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Survival Function Estimation for the Proportional Hazards Regression Model+

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ABSTRACT

The purpose of this paper is to propose the modified semiparametric estimators for survival function in the Cox's regression model with randomly censored data based on Tsiatis and Breslow estimators, and present their asymptotic variances estimates. The proposed estimators are compared to Tsiatis, Breslow, and Kaplan-Meier estimators through a small-sample Monte Carlo study. The simulation results show that the proposed estimators are preferred for small sample sizes.

1. Introduction

Cox's proportional hazards regression model (or Cox's regression model) specifies the hazard function $\lambda(t ; z)$ for the failure time of an individual with covariate vector z as

$$\lambda(t ; z) = exp(\beta' z)\lambda_0(t). \tag{1.1}$$

Here $\lambda_0(t)$ is an arbitrary unspecified baseline hazard function for an individual with covariate vector $z=0$ and $\beta' = (\beta_1, \cdots, \beta_p)$ is a vector of regression parameters.

This model is often described as Cox's semiparametric model because it illustrates in

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striking form the separation between parametric and nonparametric components. Interest of this model centers on how the distribution of failure time is affected by other variables called explanatory variables or covariates. Cox's regression model is now enjoying widespread applications in the reliability field as well as in the medical work.

While Cox's regression model has had a significant impact on the biomedical field, it has received little attention in the reliability literature. Only recently has the model been used for the analysis of software reliability, system reliability, and repairable systems by Mazzuchi et al. (1989). This model requires modern computing equipment to be applicable, so the concurrent development of modern computers has been one of the prerequisites for this methodological work.

Cox(1972, 1975) derived estimator of β and an asymptotic covariance matrix using the partial likelihood argument. The efficiency of Cox's partial likelihood function for censored data has been considered by Efron(1977), Oakes(1977), Kalbfleisch and Prentice(1980), and Kay(1979). Johnson et al.(1982) considered the small sample performance of model parameter estimators under the Cox's regression model. Breslow(1974) proposed Kaplan-Meier type semiparametric estimator for survival function under the Cox's regression model. However, Tsiatis(1981) suggested Nelson-Aalen type semiparametric estimator for the survival function and investigated a large sample study. In the reliability problems as well as medical studies, a large sample study may not be applicable because of time, cost, and other limitation. Thus much work is needed to investigate the small sample behaviors.

The purpose of this paper is as follows. In Section 2, the modified semiparametric estimators of survival function using Breslow and Tsiatis estimators are proposed, and the relationships among those modified semiparametric estimators under the Cox's regresssion model and Kaplan-Meier estimator are investigated. Furthermore the estimates for asymptotic variances of proposed survival function estimators are derived. In Section 3, the proposed estimators are compared to Tsiatis and Breslow estimators in the sense of MSE and bias through a small-sample Monte Carlo study. Finally, conclusions are given in Section 4.

2. Proposed Estimators and Their Asymptotic Variances Estimates

Throughout this paper, we will denote the lifetime for n components by $Y_1, \cdots, Y_n$. These may not all be observable due to censoring times $C_1, \cdots, C_n$ for each of components. The ramdom censoring scheme adopted in this paper assumes that, given the covariate vector $z_i = (z_{i1}, \cdots, z_{ip})'$, the Y_i and C_i are independently distributed according to the continuous distributions $F(\cdot\ ; z_i)$ and $C(\cdot\ ; z_i)$, respectively.

The observable quantities are

$$T=\min(Y,\ C) \text{ and}$$
$$\delta = I(Y \le C) = \begin{cases} 1, & \text{if } Y \le C \text{ (failure)} \\ 0, & \text{if } Y > C \text{ (censoring)}. \end{cases}$$

Therefore the observed data from n individuals consist of the vectors

$$(t_i,\ \delta_i,\ z_i),\ i=1,\ \cdots,\ n. \tag{2.1}$$

Furthermore suppose that a random sample of n individuals yields a sample with k distinct observed failure times will be denoted by $t_{(1)}<\cdots<t_{(k)}$, and $R(t_{(i)})$ will be used to represent the risk set at time $t_{(i)}$, that is, the set of individuals still under observation immediately before the i-th observation time $t_{(i)}$.

On somewhat heuristic grounds, Cox(1975) suggested the following partial likelihood for estimating β in (1.1) in the absence of knowledge about $\lambda_0(t)$:

$$L(\beta)=\prod_{i=1}^{k}\left(\frac{e^{\beta' z_{(i)}}}{\sum_{j\in R(t_{(i)})} exp(\beta' z_j)}\right), \tag{2.2}$$

where $z_{(i)}$ is covariate vector associated with the individual observed to die at $t_{(i)}$. Maximization is accomplished by setting the first partial derivatives equal to 0 so that

$$\frac{\partial \ln L(\beta)}{\partial \beta_l}=\sum_{i=1}^{k}\left(z_{il}-\frac{\sum_{j\in R(t_{(i)})} z_{jl}\, exp(\beta' z_j)}{\sum_{j\in R(t_{(i)})} exp(\beta' z_j)}\right), \tag{2.3}$$

which is easily solved by the use of the Newton-Raphson iteration method. The estimator for the regression parameters obtained from Cox's partial likelihood function, which is called the maximum partial likelihood estimator is denoted by $\hat{\beta}$.

We should like to propose the modified semiparametric estimators of survival function based on Breslow and Tsiatis estimators.

Breslow(1974) proposed the semiparametric estimator for the underlying cumulative hazard function as follows :

$$\hat{\Lambda}_0(t)=\sum_{i\,;\,t_{(i)}\le t}\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})} exp(\hat{\beta}' z_j)}, \tag{2.4}$$

where $m_{(i)}$ are the multiplicities of the death times and the outer summation is over the true death time less than or equal to t.

Tsiatis(1981) suggested the survival function estimator which combines the parameter estimator derived by Cox and the estimator of the underlying cumulative hazard function derived by Breslow(1974). Therefore Tsiatis estimator of survival function for an individual with covariate vector z is given by

$$\hat{S}_T(t\,;\,z)=\begin{cases} exp\left\{-exp(\hat{\beta}' z)\cdot \sum_{i:t_{(i)}\le t}\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})} exp(\hat{\beta}' z_j)}\right\}, & t\le t_{(k)} \\ 0 \text{ or undefined} & , t>t_{(k)}. \end{cases} \tag{2.5}$$

Remark 2.1 When $\hat{\beta}=0$, Tsiatis estimator reduces to the Nelson-Aalen type estimator calculated from the entire set of observations considered as one homogeneous sample.

Next we consider the proportion of time between t and last observed death time prior to time t, which is ignored by $\hat{S}_T(t\,;\,z)$. Therefore we obtain the modified Tsiatis estimator for the survival function as follows :

$$\hat{S}_{MT}(t\,;\,z)=\begin{cases} exp\{-exp(\hat{\beta}'z)\tilde{\Lambda}_0(t)\}, & t\leq t_{(k)} \\ 0 \text{ or undefined} & , \ t>t_{(k)} \end{cases} \tag{2.6}$$

where

$$\tilde{\Lambda}_0(t)=\sum_{i=1}^{l}\left(\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)}\right)+\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\cdot\frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})}exp(\hat{\beta}'z_j)} \tag{2.7}$$

and l is such that $t_{(l)}<t$ and $t_{(l+1)}\geq t$.

Remark 2.2 When time t is equal to last observed death time prior to time t, this modified semiparametric estimator is Tsiatis estimator. Moreover, $\hat{S}_{MT}$ is a smoothed version of $\hat{S}_T$.

However, Breslow(1974) used the Kaplan-Meier type estimator in stead of Nelson-Aalen type estimator and suggested the semiparametric estimator for the survival function as follows :

$$\hat{S}_B(t\,;\,z)=\begin{cases} \left\{\prod_{i:t_{(i)}\leq t}\left(1-\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)}\right)\right\}^{exp(\hat{\beta}'z)} & , \ t\leq t_{(k)} \\ 0 \ \text{ or undefined} & , \ t>t_{(k)} \end{cases} \tag{2.8}$$

Remark 2.3 When $\hat{\beta}=0$, (2.8) reduces to the product limit estimator of Kaplan and Meier calculated from the entire set of observations considered as one homogeneous sample.

Finally, the Kaplan-Meier type semiparametric estimator can be found in analogy with the modified Tsiatis estimator as follows :

$$\hat{S}_{MB}(t\,;\,z)=\left\{\prod_{i=1}^{l}\left(1-\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)}\right)\cdot\left(1-\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\cdot\frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})}exp(\hat{\beta}'z_j)}\right)\right\}^{exp(\hat{\beta}'z)}, \tag{2.9}$$

where l is such that $t_{(l)}<t$ and $t_{(l+1)}\geq t$.

Remark 2.4 (1) $\hat{S}_{MB}$ is a smoothed version of the Breslow estimator $\hat{S}_B$. (2) Clearly, when time t is equal to last observed death time prior to time t, the proposed modified semiparametric estimator $\hat{S}_{MB}$ is Breslow estimator $\hat{S}_B$.

Next, we consider estimating the asymptotic variances of proposed semiparametric estimators.

Theorem 2.1 Under Cox's regression model with randomly censored data, a consistent estimate of asymptotic variance of $\hat{S}_{MB}(t\,;\,z)$ is given by

$$\widehat{\mathrm{Var}}(\hat{S}_{MB}(t\,;\,z)) = \hat{S}^2_{MB}(t\,;\,z)\sum_{i=1}^{l+1}\frac{\hat{q}_i}{N_i\hat{p}_i} + (\partial\hat{S}_{MB}/\partial\hat{\beta})'\ \widehat{\mathrm{Var}}(\hat{\beta})\,(\partial\hat{S}_{MB}/\partial\hat{\beta}), \tag{2.10}$$

where

$$\hat{p}_i = \left(1 - \frac{m_{(i)}}{\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)}\right)^{exp(\hat{\beta}'z)},$$

$$\hat{p}_{l+1} = \left(1 - \frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\quad \frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})} exp(\hat{\beta}'z_j)}\right)^{exp(\hat{\beta}'z)},$$

$$\hat{q}_i = 1-\hat{p}_i, \qquad i=1,\ \cdots,\ l+1,$$

N_i is the number of j in the risk set i,

$$\widehat{\mathrm{Var}}(\hat{\beta}) \equiv (\hat{\sigma}_{pq})$$

$$= \left[\sum_{i=1}^{k}\left\{\frac{\left(\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)\right)\left(\sum_{j\in R(t_{(i)})} z_{jp}z_{jq}exp(\hat{\beta}'z_j)\right)}{\left(\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)\right)^2} - \frac{\left(\sum_{j\in R(t_{(i)})} z_{jp}exp(\hat{\beta}'z_j)\right)\left(\sum_{j\in R(t_{(i)})} z_{jq}\ exp(\hat{\beta}'z_j)\right)}{\left(\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)\right)^2}\right\}\right]^{-1}, \tag{2.11}$$

and

$$\partial\hat{S}_{MB}(t\,;\,z)/\partial\hat{\beta}_p = \hat{S}_{MB}(t\,;\,z)\cdot e^{\hat{\beta}'z}\Bigg[z_p\cdot$$

$$ln\left(\prod_{i=1}^{l}\left(1-\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)}\right)\cdot\left(1-\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\cdot\frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})} exp(\hat{\beta}'z_j)}\right)\right)$$

$$+\left(\sum_{i=1}^{l}\frac{m_{(i)}\sum_{j\in R(t_{(i)})} z_{jp}exp(\hat{\beta}'z_j)}{\left(1-\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)}\right)\left(\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)\right)^2}\right.$$

$$\left.+\frac{\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\cdot m_{(l+1)}\cdot\sum_{j\in R(t_{(l+1)})} z_{jp}exp(\hat{\beta}'z_j)}{\left(1-\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\ \frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})} exp(\hat{\beta}'z_j)}\right)\left(\sum_{j\in R(t_{(l+1)})} exp(\hat{\beta}'z_j)\right)^2}\right)\Bigg] \tag{2.12}$$

Proof. By a first-order Taylor series expansion we obtain that

$$\hat{S}_{MB}(t\,;\,z)=\hat{S}_{MB}(t\,;\,z)|_{\beta}+(\hat{\beta}-\beta)'\partial\hat{S}_{MB}/\partial\hat{\beta}|_{\hat{\beta}=\beta},$$

where $\hat{S}_{MB}(t\,;\,z)|_{\beta}$ is $\hat{S}_{MB}(t\,;\,z)$ evaluated at β and $\partial\hat{S}_{MB}|\partial\hat{\beta}|_{\hat{\beta}=\beta}$ is the vector of first partial derivatives of $\hat{S}_{MB}(t\,;\,z)$ with respect to β.

Thus

$$\mathrm{Var}(\hat{S}_{MB}(t\,;\,z))=(\partial\hat{S}_{MB}/\partial\hat{\beta})'|_{\hat{\beta}=\beta}\ Var(\hat{\beta})\,(\partial\hat{S}_{MB}/\partial\hat{\beta})|_{\hat{\beta}=\beta}+Var(\hat{S}_{MB}(t\,;\,z)|\beta).$$

Here the first term gives the variation due to the estimation of β by $\hat{\beta}$ and the second term that due to the estimation of the integrated hazard function.

Cox's partial likelihood theory asserted that

$$\widehat{Var}(\hat{\beta})\equiv(\hat{\sigma}_{pq})=\left[\sum_{i=1}^{k}\left\{\frac{\left(\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)\right)\left(\sum_{j\in R(t_{(i)})}z_{jp}z_{jq}exp(\hat{\beta}'z_j)\right)}{\left(\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)\right)^2}-\frac{\left(\sum_{j\in R(t_{(i)})}z_{jp}exp(\hat{\beta}'z_j)\right)\left(\sum_{j\in R(t_{(i)})}z_{jq}exp(\hat{\beta}'z_j)\right)}{\left(\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)\right)^2}\right\}\right]^{-1}$$

is a consistent estimate of asymptotic variance of $\hat{\beta}$ which is given by minus the second derivative of the logarithm of the partial likelihood function.

The asymptotic variance estimate of $\hat{S}_{MB}(t\,;\,z)$ given β is given by Greenwood's formula. This complete the proof.

By a similar method of Theorem 2.1, we get the following result.

Corollary 2.1

$$\widehat{Var}(\hat{S}_{MT}(t\,;\,z))=\hat{S}_{MT}(t\,;\,z)\sum_{i=1}^{l+1}\frac{\hat{q}_i}{N_i\hat{p}_i}+(\partial\hat{S}_{MT}(t\,;\,z)/\partial\hat{\beta})'\,\widehat{Var}(\hat{\beta})\,(\partial\hat{S}_{MT}(t\,;\,z)/\partial\hat{\beta}),\qquad(2.13)$$

where

$$\hat{p}_i=exp\{-m_{(i)}exp(\hat{\beta}'z)/\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)\},\ \ i=1,\ \cdots,\ l,$$

$$\hat{p}_{l+1}=exp\left\{-exp(\hat{\beta}'z)\left(\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\ \frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})}exp(\hat{\beta}'z_j)}\right)\right\},$$

$$\hat{q}_i=1-\hat{p}_i,\ \ i=1,\ \cdots,\ l+1,$$

and

$$\partial\hat{S}_{MT}(t\,;\,z)/\partial\hat{\beta}_p=\hat{S}_{MT}(t\,;\,z)\cdot exp(\hat{\beta}'z)\cdot$$

$$\left[\left(\sum_{i=1}^{l} m_{(i)}\frac{\sum_{j\in R(t_{(i)})} z_{jp}exp(\hat{\beta}'z_j)}{\left(\sum_{j\in R(t_{(i)})} exp(\hat{\beta}'z_j)\right)^2}+\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\cdot m_{(l+1)}\frac{\sum_{j\in R(t_{(l+1)})} z_{jp}exp(\hat{\beta}'z_j)}{\left(\sum_{j\in R(t_{(l+1)})} exp(\hat{\beta}'z_j)\right)^2}\right)\right.$$

$$\left.-z_p\cdot\left(\sum_{i=1}^{l}\frac{m_{(i)}}{\sum_{j\in R(t_{(i)})}exp(\hat{\beta}'z_j)}+\frac{t-t_{(l)}}{t_{(l+1)}-t_{(l)}}\,\frac{m_{(l+1)}}{\sum_{j\in R(t_{(l+1)})}exp(\hat{\beta}'z_j)}\right)\right] \qquad (2.14)$$

3. A Monte Carlo Study

In this section we intend to compare the performances of five estimators $\hat{S}_T$, $\hat{S}_{MT}$, $\hat{S}_B$, $\hat{S}_{MB}$, and $\hat{S}_{KM}$ for survival function in terms of MSE and bias via a Monte Carlo study. Furthermore, Monte Carlo simulations were carried out to investigate the effects of varing the lifetime distributions, censoring rates, covariates, and sample sizes.

Trials were done 500 times. For each combinations of lifetime distributions(exponential regression model, Weibull regression model with DFR, Weibull regression model with IFR), mission time $t(t: S(t\,;\,z)=0.2(0.3)0.8)$, sample sizes($n=10$, 20, 30), censoring rates(about 10%, 30%, 50%, and uncensored case), and covariates(0, 1), the MSE and bias were computed. The standard error (SE) was also obtained for each MSE. These simulations were performed on CYBER-170/835 using IMSL.

From Tables 2-4, we can observe the following facts :

(1) Modified Tsiatis estimator $\hat{S}_{MT}$ is better than Tsiatis estimator $\hat{S}_T$ in the middle region and upper tails of lifetime distributions by means of MSE.

(2) In the case of heavy censoring rates, MSEs of Tsiatis estimator $\hat{S}_T$ are smaller than those of Breslow estimator $\hat{S}_B$ in the lower tails and middle region of lifetime distributions. In the uncensored case, however, $\hat{S}_T$ are better than $\hat{S}_B$ at almost points in time.

(3) When failare time t approaches to the upper tails from in the middle region of lifetime distributions, $\hat{S}_{MB}$ performs better than $\hat{S}_B$.

(4) As censoring rates increase, performances of modified Breslow estimator $\hat{S}_{MB}$ become better than those of modified Tsiatis estimator $\hat{S}_{MT}$ in the upper tails of lifetime distributions. In the case of no censoring, however, $\hat{S}_{MT}$ performs better than $\hat{S}_{MB}$ at almost points in time.

(5) Under the Cox's regression model, as expected, $\hat{S}_{KM}$ is worse than the semiparametric estimators.

(6) It is not surprizing that varing lifetime distributions gives no essential changes in the simulation results.

4. Conclusions

In this paper, the modified semiparametric estimators of survival function using Tsiatis and

Breslow estimators were suggested, and the relationships among $\hat{S}_T$, $\hat{S}_{MT}$, $\hat{S}_B$, $\hat{S}_{MB}$, and $\hat{S}_{KM}$ were investigated. Furthermore, a small-sample Monte Carlo study was carried out to investigate the performances of those semiparametric estimators and nonparametric K-M estimator. This simulation results showed that $\hat{S}_{MT}$ and $\hat{S}_{MB}$ are better than $\hat{S}_T$ and $\hat{S}_B$, respectively, in the middle region and upper tails of lifetime distributions. As censoring rates increase, performances of $\hat{S}_{MB}$ becomes better than those of $\hat{S}_{MT}$ in the sense of MSE. In the uncensored case $\hat{S}_{MT}$ performs better tham $\hat{S}_{MB}$ at almost points in time. From simulation results we recommend the modified semiparametric estimators in the middle region and upper tails of lifetime distributions. Finally, we estimated the asymptotic variances of proposed semiparametric estimators.

Table 1. The Lifetime and Censoring Distributions

Lifetime Distribution	Censoring Distribution	Censoring Rate
A $_a$Exp(1)	D $_a$Exp(0.43)	30%
$_b$Exp(1.11)	$_b$Exp(1.11)	50%
B $_a$*Weib*(1, 0.5)	E Unccnsored	0%
$_b$*Weib*(1.22, 0.5)	Case	
C $_a$*Weib*(1, 2)	F $_a$Unif(0, 8.86)	10%
$_b$*Weib*(1.05, 2)	$_b$Unif(0, 2.81)	30%

a and *b* represent the distributions for individuals with covariates 0 and 1, respectively.

Table 2. Comparisons of $\hat{S}_T$, $\hat{S}_{MT}$, $\hat{S}_B$, $\hat{S}_{MB}$, and $\hat{S}_{KM}$ with Lifetime Distribution$=A$ and Censoring Distribution$=D$

Mission time t : $S(t\,;\,0)=0.8,\ 0.5,\ 0.2$ and Covariate$=0$

SAMPLE SIZE		10			20			30		
MISSION TIME		.2230	.6930	1.6090	.2230	.6930	1.6090	.2230	.6930	1.6090
$\hat{S}_T$	MSE	.0220	.0670	.0506	.0129	.0286	.0310	.0078	.0185	.0222
	BIAS	.0057	−.0118	−.0604	−.0012	−.0005	−.0391	.0065	.0129	−.0052
	SE	.0022	.0036	.0023	.0011	.0019	.0015	.0005	.0012	.0011
$\hat{S}_{MT}$	MSE	.0250	.0631	.0446	.0135	.0280	.0292	.0078	.0180	.0211
	BIAS	−.0304	−.0459	−.0750	−.0224	−.0186	−.0472	−.0088	.0003	−.0115
	SE	.0024	.0036	.0019	.0012	.0019	.0014	.0006	.0011	.0011
$\hat{S}_B$	MSE	.0250	.0717	.0459	.0139	.0310	.0294	.0081	.0194	.0212
	BIAS	−.0059	−.0404	−.0768	−.0071	−.0178	−.0591	.0029	.0018	−.0248
	SE	.0024	.0036	.0021	.0012	.0021	.0015	.0006	.0012	.0010
$\hat{S}_{MB}$	MSE	.0290	.0691	.0413	.0148	.0309	.0281	.0082	.0192	.0204
	BIAS	−.0427	−.0738	−.0908	−.0286	−.0358	−.0664	−.0124	−.0108	−.0306
	SE	.0026	.0036	.0017	.0013	.0021	.0014	.0006	.0012	.0010
$\hat{S}_{KM}$	MSE	.0321	.0633	.0504	.0179	.0308	.0276	.0116	.0211	.0198
	BIAS	−.0128	−.0218	−.0445	−.0046	−.0149	−.0300	−.0038	.0028	.0004
	SE	.0020	.0035	.0027	.0014	.0019	.0015	.0008	.0013	.0012

Mission time t : $S(t\,;\,1)=0.8,\ 0.5,\ 0.2$ and Covariate$=1$

SAMPLE SIZE		10			20			30		
MISSION TIME		.2020	.6270	1.4560	.2020	.6270	1.4560	.2020	.6270	1.4560
$\hat{S}_T$	MSE	.0240	.0627	.0467	.0118	.0287	.0352	.0083	.0190	.0242
	BIAS	.0038	.0005	−.0585	.0026	.0031	−.0286	.0032	.0044	−.0086
	SE	.0020	.0034	.0022	.0009	.0019	.0017	.0005	.0011	.0013
$\hat{S}_{MT}$	MSE	.0274	.0584	.0415	.0122	.0280	.0330	.0084	.0188	.0232
	BIAS	−.0338	−.0368	−.0737	−.0193	−.0157	−.0372	−.0127	−.0086	−.0150
	SE	.0023	.0034	.0018	.0010	.0018	.0016	.0006	.0011	.0012
$\hat{S}_B$	MSE	.0255	.0629	.0432	.0122	.0289	.0321	.0085	.0193	.0225
	BIAS	−.0054	−.0254	−.0775	−.0023	−.0124	−.0497	.0000	−.0056	−.0270
	SE	.0021	.0035	.0020	.0009	.0019	.0014	.0005	.0011	.0011
$\hat{S}_{MB}$	MSE	.0295	.0603	.0384	.0127	.0287	.0305	.0086	.0193	.0217
	BIAS	−.0437	−.0638	−.0930	−.0244	−.0311	−.0575	−.0160	−.0186	−.0329
	SE	.0024	.0035	.0016	.0010	.0019	.0013	.0006	.0011	.0011
$\hat{S}_{KM}$	MSE	.0400	.0975	.0500	.0195	.0447	.0450	.0130	.0283	.0361
	BIAS	−.0053	−.0600	−.1160	−.0058	−.0217	−.0810	.0060	−.0108	−.0628
	SE	.0035	.0046	.0028	.0013	.0030	.0020	.0008	.0019	.0014

Table 3. Comparisons of $\hat{S}_T$, $\hat{S}_{MT}$, $\hat{S}_B$, $\hat{S}_{MB}$, and $\hat{S}_{KM}$ with Lifetime Distribution$=B$ and Censoring Distribution$=E$

Mission time t : $S(t\ ;\ 0)=0.8,\ 0.5,\ 0.2$ and Covariate$=0$

SAMPLE SIZE		10			20			30		
MISSION TIME		.0500	.4800	2.5900	.0500	.4800	2.5900	.0500	.4800	2.5900
$\hat{S}_T$	MSE	.0175	.0371	.0322	.0105	.0193	.0132	.0064	.0129	.0104
	BIAS	.0126	.0236	.0344	-.0009	.0002	.0085	.0022	.0127	.0172
	SE	.0013	.0020	.0022	.0009	.0012	.0010	.0004	.0008	.0007
$\hat{S}_{MT}$	MSE	.0197	.0367	.0255	.0112	.0195	.0122	.0065	.0126	.0094
	BIAS	-.0213	-.0190	-.0081	-.0217	-.0218	-.0143	-.0122	-.0027	.0009
	SE	.0016	.0021	.0016	.0010	.0012	.0009	.0005	.0008	.0006
$\hat{S}_B$	MSE	.0198	.0410	.0301	.0113	.0208	.0137	.0066	.0134	.0104
	BIAS	.0025	-.0006	.0038	-.0061	-.0125	-.0113	-.0011	.0046	.0040
	SE	.0016	.0022	.0020	.0010	.0013	.0010	.0004	.0008	.0007
$\hat{S}_{MB}$	MSE	.0228	.0424	.0261	.0121	.0215	.0133	.0069	.0133	.0099
	BIAS	-.0322	-.0436	-.0384	-.0272	-.0345	-.0333	-.0156	-.0109	-.0120
	SE	.0019	.0023	.0015	.0011	.0014	.0008	.0005	.0008	.0006
$\hat{S}_{KM}$	MSE	.0304	.0466	.0327	.0170	.0255	.0155	.0116	.0157	.0116
	BIAS	-.0060	-.0070	-.0012	-.0024	-.0080	-.0132	-.0008	.0027	.0034
	SE	.0019	.0026	.0021	.0012	.0016	.0010	.0008	.0010	.0008

Mission time t : $S(t\ ;\ 1)=0.8,\ 0.5,\ 0.2$ and Covariate$=1$

SAMPLE SIZE		10			20			30		
MISSION TIME		.0410	.3930	2.1210	.0410	.3930	2.1210	.0410	.3930	2.1210
$\hat{S}_T$	MSE	.0208	.0355	.0281	.0093	.0159	.0128	.0059	.0127	.0097
	BIAS	.0056	.0191	.0310	.0070	.0120	.0171	.0066	.0164	.0190
	SE	.0015	.0020	.0018	.0006	.0010	.0009	.0004	.0008	.0006
$\hat{S}_{MT}$	MSE	.0234	.0347	.0219	.0093	.0159	.0116	.0059	.0124	.0089
	BIAS	-.0290	-.0248	-.0091	-.0141	-.0109	-.0047	-.0095	.0009	.0038
	SE	.0017	.0021	.0013	.0007	.0010	.0008	.0004	.0007	.0006
$\hat{S}_B$	MSE	.0220	.0362	.0263	.0095	.0159	.0120	.0060	.0126	.0091
	BIAS	-.0027	-.0021	-.0097	.0025	.0000	-.0047	.0036	.0089	.0064
	SE	.0016	.0021	.0019	.0006	.0010	.0008	.0004	.0007	.0006
$\hat{S}_{MB}$	MSE	.0251	.0371	.0228	.0097	.0164	.0117	.0061	.0125	.0086
	BIAS	-.0379	-.0483	-.0512	-.0189	-.0230	-.0260	-.0126	-.0068	-.0086
	SE	.0018	.0022	.0014	.0007	.0010	.0007	.0004	.0007	.0005
$\hat{S}_{KM}$	MSE	.0315	.0452	.0309	.0161	.0221	.0142	.0103	.0160	.0117
	BIAS	.0021	-.0054	-.0040	-.0013	-.0052	-.0032	.0013	.0112	.0072
	SE	.0017	.0026	.0019	.0010	.0014	.0010	.0006	.0010	.0008

Table 4. Comparisons of $\hat{S}_T$, $\hat{S}_{MT}$, $\hat{S}_B$, $\hat{S}_{MB}$, and $\hat{S}_{KM}$ with Lifetime Distribution = C and Censoring Distribution = F

Mission time t : $S(t\,;\,0)=0.8,\ 0.5,\ 0.2$ and Covariate = 0

SAMPLE SIZE		10			20			30		
MISSION TIME		.4720	.8330	1.2690	.4720	.8330	1.2690	.4720	.8330	1.2690
$\hat{S}_T$	MSE	.0195	.0418	.0387	.0110	.0238	.0168	.0067	.0142	.0126
	BIAS	.0122	.0287	.0322	.0092	.0132	.0163	.0050	.0149	.0200
	SE	.0014	.0023	.0023	.0007	.0013	.0011	.0005	.0009	.0009
$\hat{S}_{MT}$	MSE	.0219	.0392	.0314	.0113	.0229	.0148	.0068	.0137	.0115
	BIAS	−.0260	−.0139	.0007	−.0137	−.0086	.0012	−.0105	.0007	.0097
	SE	.0016	.0023	.0018	.0008	.0013	.0009	.0005	.0009	.0008
$\hat{S}_B$	MSE	.0222	.0464	.0346	.0117	.0254	.0164	.0070	.0147	.0126
	BIAS	.0012	.0007	−.0010	.0039	−.0013	−.0072	.0014	.0054	.0029
	SE	.0018	.0025	.0022	.0008	.0014	.0010	.0005	.0009	.0009
$\hat{S}_{MB}$	MSE	.0257	.0454	.0294	.0122	.0250	.0151	.0072	.0144	.0118
	BIAS	−.0379	−.0417	−.0303	−.0193	−.0230	−.0212	−.0142	−.0088	−.0069
	SE	.0020	.0026	.0016	.0009	.0014	.0009	.0006	.0006	.0008
$\hat{S}_{KM}$	MSE	.0322	.0516	.0365	.0166	.0296	.0181	.0120	.0170	.0138
	BIAS	−.0085	−.0091	−.0023	.0022	−.0010	−.0068	.0026	.0024	.0040
	SE	.0020	.0029	.0023	.0010	.0017	.0011	.0008	.0011	.0009

Mission time t : $S(t\,;\,1)=0.8,\ 0.5,\ 0.2$ and Covariate = 1

SAMPLE SIZE		10			20			30		
MISSION TIME		.4490	.7920	1.2070	.4490	.7920	1.2070	.4490	.7920	1.2070
$\hat{S}_T$	MSE	.0235	.0438	.0435	.0104	.0249	.0207	.0068	.0168	.0141
	BIAS	.0041	.0194	.0256	.0084	.0143	.0223	.0061	.0160	.0240
	SE	.0019	.0024	.0026	.0006	.0015	.0013	.0004	.0010	.0010
$\hat{S}_{MT}$	MSE	.0276	.0419	.0365	.0109	.0242	.0187	.0068	.0163	.0130
	BIAS	−.0377	−.0249	−.0028	−.0163	−.0079	.0079	−.0110	.0011	.0137
	SE	.0022	.0024	.0021	.0007	.0015	.0012	.0005	.0010	.0009
$\hat{S}_B$	MSE	.0250	.0438	.0367	.0108	.0250	.0187	.0070	.0168	.0122
	BIAS	−.0049	−.0057	−.0120	.0037	.0012	−.0025	.0029	.0074	−.0021
	SE	.0021	.0024	.0021	.0006	.0015	.0011	.0005	.0010	.0009
$\hat{S}_{MB}$	MSE	.0297	.0434	.0319	.0115	.0248	.0173	.0070	.0165	.0122
	BIAS	−.0475	−.0501	−.0382	−.0213	−.0210	−.0160	−.0143	−.0076	−.0021
	SE	.0023	.0025	.0017	.0008	.0016	.0010	.0005	.0010	.0008
$\hat{S}_{KM}$	MSE	.0371	.0676	.0526	.0157	.0345	.0265	.0117	.0212	.0198
	BIAS	.0023	−.0088	−.0156	.0037	−.0035	−.0075	.0006	.0076	.0043
	SE	.0026	.0037	.0032	.0009	.0021	.0014	.0007	.0013	.0012

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