \{\mu^1, \cdots, \mu^{i-1}, \mu^{i+1}, \cdots, \mu^n\})$. The penalty probability f( ) is determined by $\{\mu^i\}_i$, and by the penalty rule p of the contract R that specifies the total number n of agents in the contest and the number m of the agents in the contest upon whom the penalty payoff D is going to be imposed. Thus the penalty rule p can be described as (n, m). Let us denote by R((n, m), F, D) what I call a multi-agent rank-order contract that pays fixed payoff F and imposes penalty payoff D upon agents by a rule (n, m).

Then the problem becomes what pair ((n, m), F, D) of parameters can resolve the two informational asymmetry problems efficiently. Lazear and Rosen (1981) suggested a two-agent rank-order contract that can induce agents to choose the first-best effort levels under pure moral hazard (no adverse selection). But their contract has a property that one of the two contestants is going to be penalized. We will see later that because of this property their contract cannot support the first-best outcome when heterogeneous agents have private information not only about their choices of effort levels but also about their ability levels.

In the next two sections, we will see how a certain type of multi-agent rank-order contract $R((n, m), F, D)$ can yield the first-best outcome when agents are risk-neutral, and how a certain type of the contract $R((n, m), F, D)$ can approximate the first-best result arbitrarily. Before proving these points, here are some notations that will be used in this model :

$$R = R(p, F, D) = R((n, m), F, D)$$

$\bar{R}_z$; first-best R for z-type agent under pure moral hazard

$\hat{R}_z$; first-best R for z-type agent under moral hazard and adverse selection

$\bar{R} = \{\bar{R}_z\}_z$

$\hat{R} = \{\hat{R}_z\}_z$

III. Risk-Neutral Agent

In this section, the utility of income for agent is assumed to be linear in x. And without loss of generality, we can assume that $u(x) = x$. Then the first-best effort level for z-type agent, $\hat{\mu}_z$, can be characterized by

$$V = C'(\hat{\mu}_z ; z). \tag{2}$$

The first-best result will be established as Follows. first, in the subsection (A) a set $\bar{R}_z$ of first-best contracts for different types of agents under pure moral hazard will be characterized. Then in the subsection (B), the privately informed heterogeneity of agents will be introduced to show that there exists a set $\bar{R}_z$ of contracts for different types of agents such that once these contracts are offered, different types of agents will self-select by choosing different contracts.

(A) Set of First-Best Contracts $\bar{R}$ under Pure Moral Hazard

In this subsection we will assume that an employer can identify the type of individual agent. Therefore we will only have to consider the incentive problems of homogeneous agents (of the same ability z) under a contract R in order to characterize $\bar{R}$. The problem for an i-th agent of type z under a contract R is

$$\underset{\mu}{\text{Max}}\ F - f(\mu, \mu^{-i}; R)D - C(\mu; z). \tag{3}$$

Since the optimal choice for each agent depends upon the choices of the other agents, we will consider a Nash equilibrium choices of effort levels by the agents as their optimal reactions to the contract. Then the question we need to consider is the existence, the uniqueness and the symmetry of the Nash equilibrium.

Instead of characterizing the restrictive conditions for the non-existence of non-symmetric Nash equilibrium under a multi-agent rank-order contract R I would rather choose to assume as in the existing literature that if the unique symmetric Nash equilibrium $\{\mu_z\}$ exists for a group of z-type agents under R, then each z-type agent chooses the same effort level μ_z, To check whether or not there exixts a unique symmetric Nash equilibrium exists under R, we need to examine the problem (3) when all the other agents choose the same effort levels μ'. This requires specification of the penalty probability $f(\mu^i, \mu'; R)$ for the i-th agent in the contest who chooses μ^i.

Let us suppose all the agents other than the i-th agent choose the same effort levels μ'. And suppose the resulting observed performances are $(y_1, y_2, \cdot\cdot, y_{i-1}, y_{i-1}, \cdot\cdot, y_n)$, where $y_j = \mu' + \varepsilon_j$ for $j \neq i$. Let us rearrange these y's in the ascending order $(y^1, y^2, \cdots, y^{n-1})$ such that $y^j < y^{j+1}$ for $j = 1, \cdots, n-2$. Then, when the i-th agent who chooses his effort level μ^i is penalized it should be case that $y_i < y^n$, where $y_i = \mu^i + \varepsilon_i$. Now let us fix the agent fot the moment, say the k-th agent $(k \neq i)$, such that $y_k = y_n$. Then we can see that for a given y_k

$$\Pr(y_k = y^m \mid \varepsilon_k = \varepsilon) = \frac{(n-2)!}{(n-1-m)!\,(m-1)!} [1-H(\varepsilon)]^{n-1-m} H(\varepsilon)^{m-1} = \frac{(n-2)}{(n-1-m)!\,(m-1)!} H(-\varepsilon)^{n-1-m} H(\varepsilon)^{m-1}, \tag{4}$$

because $h(\varepsilon)$ is assumed to be symmetric at $\varepsilon = 0$. Since (4) holds for every possible ε_k, and since the probability that $y_i < y_k$ is $H(-s + \varepsilon_k)$ (where $s = \mu^i - \mu'$), we have

$$\Pr(y_i < y_k, y_k - y^m) = \frac{(n-2)!}{(n-1-m)!\,(m-1)!} \int_{-\infty}^{\infty} h(\varepsilon) H(-\varepsilon)^{n-1-m} H(\varepsilon)^{m-1} H(-s + \varepsilon) d\varepsilon \tag{5}$$

Since any y_j $(j \neq i)$ could be y^m, the penalty probability $f(s; R)$ for the i-th agent will be

$$f(s, R) = (n-1) \Pr(y_i < y_k, y_k = y^m)$$
$$= F(n, m) \int_{-\infty}^{\infty} h(\varepsilon) H(-\varepsilon)^{n-1-m} H(\varepsilon)^{m-1} H(-s + \varepsilon) d\varepsilon \tag{6}$$

where $F(n, m) = \frac{(n-1)!}{(n-1-m)!(m-1)!}$ Then we can point out the following basic properties of $f(s; R)$.

Lemma 1

The followings are true about $f(s; r)$:

i) $f(0; R) = m/n$

ii) $f(s : R)$ is decreasing in s and n, while it is increasing in m.

The proof of this lemma is delegated to the appendix. In the next lemma we can establish another important property of the penalty probability $f(s; R)$

that plays an essential role for the main result in this paper.

Lemma 2

Suppose that $n > 2m$, i.e., that more than half of the contestants are penalized. Then the penalty probability $f(s\,;R)$ is convex over a certain interval of s, $[-c,\ c]$ for some $c > 0$. If $\alpha(n, m\,; H)$ is the maximum of c for a given (n, m), $\frac{\delta\alpha}{\delta m} > 0$ and $\frac{\delta\alpha}{\delta m} < 0$.

<proof>

$$f_{ss}(s\,;R) = F(n,m)\int_{-\infty}^{\infty} L(\varepsilon)H(-\varepsilon)^{n-m}H(\varepsilon)^{m}h'(-s+\varepsilon)d\varepsilon, \qquad (7)$$

where $L(\varepsilon) = \frac{h(\varepsilon)}{H(-\varepsilon)H(\varepsilon)}$. Note that $L(\varepsilon)$ is symmetric at $\varepsilon = 0$. We can rewrite $f_{ss}(s\,;R)$ as follows :

$$f_{ss}(s\,;R) = G_1\ (s, (n,m)) + G_2\ (s\,;(n,m)), \qquad (8)$$

where

$$G_1\ (s, (n,m)) = \int_{-\infty}^{\infty} L(\varepsilon)H(-\varepsilon)^{n-m}H(\varepsilon)^{m}h'(-s+\varepsilon)d\varepsilon,$$

$$G_2\ (s, (n,m)) = \int_{-\infty}^{\infty} L(\varepsilon)H(-\varepsilon)^{n-m}H(\varepsilon)^{m}h'(-s+\varepsilon)d\varepsilon.$$

Since $h'(-s+\varepsilon) > 0$ for $\varepsilon < s$, and $h'(-s+\varepsilon) < 0$ for $\varepsilon > s$, we have $G_1\ (s, (n,m)) > 0$ and $G_2\ (s\,;(n,m)) < 0$. if we define $G(s, (n,m)) = -G_1/G_2$, then $f_{ss}((s\,;R) \geqq (\text{or} \leqq)\ 1$. Since $G(s, (n,m)) = -\frac{G_1/H(-\varepsilon)^{n-m}H(\varepsilon)^m}{G_2/H(-\varepsilon)^{n-m}H(\varepsilon)^m}$,

$$G(s, (n,m)) = -\frac{\int_{-\infty}^{s} L(\varepsilon)Q(s,\varepsilon,(n,m))h'(-s+\varepsilon)d\varepsilon}{\int_{s}^{\infty} L(\varepsilon)Q(s,\varepsilon,(n,m))h'(-s+\varepsilon)d\varepsilon},$$

where

$$Q(s,\varepsilon,(n,m)) = \left[\frac{H(-\varepsilon)}{H(-s)}\right]^{n-m}\left[\frac{H(\varepsilon)}{H(s)}\right]^{m}.$$

Note that $|h'(\varepsilon)|$ is symmetric at $\varepsilon = 0$ and that when $\varepsilon <$ (or $>$) s, $Q(s,\varepsilon,(n,m))$ is increasing in (or decreasing in) n and decreasing (or decreasing in) m. So we have

$$\frac{\delta G}{\delta n} > 0 \text{ and } \frac{\delta G}{\delta m} < 0 \text{ for any given s.} \qquad (9)$$

And if $s = 0$ and $n = 2m$, $G(0, (n, m)) = 1$ so that $f_{ss}(0 ; R) = 0$. So it is clear from (9) that when $n > 2m$, $f_{ss}(0 ; R) > 0$. Since every function is continuous and differentiable in this model, there exists c (> 0) such that $f_{ss}(s ; R) > 0$ for $s \varepsilon [-c, c]$ when $n > 2m$. Finally from (9) we have that $\frac{\delta \alpha}{\delta n} > 0$, $\frac{\delta \alpha}{\delta m} < 0$.

Lemma 2 shows that the penalty probability $f(s ; R)$, which is depicted in Figure 1, will be convex over the wider range of s around 0 as smaller fraction of the larger group of contestants is penalized. The following lemma, which establishes the property of $f(s ; R)$ in the limiting case where n is a continuum, demonstrates that this range of s, $[-\alpha, \alpha]$, can be made as wide as we want. Let us denote by $R(q, F, D)$ a continuum-agent rank -order contract that imposes the penalty payoff D upon the fraction q of the agents in the same contest. Then we have

Lemma 3

i) $f(s ; R(q, F, D)) = H(-d(q)-s)$, where $q = H(-d(q))$.

ii) $\frac{\delta \alpha (q ; H)}{\delta q} < 0$.

iii) $\alpha (q ; H) \rightarrow \infty$ as $q \rightarrow 0$.

<proof>

If the unmber of the contestants is continuum, the frequency distrbution of y's for the agents who choose μ' will be just the distribution of $(\mu' + \varepsilon)$. If $(\mu' - d(q))$ is in the 100q percentile from the bottom of the frequency distribution of y's, the penalty probability $f(s ; R(q, F, D))$ for the i-th agent who chooses μ^i (so that $s = \mu^i - \mu'$) to generate y_i $(= \mu^i + \varepsilon_i)$ is $\Pr(y_i < \mu' - d(q))$, which is equal to $H(-d(q)-s)$.

By the assumtion A1, $H(\varepsilon)$ is convex for $\varepsilon < 0$. Since $d'(q) < 0$ and $d(q) \rightarrow \infty$ as $q \rightarrow 0$, we can get ii) and iii).

The penalty probability $f(s ; R(q, F, D))$ under a continuum-agent rank-order contract $R(q, F, D)$ is depicted in Figure 2. From the fact that $q = m/n$ and from lemma 3, we can see that the properties of the penalty probability $f(s ; R((n, m), F, D))$ specified in lemma 2 will hold independently of whether n and m are finite or infinite. We can also see that $\alpha(n, m ; H)$ can go to infinity as n goes to infinity for any given m.

Given the penalty probability $f(s ; R)$, we can see that the first-order condition for (3) when all the other agents choose the same effort levels μ' becomes

$$-Df_s(s ; R) - C'(\mu^i ; z) - D F(n,m) \int_{-\infty}^{\infty} L(\varepsilon) Q^0(S, \varepsilon, (n,m)) h(-s+\varepsilon)\alpha\varepsilon - C'(\mu^i ; z) = 0, \tag{10}$$

where $Q^0(s, \varepsilon, (n,m)) = H(-\varepsilon)^{n-m} H(\varepsilon)^m$, So a symmetric nash equilibrium $\{\mu_z\}$ under $R((n,m)F, D)$, if it exists, can be characterized by

$$D F(n,m) \int_{-\infty}^{\infty} L(\varepsilon) Q^0(0, \varepsilon, (n,m)) h(\varepsilon) d\varepsilon - C'(\mu_z ; z). \tag{11}$$

Then we can see from (11) that the symmetric Nash equilibrium $\{\mu_z\}$ under $R((n,m), F, D)$ is unique. And the second-order vondition for (10) will be

$$D F(n,m) \int_{-\infty}^{\infty} L(\varepsilon) Q^0(s, \varepsilon, (n,m)) h'(-s+\varepsilon) d\varepsilon + C''(\mu^i ; z) > 0 \tag{12}$$

for any s given (n, m, D), which is assumed to be satisfied. Note from lemma

2 that the second-order condition (12) is satisfied locally and that it is satisfied for the wider range of s as n gets larger and m gets smaller. Then we can establish the following lemma regarding the existence of the Nash

equilibrium $\{\mu_z\}$

Lemma 4

Assume A1 and (12). Then there exists a unique symmetric Nash equilibrium $\{\mu_z\}$ under $R((n, m), F, D)$, which satisfies (11).

Finally, when the effort level μ' chosen by all the other agents changes such that $|s| < \alpha$, we can see from lemma 3 that

$$0 < \frac{\delta\mu^i}{\delta\mu} < 1. \tag{13}$$

So far we have established that a multi-agent rank-order contract $R((n, m), F, D)$ generates a unique symmetric nash equilibrum. And the symmetric Nash equilibrium under $R((n, m), F, D)$ will also vary as the parameters $((n, m), F, D)$ change. Figure 3 shows how the symmetric Nash equilibrium $\{\mu_z\}$ for a given pair $((n, m), F, D)$ of parameters is determined and how $\{\mu_z\}$ can change as one of the parameters, D, for example, changes.

Now we are ready to characterize a set of first-best contracts $\bar{R}$ ($= \{\bar{R}_z\}_z$) for different types of agents under pure moral hazard. The first-best contract $\bar{R}_z$ for z-type agent under pure moral hazard is the one which generates a symmetric Nash equilibrium $\hat{\mu}_z$ such that $\hat{\mu}_z = \hat{\mu}_z$ and the employer gets zero profit. If $\bar{R}_z = R((n, m), F_z, D_z)$, then the pair (n, m), F_z, D_z) of parameters needs to satisfy the following conditions.

$$C'(\hat{\mu}_z ; Z) = V = D_z \; F(n/m) \int_{-\infty}^{\infty} L(\varepsilon) Q^0 (0, \varepsilon, (n, m)) h(\varepsilon) d\varepsilon \tag{14}$$

and

$$F_z = V\mu_z + \frac{m}{n} D_z \tag{15}$$

Once the penalty parameters $((n, m), D_z)$ are set by (14) for each z to induce the agents to choose the first-best effort levels, the expected penalty $f(0 ; \bar{R}_z) D_z$ for z-type agent will be determined as $(m/n) D_z$ because the agents choose the same $\hat{\mu}_z$ under $\bar{R}_z$. Then the fixed payoff F_z for him under the contract can be set by zero profit condition (15). But we can see that there are many pairs $((n, m), F_z, D_z)$ that satisfy (14) and (15) for each z. This implies that there are many sets $\bar{R}$'s of first-best contracts $\{\bar{R}_z\}_z$ under pure moral hazard. Let us denote by Γ the set of $\bar{R}$'s.

(B) Set of First-Best Contracts $\hat{R}$ under moral hazard and adverse selection

Clearly, a set of first-best contracts $\hat{R}$ under the two informatioal asymmetries should be an element of Γ. So the question is whether there exists $\bar{R} \varepsilon \Gamma$ that can satisfty the self-selection constraint under adverse selection circumstance. Let us choose one $\bar{R}$ amng Γ, which is denoted by $\hat{R}(= \{\hat{R}_z\}_z = \{R((\hat{n}_z, \hat{m}_z), \hat{F}_z), \hat{D}_z)\}_z)$, such that

$$\hat{R} \varepsilon \Gamma$$

$$\hat{n}_z = \hat{n} \text{ and } \hat{m}_z = \hat{m} \text{ for all } z$$

$$\alpha(\hat{n}, \hat{m}; H) > X, \text{ where } X = \hat{\mu}_2 - \hat{\mu}_1 \tag{16}$$

where X is the difference in the first-best effort levels $\hat{\mu}_2$, $\hat{\mu}_1$ between the highest ability(z_2) and the lowest ability(z_1) agents. The set of contracts $\hat{R}$ has two important properties. One is that the penalty parameters $(\hat{n}, \hat{m}, \hat{D})$ are the same for all types of agents by (14), (16). This implies that once different types of agents self-select, the expected penalty $f(0; \hat{R}_z)\hat{D}$ will be the same for all types of agents.

This also implies that then the difference in the fixed payoff $\hat{F}_z$ between the two different Z's will be exactly the same as the difference in the first-best output levels $V\hat{\mu}_z$'s between the two different types of agents, because the fixed payoff is determined by the zero profit condition (15). This property, which simplifies the analysis, is not essential for the first-best result in this paper. the other property, which plays an important role for the first-best result, is that the penalty probability $f(s; \hat{R}_z)$ is convex in μ^{i} or in s for s ε [-X, X] by (16) and by lemma 2, 3. Before proving the first-best result for $\hat{R}$, let us introduce the following lemma.

Lemma 5

Suppose that an agent of type z joins a contest under one contract of $\hat{R}$, where all the other agents choose $\mu' \varepsilon [\hat{\mu}_1, \hat{\mu}_2]$. Then the optimal effort level μ_z for this agent will be in between $\hat{\mu}_z$ and μ'.

<proof>

If $\mu' = \hat{\mu}_z$, $\mu_z = \hat{\mu}_z$ shince $\{\mu_z\}$ is the Nash equilibrium under the contract. If $\mu' \neq \hat{\mu}_z$, then $\mu_z \ \varepsilon\ [\hat{\mu}_z, \mu']$ when $\mu' > \hat{\mu}_z$, and $\mu_z \ \varepsilon\ [\mu', \hat{\mu}_z]$ otherwise by (13) (see Figure 4). This is because $(\mu' - \hat{\mu}_z)$ ε [-X, X] over which the penalty probability under the contract is convex in s. Q. E. D..

Now we are ready to state the main result of this paper.

Proposition 1

A set of contracts $\hat{R}$ defined by (16) yields the first-best outcome under the two infotmational asymmetries when agents are risk-neutral.

<proof>

To prove this proposition, let us pick up any two different ability levels z_a and z_b $(z_1 \leqq z_a < z_b \leqq z_2)$ and see whether an agent of ability z_i would have any incentive to change his contract from $\hat{R}_i$ to $\hat{R}_k$ (i, k = a, b) given that all the other agents self-select, where $\hat{R}_i$ (or $\hat{R}_k$) = $\hat{R}_z$ for $z = z_i$ (or = $\hat{R}_z$ for $z = z_k$). First, let us check the case for an agent of ability z_a. The expected utility $EU_a(\hat{R}_b)$ that he can get from choosing $\hat{R}_b$ that the agents of ability z_b choose will be

$$EU_a(\hat{R}_b)$$

$$= \underset{\mu}{\text{Max}}\ \hat{F}_b - f(\mu - \hat{\mu}_b\ ;\ \hat{R}_b)\ \hat{D} - C(\mu\ ;\ z_a)$$

$$= V\hat{\mu}_b + [f(0 ; \hat{R}_b) - f(\mu_a - \hat{\mu}_b ; \hat{R}_b)]\hat{D} - C(\mu_a ; z_a) \text{ by (15)},$$

where $\hat{\mu}_i$ is the first-best level of effort for an agent of type i (i = a, b) and μ_a is the optimal choice of effort level for the deviant. Note that $\mu_a \varepsilon [\hat{\mu}_a, \hat{\mu}_b]$ by lemma 5. Since $f(\mu - \hat{\mu}_b ; \hat{R}_b)$ is convex in μ for $\mu \varepsilon [\hat{\mu}_a, \hat{\mu}_b]$ and since $\hat{D} = -V/f_s(0 ; \hat{R}_b)$ by (14), we have

$$EU_a(\hat{R}_b) < V\mu_a - C(\mu_a ; z_a)$$

$$< V\hat{\mu}_a - C(\hat{\mu}_a ; z_a) \text{ because } \hat{\mu}_a = \underset{\mu}{\text{Argmax}} \{V\mu - C(\mu ; z_a)\}$$
$$= EU_a(\hat{R}_a),$$

which is his expected utility from not deviating from $\hat{R}_a$. (See also Figure 5) Similarily, we can show that

$$EU_b(\hat{R}_a)$$

$$= \text{Max } \hat{F}_a - f(\mu ; \hat{\mu}_a, \hat{R}_a)\hat{D} - C(\mu ; z_b)$$

$$= V\hat{\mu}_a + [f(0 ; \hat{R}_a) - f(\mu_b - \hat{\mu}_a ; \hat{R}_a)] D - C(\mu_b ; z_b) \text{ by}$$

(15)

$$< V\mu_b - C(\mu_b ; z_b)$$

$$< V\hat{\mu}_b - C(\hat{\mu}_b ; z_b)$$

$$= EU_b(\hat{R}_b). \qquad \text{Q. E. D..}$$

The intuition behind this result comes from the fact that the penalty probability $f(\mu - \mu_z ; \hat{R}_z)$ is convex in the effort level μ of the deviant agent, and from the fact that when his effort level is $\hat{\mu}_z$ the marginal increase in the expected penalty is v. This implies that the expected reward a deviant agent can get is less than the output he produces, so that his expected utility will be less than the first-best utility that he could have enjoyed if he did not deviate. so the convexity of the penalty probability over the relevant interval plays an essential role for the first-best result. We have shown that it is the penalty rule of penalizing a small portion of the conterstants (larger n and smaller m, or smaller q) that leads to the convexity of the penalty probability over the relevant interval. Note that the rank-order contract in Lazear & Rosen(1981) is just a R((n, m), F, D) where n = 2 and m = 1. therefore the penalty probability under the contract is not going to be convex over the relevant interval (i.e., $\alpha = 0$) (as was shown in the proof of lemma 2)

Final remark. If the required portion of the contestants that is to be penalized is small, then the required penalty needs to be large to induce agents to choose the first-best effort levels. In this case we may have to worry about the situation where the required penalty payoff is larger than the fixed payoff and where agents are constrained by the imperfect capital market. If the unemployment rate is positive, however, an employer can fire the agents of lower performances, which can make the actual penalty higher than the fixed payoff as in Shapiro & Stiglitz(1984).[5)] But then the first-best outcome is not going to be attainable any longer. Thus the constraint of the infeasibility of some large penalty may limit the scope of the first-best result in this model. It is shown in the next section, however, that if u(x) is concave and $u(x) \rightarrow -\infty$ as $x \rightarrow 0$ so that the feasibility constraint diappears, we can approximate the first-best outcome arbitrarily by a certain from of continuum-agent rank-order contract R(q, F, D).

IV. Risk-Averse Agent

Let us suppoes that the utility function of agent is additively separable in income and effort, i.e.,

$$U(x, \mu ; z) = u(x) - c(\mu ; z),$$

where $C(\mu ; z)$ satisfies the previous conditions, and $u' > 0$, $u'' < 0$. In this case, the first-best effort level $\hat{\mu}_z$ and the first-best utility level (given the zero profit condition) $\hat{U}_z$ for the z-type agent will be

$$Vu'(V\hat{\mu}_z) = C'(\hat{\mu}_z ; z),$$
$$\hat{U}_z = u(V\hat{\mu}_z) - C(\hat{\mu}_z ; z). \tag{17}$$

It has been established that as long as the employer's observation of the agent's performance entails an error ε, the first-best outcome cannot be supported even under pure moral hazard when u( ) is concave. Mirrless (1974), however, showed that under some conditions, there exists a contract yielding the outcome that approximates the first-best one arbitrarily under moral hazard context. We will be dealing with in this section whether there exists a multi-agent rank-order contract that can achieve the Mirrless' result of the almost-first-best outcome when we have both moral hazard and adverse selection problems simultaneously. It should be noted that it is not obvious whether the Mirrless' result holds under both of the two informational asymmetries although it is true either for moral hazard case or for adverse selection case. This is because the size of penalty to induce the first-best effort level from agent may not be sufficient for the self -selection constraint among different types of agents to hold. In other words, moral hazard problem imposes additional constraint upon the set of contracts that should solve the adverse selection problem as well.

As in Mirrless(1974), I will assume the following.

B1 : $u(x) \rightarrow -\infty$ as $x \rightarrow 0$.

This assumption B1 enables us to set the penalty in terms of utility as large as possible without worrying about the capital constraint of agents under imperfect capital market.

Since the relevant penalty probability needs to be very small in this case, we are going to have to focus on the case where the size n of the group is very large or a continuum, i,e., on the set of continuum, -agent rank-order contracts $R(q, F, D)$. Let us consider a set of these contracts $\tilde{R}(= \{\tilde{R}_z\}_z = R(q_z, F_z, D_z))$ such that

$$q_z = q^0 \text{ for all } z$$

$$h(-d(q^0))D_z = Vu'(V\hat{\mu}_z)$$

$$F_z = u(V\hat{\mu}_z), \tag{18}$$

where $q^0 \to 0$ (or $d(q^0) \to \infty$). Given q^0, the penalty payoff D_z is determined so as to induce the agent of type z to choose $\hat{\mu}_z$ in the symmetric Nash equilivrium under $\tilde{R}_z$ as we can see in (18). Note that since q^0 is very small D_z should be very large, which is possible by B1. And the fixed payoff F_z is set just equal to the utility of the first-best output $V\hat{\mu}_z$.

Let us assume the following condition.

$$\text{B2}: \lim_{d \to \infty} \frac{h(-d)}{H(-d)} = \infty.$$

In fact, the assumption B1 and B2 are what Mirrless(1974) requires for the almost-first-best result under pure moral hazard. Since

$$D_z = \frac{Vu'(V\hat{\mu}_z)}{h(-d(q^0))},$$

we have by B2

$$H(-d(q^0))\, D_z \to 0 \tag{19}$$

since q^0 approaches zero. Then the expected utility of a z-type agent under $\tilde{R}_z$, $EU_z(\tilde{R}_z)$, will be

$$EU_z(\tilde{R}_z) = u(V\hat{\mu}_z) - H(-d(q^0))D_z - C(\hat{\mu}_z; z)$$

$$- \hat{U}_z \text{ by (19)}.$$

Also the expected profit of the employer is approximately zero because the expected penalty

$$H(-d(q^0))\ (V\hat{\mu}_z - W_z) \to 0 \text{ since } q^0 \to 0,$$

where $u(V\hat{\mu}_z) - u(W_z) = D_z$ .So the set of contracts $\tilde{R}$ achieves the almost-first-best outcome under pure moral hazard given B1 and B2.

Then the remaining question is whether the set of contracts $\tilde{R}$ can satisfy the self-selection constraint when an individual agent has private information about his ability level. Let us introduce another assumption B3, which can support the almost-first-best result when we have both moral hazard and adverse selection simultaneously.

$$\text{B3}: \lim_{d \to \infty} \frac{h'(-d+x)}{h(-d)} \geqq |t| \text{ for any } x \in [0, X],$$

where $t = \text{Max}\ \{Vu''(b)/u'(a) \mid a, b \in [V\hat{\mu}_1, V\hat{\mu}_2]\}$ and $X = \hat{\mu}_2 - \hat{\mu}_1$. This assumption implies that over the relevant region the curvature of H ( ) is greater than that of u( ). Since t and X are bounded and $x > 0$, any distribution which has the property that $h'(-d)/h(-d) \to \infty$ as $d \to \infty$ will satisfy B3 by A1. And note that any normal distribution satisfies this condition. Now we can establish the following.

Proposition 2

Assume A1 and B1-3. Then a set of contracts $\tilde{R}$ approximates the first -best outcome arbitrarily both under moral hazard and under adverse selection when agents are risk-averse.

<proof>

Now consider the two types of agents-agents of low ability z_a and agents of high ability z_b . Suppose that initially low ability agebts choose $\tilde{R}_a$ ($= \tilde{R}_z$ for z_a type) and high ability agents choose $\tilde{R}_b$ (- $\tilde{R}_z$ for z_b type). And suppose that an agent of low ability agent tries to choose $\tilde{R}_b$. The expected utility $EU_a(\tilde{R}_b)$ that he can get from changing his contract to $\tilde{R}_b$ is going to be

$$EU_a(\tilde{R}_b) = \underset{\mu}{\text{Max}}\ u(V\hat{\mu}_b) - H(-d(q^0)-(\mu-\hat{\mu}_b))D_b - C(\mu\,;z_a)$$

$$= u(V\hat{\mu}_b) - H(-d(q^0)-(\mu_a - \hat{\mu}_b))D_b - C(\mu_a\,;z_a)$$

$$= u(V\mu_a) - P(\mu_a\,;z_a),$$

where μ_a is the optimal choice of effort for the deviant under $\tilde{R}_b$, and

$$P(\mu_a) = u(V\mu_a) - u(V\hat{\mu}_b) + H(-d(q^0)-(\mu_a-\hat{\mu}_b))D_z\,.$$

Note that $\mu_a\ \epsilon\ [\hat{\mu}_a, \hat{\mu}_b]$ as befote. Since

$$P'(\hat{\mu}_b) = 0$$

and

$$P''(\mu_a) = V^2\ u''(V\mu_a) + \frac{h'(-d(q^0)-(\mu_a-\hat{\mu}_b))}{h(-d(q^0))}\ Vu'(V\hat{\mu}_b) > 0 \text{ by}$$

B3,

$P'(\mu_a) = 0$ for $\mu_a < \hat{\mu}_b$, which implies $P(\mu_a) < \hat{\mu}_b$ because

$$P(\hat{\mu}_b) = \frac{H(-d(q^0))}{h(-d(q^0))}\ Vu'(V\hat{\mu}_b) \geqq 0.$$

Thus we have

$$EU_a(\tilde{R}_b) < u(V\mu_a) - C(\mu_a\,;z_a)$$

$$< u(V\hat{\mu}_a) - C(\hat{\mu}_a\,;z_a)$$

$$\sim EU_a(\tilde{R}_a).$$

Next when an agent of type z_b devoates from $\tilde{R}_b$ to $\tilde{R}_a$, we have

$$EU_b(\tilde{R}_a) = u(V\hat{\mu}_a) - H(-d(q^0)-(\mu_b-\hat{\mu}_a))D_a - C(\mu_b\,;z_b)$$

$- u(V\hat{\mu}_a) - C(\mu_b\,;z_b)$ since $\mu_b > \hat{\mu}_a$ and $H(\ )D_a \rightarrow 0$

$$< u(V\mu_b) - C(\mu_b\,;z_b)$$

$$< u(V\hat{\mu}_b) - C(\hat{\mu}_b ; z_b)$$

$$\sim EU_b(\tilde{R}_b),$$

where μ_b is the optimal choice of effort for the deviant under $\tilde{R}_a$. Q. E. D..

What keeps a low ability agent from deviating to high ability contract is the fact that the expected penalty payoff for deviant is greater than the difference between the fixed payoff $u(V\hat{\mu}_b)$ under the high ability contract and the utility $u(V\mu_a)$ of the output $V\mu_a$ prodced by the deviant.[6] This is made possible by the assumption B3, which makes the curvature of H( ) greater than that of u( ) without affecting the agents' choices of effort levels. Note that if it were not for moral hazard constraint, or if it were not for adverse selection constraint, we do not need the additional assumption B3. Without moral hazard constraint, we couldjust set D_z to be high enough to get the first-best self-selection, as is in Nalebuff & Scharstein (1987). In fact, proposition 2 generalizes the Mirrless' result to the case where we have both moral hazard and adverse selection simultaneously.

The above proposition also has an interesting implication. Suppose a large group of heterogeneous workers forms a union. Suppose, for simplicity's sake, that there are only two ability levels z_1, z_2. The number of workers for each type is n_1, n_2, and an employer does not know an individual type of worker. Suppose the union has a utilitarian objective function[7] such that

$$W(U_1^1, \cdots ; U_1^2, \cdots) = \frac{U_1^1 + \cdots + U_{n_1}^1}{n_1} + \frac{{}_1^2 + \cdots + U_{n_2}^2}{n_2}, \qquad (20)$$

where U_i^1 is the utility of i-th worker of type z_1, and U_i^2 is the utility of i-th worker of type z_2 .If the individual utility function is concave in income, the union will demand for equal pay for equal job (equal contract) as well as for pay increase to maximize its welfare. It has been believed, however, that equal pay would not give agents sufficient incentives to work hard and that high ability workers wold be adversely affected by the equal pay structre because it could not sort out different types of workers (Lazear (1989)). Here we can establish the following proposition.

Proposition 3

Assume B1–B3. Suppose that a union has a continuum umbers and gas an objective function (20). Then a set of contracts $\tilde{R}$ $(-\{\tilde{R}_1, \tilde{R}_2\})$ will achieve the almost–first–best welfare level for the union given the zero profit for the employer.

<proof>

From proposition 2 different types of agents will self–select by choosing different contracts given $\tilde{R}$, and then choose their first–best effort levels $\hat{\mu}_1$, $\hat{\mu}_2$. Then a small portion q^0 of the agents in each contest is going to be penalized very heavily. The resulting welfare for the union will be

$$W(U_1^1, \cdots, U_1^2, \cdots ;\ \tilde{R})$$

$$= [u(V\hat{\mu}_1) - q^0 D_1 - C(\hat{\mu}_1 ; z_1)] + [u(V\hat{\mu}_2) - q^0 D_2 - C(\hat{\mu}_2 ; z_2)]$$

$$- [u(V\hat{\mu}_1) - C(\hat{\mu}_1 ; z_1)] + [u(V\hat{\mu}_2) - C(\hat{\mu}_2 ; z_2)],$$

(since $q^0 = H(-d(q^0))$ and $D_i = Vu'(V\hat{\mu}_i)/h(-d(q^0))$)

which are the sum of the first–best utility levels for the two types of agents. Q.E.D..

The proposition 3 implies that under $\tilde{R}$, there is no conflict between the union's demand for pay equality, and the incentive pay structure under the two informational asymmetries, although many believe that the conflict does exist.

V. Common Shock θ Among Agents

An important case where employer prefers rank–order contracts to piece–rate contracts–wich is formulated in Nalebuff & Stiglitz(1983) and in Green & Stokey(1983) –, is where there is a common shock θ in the outputs of agents or in the effort cost function C(.). Let us suppose that there is a common shock θ among the agents in the same contest such that the effort cost function $\hat{C}(\mu ; z, \theta)$ satisfies (1) and

$$\hat{C}_{\theta} > \hat{C}_{\theta\mu} > 0 \text{ for all } z. \tag{21}$$

Condition (21) impiles that high θ means more difficult work environment. Suppose that after he enters the contest each agent in the same contest has preivate information about θ. And let θ be distributed within the finite interval $[\theta_1, \theta_2]$ with the distribution function $F(\theta)$, which is a common knowledge. Then the first–best effort level $\hat{\mu}(z, \theta)$ for the agent of type z given θ can be described by

$$Vu' \ (V\hat{u}(z, \theta)) = \hat{C}_{\mu} (\mu ; z, \theta). \tag{22}$$

So by (22) $\hat{\mu}(z, \theta)$ decreases as θ increases. And the first–best utility $\hat{U}_z$ for the type z agent is going to be

$$\hat{U}_z = u(V\hat{\mu}(z)) - E_{\theta}\{\hat{C}(\hat{\mu}(z, \theta) ; z)\},$$

where $\hat{\mu}(z) = E_{\theta}\{\hat{\mu}(z, \theta)\}$.

If an employer uses a quota scheme, which is a piece–rate contract that offers two levels of wages based on individual y_i 's the first–best outcome under pure moral hazard would not be achieved even when agents are risk–neutral. Suppose, for example, that θ turns out to be low. Then the first–best effort level $\hat{\mu}(z, \theta)$ will be high by (21) and (22). Since the minimum performance level that the higher wage is paid for by the quota–scheme is set independently of θ, each agent would choose the effort level lower than $\hat{\mu}(z, \theta)$. In the next two subsections, it will be shown that certain types of multi-agent rank-order contracts can achieve the first-best outcome both for the risk-neutral agents and for the risk-averse agents.

(1) Risk-Neutral Agents

In this case, we can set without loss of generality u(x) to be x. Then we can redefine a set of contracts $\hat{R}$ to be the one that satisfies (16) except that the fixed payoff $\hat{F}_z$ is equal to $(V\hat{\mu}(z) + f(0;\hat{R}_z)\hat{D})$ insetead of $(V\hat{\mu}_z + f(0:\hat{R}_z)\hat{D})$ and that $\alpha(n, m; H) > X^{'}$, where $X^{'} = \hat{\mu}(z_2, \theta_1) - \hat{\mu}(z_1, \theta_2)$. Although all the parameters of $\hat{R}_z$ do not depend upon θ, each type of agent will choose the first-best effort level $\hat{\mu}(z, \theta)$ for each θ under pure moral hazard because

$$\hat{C}_{\mu}(\hat{\mu}(z, \theta); z, \theta) = f_s(0; \hat{R}_z)\hat{D} = V \text{ for given } \theta.$$

Now let us check whether $\hat{R}$ satisfies the self-selection constraint. Suppose that initially each type of agent chooses the right contract $\hat{R}_z$. And suppose low ability z_a agent deviates to high ability(z_b) conract $\hat{R}_b$. Since f( ) is convex in s by lemma 2, his expected utility under $\hat{R}_b$ will be

$$\begin{aligned}
EU_a(\hat{R}_b) &= E_\theta[V\hat{\mu}(z_b, \theta) + \hat{D}E_\theta[f(\mu(z_a, \theta) - (\hat{\mu}(z_b, \theta); \hat{R}_b) \\
&\quad - f(0; \hat{R}\ \ b) - E_\theta\{\hat{C}(\mu(z_z, \theta); z_a, \theta)\} \\
&< E_\theta\{V_\mu(z_a, \theta)\} - E\theta\{\hat{C}(\mu(z_a, \theta); z_a, \theta)\} \\
&\quad \text{because } \hat{D} = -V/f_s(0; \hat{R}_b) \\
&< E_\theta\{V\hat{\mu}(z_a, \theta)\} - E_\theta\{\hat{C}(\hat{\mu}(z_a, \theta)\} \\
&\quad - EU_a(\hat{R}_a),
\end{aligned}$$

where $\mu(z_a, \theta)$ is the optimal choice of the effort for the deviant under $\hat{R}_b$ given θ. Simiarily we can also show that high ability agent would not deviate. Thus we can state the following.

proposition 4

Suppose that there is a privately−informed common shock among the agents in the same contest such that the effort cost function for the agent satisfies (1), (21). Then the set of contracts $\hat{R}$ can achieve the first−best outcome under the two informational asymmetries when agents are risk−neutral.

(2) Risk−Averse Agents

Assume B1−B3. And let us redefine a set of contracts $\tilde{R}$ $(-\{\tilde{R}_z\}_z)$ to be the one such that

$$q_z = q^0 \qquad \text{for all } z$$
$$h(-d(q^0))D_z = Vu'(V\hat{\mu}(z))$$
$$F_z = u(V\hat{\mu}(z)), \tag{23}$$

where $q^0 \rightarrow 0$. Then we can see that under pure moral hazard each type of agent chooses his first−best effort level for any given θ, and that each type of agent almost gets his first−best utility by B2 because

$$EU_z(\tilde{R}_z) \rightarrow u(V\hat{\mu}(z)) - E_\theta\{\hat{C}(\hat{\mu}(z,\theta);z,\theta)\}.$$

To see whether $\tilde{R}$ satisfies the self−selection constraint, as in the proof of proposition 2, let us consider the two first−best contracts $\tilde{R}_a$, $\tilde{R}_b$ for z_a−type and z_b−type agents, respectively. Then suppose a z_a−type agent changes hs contract to $\tilde{R}_b$. Then his expected utility under $\tilde{R}_b$, $EU_a(\tilde{R}_b)$, will be

$$EU_a(\tilde{R}_b) = E_\theta[u(V\hat{\mu}(z_b) - D_b H\{-d(q_0) - (\mu(z_a,\theta)$$
$$-\hat{\mu}(z_b,\theta)\} - \hat{C}(\mu(z_a,\theta);z_a,\theta)]$$

$$=u(V\mu(z_a))-E_\theta\{P(\mu(z_a,\theta);F_\theta\{\hat{C}(\mu(z_a,\theta)z_a,\theta)\}.$$

where $P(\mu(z_a,\theta);F(\theta)-u(V\mu(z_a))-u(V\hat{\mu}(z_b))+D_b$

$$H\{-d(q_0)-(\mu(z_a,\theta)-\hat{\mu}(z_b,\theta)\}.$$

$P(\hat{\mu}(z_b,\theta);F(\theta)>0$ because then $\mu(z_a)=\hat{\mu}(z_b)$. Also we have

$$P'(\hat{\mu}(z_b,\theta);F(\theta))=Vu'(V\hat{\mu}(z_b))F(\theta)-Vu'(V\hat{\mu}(z_b))\frac{h(-d(q^0))}{h(-d(q^0))}f(\theta)=0 \quad \text{for all } \theta,$$

and

$$P''(\mu(z_a,\theta);F(\theta))=[V^2u''(V\mu(z_a))f(\theta)-Vu'(V\hat{\mu}(z_b)\frac{h(-d(q^0)-\Delta)}{h(-d(q^0)}]f(\theta)>0 \text{ by B3,}$$

where $\Delta-\mu(z_a,\theta)-\hat{\mu}(z_b,\theta)<0$. These imply than $P(\mu(z_a,\theta);F(\theta))>0$ for any θ since $\mu(z_a,\theta)<\hat{\mu}(z_b,\theta)$. So it follows that

$$EU_a(\tilde{R}_b)<u(V\mu(z_a))-E_\theta\{\hat{C}(\mu(z_a,\theta);z_a,\theta)\}$$
$$<EU_a(\tilde{R}_a).$$

These arguments have established the following proposition.

Proposition 5

Suppose that there is a privately-informed common shock among the agents in the same contest such that the effort cost function for the agent satisfies (1), (21). Then the set of contracts $\tilde{R}$ defined in (23) can approximate the first-best outcome under the two informational asymmetries when agents are risk-averse

VI. Concluding Remarks

In this paper, a type of multi−agent rank−order contract is analyzed. It is shown that a contract that penalizes the smaller fraction more heavily is more likely to achieve the first−best efficiency under both moral hazard and adverse selection. The first−best outcome under the two concurrent informational asymmetries can be supported by a milti−agent rank−order contract when agents are risk−neutral, while it can be approximated arbitrarily as the fraction of the penalized gets smaller (together with an appropriate increase in the penalty) when agents are risk−averse, The result in Mirrless(1974), when extended to the two concurrent informational asymmetry circumstances, is shown to support the latter finding. These first−best results are shown to continue to hold evern when thd agents in the same contest are faced with a privately−informed common shock.

A couple of interesting implications can be earned from these findings. First, the efficient contracts considered in this paper are consistent with the current promotion structure within a firm in several Asian countries (Korea, Japan). In these countries, the promotion rate is relatively high, while the cost of not being promoted is also high as Aoki(1988) indicates.

Although there could be various other explanations of the current promotion structure in these countries, the results in this paper disclose the efficiency of this structure in controlling various incentive problems on the part of agents. Second, this type of contracts enables a manager of a large grop of workers to deal effectively not only wih the incentive problems of individual workers to deal effectively not only with the incentive provlems of individual workers but also with the union's demand for pay equality. This implication contradicts to the existing belief that the pay equality demanded by union would adversely affect the incentives of workers. In this respect, this argument probably would shed a new light on another aspect of the relationship between the pay equality and the incentive pay structure under the two informational asymmtries.

Appendix

Proof of Lemma 1

i)

$$
\begin{aligned}
& f(0;R) \\
& = F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-1}H(\varepsilon)d\varepsilon \\
& = F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m}d\varepsilon \\
= {} & F(n,m)\left[\left[\frac{1}{m+1}H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m}\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} h(-\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m+1}d\varepsilon\right] \\
= {} & F(n,m)\frac{n-1-m}{m+1}\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m+1}d\varepsilon \\
= {} & F(n,m)\frac{n-1-m}{m+1}\frac{n-2-m}{m+2}\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-3-m}H(\varepsilon)^{m+2}d\varepsilon
\end{aligned}
$$

$$
\begin{aligned}
= {} & F(n,m)\frac{n-1-m}{m+1}\frac{n-2-m}{m+2}\cdots\frac{1}{n-1}\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1} \\
= {} & F(n,m)\frac{n-1-m}{m+1}\cdots\frac{1}{n-1}\frac{1}{n} \\
= {} & \frac{m}{n}.
\end{aligned}
$$

ii)

$$
f'(s;R) = -F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-1}h(-s+\varepsilon)d\varepsilon < 0.
$$

Since f(s;R) + (1-f(s;R)) = 1, we have for any (n,m)

$$
F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-1}d\varepsilon = 1. \qquad (A)
$$

Now let us define $\beta(n,m)$ such that

$$F(n-1,m) = \beta(n,m)F(n,m).$$

Then

$$f(s;R((n-1,m),F,D))$$

$$= F(n-1,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m-1}H(-s+\varepsilon)d\varepsilon$$

$$= F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m-1}\beta(n,m)H(-s+\varepsilon)d\varepsilon \quad \textit{since } \beta \textit{ is independent of } \varepsilon$$

$$> F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m-1}H(-\varepsilon)H(-s+\varepsilon)d\varepsilon$$

$$\textit{since } \int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m-1}\beta(n,m) = \int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-2-m}H(\varepsilon)^{m-1}H(-\varepsilon)d\varepsilon \quad \textit{by } (A)$$

$$= f(s;R((n,m),F,D)).$$

Similarily let us define $\beta'(n,m)$ such that

$$F(n,m-1) = \beta'(n,m)F(n,m-1).$$

Then we have

$$f(s;R(n,m-1))$$

$$= F(n,m-1)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-2}H(-s+\varepsilon)d\varepsilon$$

$$= F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-2}\beta'(n,m)H(-s+\varepsilon)d\varepsilon$$

$$< F(n,m)\int_{-\infty}^{\infty} h(\varepsilon)H(-\varepsilon)^{n-1-m}H(\varepsilon)^{m-2}H(\varepsilon)H(-s+\varepsilon)d\varepsilon \qquad \textit{by } (A)$$

$$= f(s;R(n,m)).$$

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Footnote

1) A supporting evidence for this belief, which is suggested by M. Aoki (1988), is the fact that the separation rate fot the midcareer employees in Japan is relatively lower that of the United States.

2) Aoki(1988) argued that the midcareer separation involves substantial penalty in Japan for the following reasons: i) low separation payment, the amount of which varies considerably with the lenth of tenure within a firm, ii) few job vacancies for the midcareer agents since an employer values highly on the firm-specific human capital of an agent, iii) the reputation effect, i.e., that of conveying negative information about the job-changer in his midcareer to the potential employers. I think that Aoki's argument is also true in Korea to a large extent.

3) A simple form of the first-best piece-rate contract is the one that pays an agent as much as his observed performance Vy.

4) If pays are based upon the rankings of performances of agents, the total payment by an employer is fixed so that the adverse incentive on the part of employer will disappear, as has been pointed out by Bhattacharaya (1984).

5) Using the arguments in Shapiro & Stiglitz(1984), we can see that the unemployment pool as well as $\hat{D}$ can make the actual penalty sufficient enough to control the incentive problems of agents.

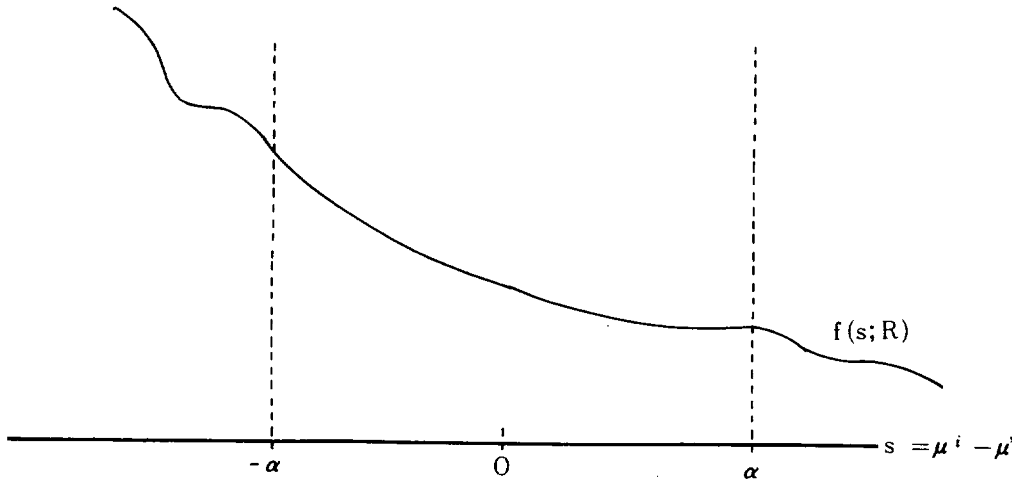
6) Although the deviant can expect the higher fixed payoff $u(V\hat{\mu}_b)$ by deviation, his total payoff (fixed payoff minus penalty payoff) will be less than the utility of the output $V\mu_a$ produced by him.

7) If the size of the union $(n_1 + n_2)$ is taken as exogeneous, the union's objective function (20) is equivalent to the utilitarian social welfare function

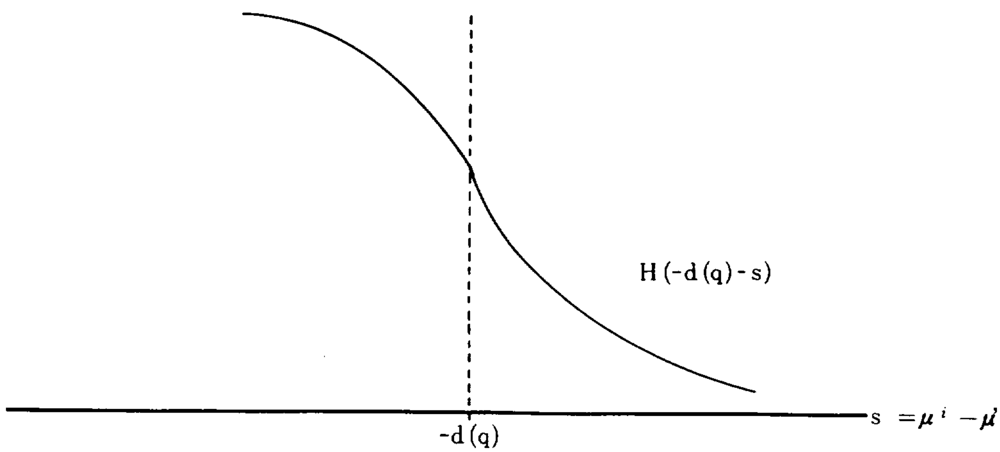
objective function (20) is equivalent to the utilitarian social welfare funtion.

$$W = \sum_{i-1}^{n_1 + n_2} U_i$$

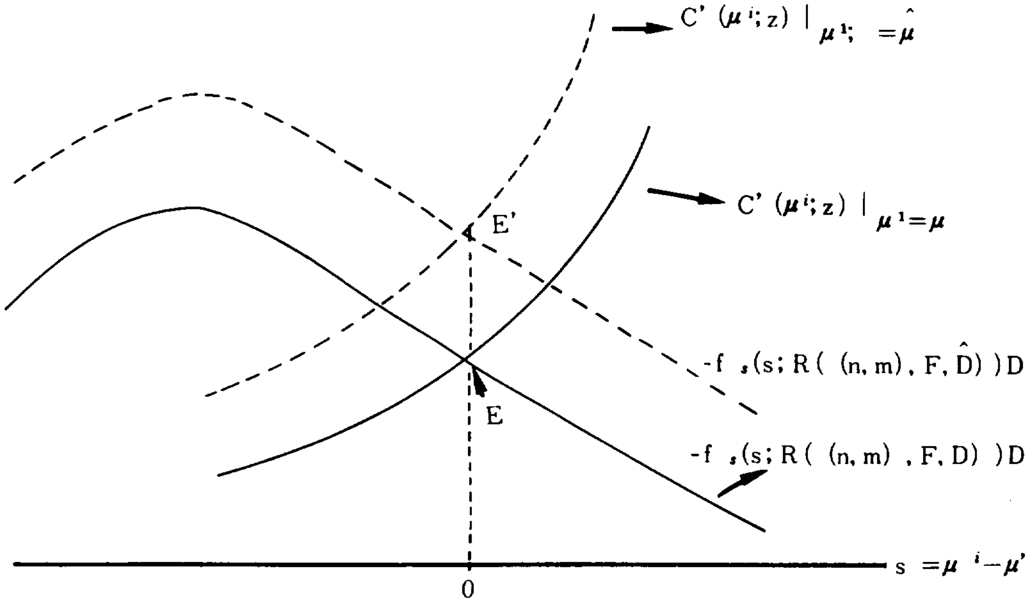
The funtion (20) is one of the most popular forms of the union objective that are taken in the related literature.



<Figure 1>Penalty Probability $f(s; R((n, m), F, D))$ when $n>2m$

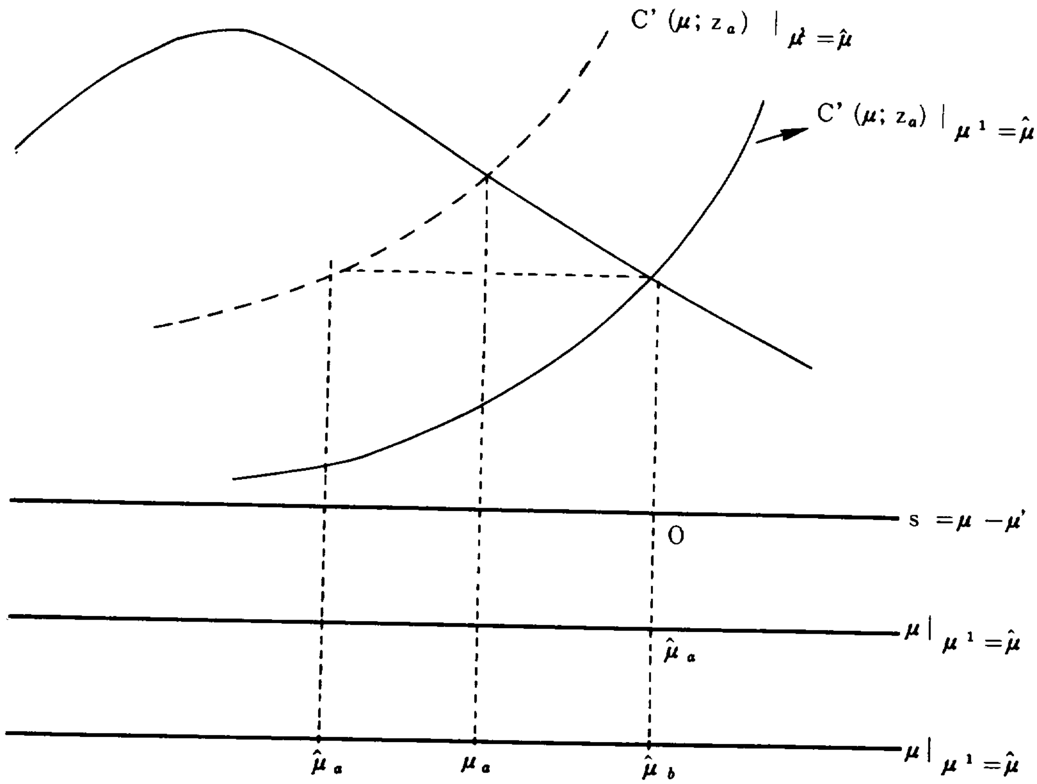


<Figure 2>Penalty Probability $f(s; R(q, F, D))$

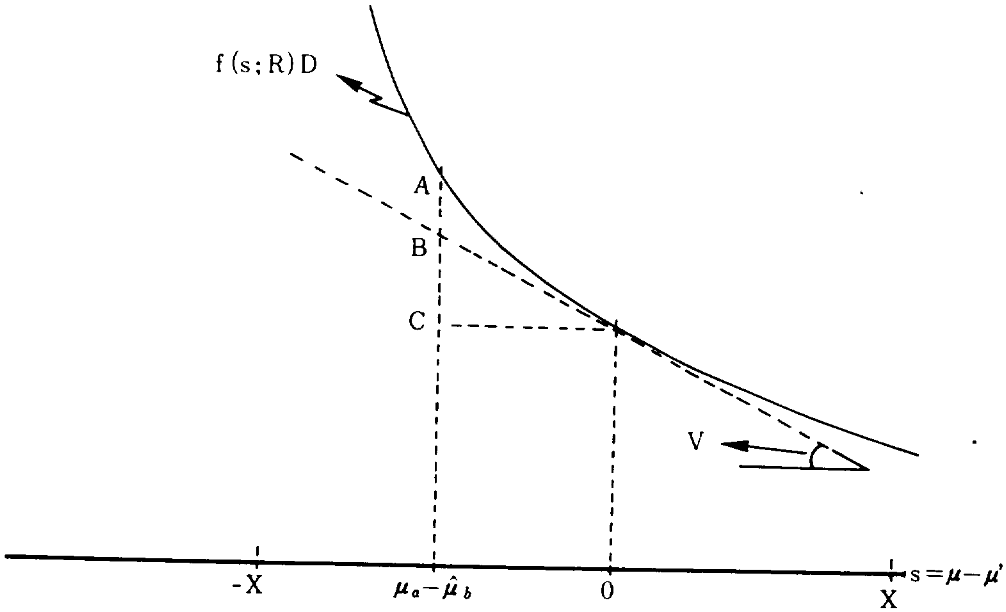


E: { μ z }   E' : { μ̂ z }

<Figure 3> Symmetric Nash Equilibria E, E'



<Figure 4>Optimal choice μ_a for z_a-type agent given $\mu' = \hat{\mu}_b$



AC: △penalty, BC: $V\hat{\mu}_b - V\mu_a$

<Figure 5> Penalty Increase for the Deviant