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QUANTUM STATISTICAL MECHANICS FOR COUPLED BOSON-FERMION MODELS IN LATTICE SPACES*

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1. Introduction ; Notation, Models and Main Results

We study quantum statistical mechanics for a class of coupled boson-fermion models in lattice spaces.The class includes quantum polyacetylene model and Yukawa type interactions in lattice spaces.In order to investigate some interesting properties of polyacetylene such as soliton-like excitations of phonons and fractional charges,the polyacetylene model has been studied intensively by many authors [2,3,6,8-9]. Recently one of the present authors studied a semi-classical version of the model and established the existence and uniqueness of the Gibbs state [6].The aim of this paper is to construct the thermodynamic limit theory for a class of coupled boson-fermion model which inclues the quantum polyacetylene model.

In order to exhibit our motivation,let us introduce the quantum polyacetylene model [8]. The Hamiltonian for the model in a bounded region $\Lambda = \{-n, -n+1, \dots, n\} \subset \mathbf{Z}$ is given by

$$H = H_B + H_F \tag{1.1}$$

with

$$H_B = \sum_{i=-n}^{n} \frac{1}{2m}\left(-\frac{\partial^2}{\partial x_i{}^2}\right) + \sum_{i=-n}^{n-1} \frac{\omega^2}{2}(x_i - x_{i+1})^2 \tag{1.2}$$

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$$H_F = \sum_{i=-n}^{n+1} \sum_{s=\pm\frac{1}{2}} [C^*_{s,i} C_{s,i+1} + C^*_{s,i+1} C_{s,i}][t - g(x_i - x_{i+1})]$$

where m is the mass of CH of polyacetylene $(CH)_x$,ω is a positive constant, and t and g are real constants. The indices $s = \pm\frac{1}{2}$ stand for the spins of fermions in $C - C$ bonds, and one imposes

$$\{C^*_{s,i}, C_{s',j}\} = \delta_{ss'}\delta_{ij} \tag{1.3}$$

where $\{A, B\} = AB + BA$. In order to make the model well-defined, one has to add appropriate boundary terms in (1.2). For an instance, see Example 1 in this section. We generalize the above model as follows : Let $\Lambda \subset \mathbf{Z}^\nu$ be a bounded region in a ν -dimensional lattice space Z^ν . Denote the cardinality of Λ by $|\Lambda|$. For any $i \in \mathbf{Z}^\nu$, let $\mid i \mid$ be the Euclidean norm of i. The local Hilbert space is given by

$$\mathcal{H}_\Lambda = \mathcal{H}_{\Lambda,B} \otimes \mathcal{H}_{\Lambda,F} \tag{1.4}$$

where the Hilbert spaces for bosons and fermions are given by

$$\begin{aligned} \mathcal{H}_{\Lambda,B} &= L^2(\mathbf{R}^{|\Lambda|}) \\ \mathcal{H}_{\Lambda,F} &= \mathbf{C}^2 \otimes \mathbf{C}^{2^{|\Lambda|}} \end{aligned} \tag{1.5}$$

Let M and A be integral operators (matrices) acting on $l^2(\mathbf{Z}^\nu)$. We assume that M and A are positive and real matrices respectively. Next, let $D^{(1)}$ and $D^{(2)}$ be bounded (self-adjoint) integral operators (matrices) acting on $\mathbf{C}^2 \otimes l^2(\mathbf{Z}^\nu)$ with matrix elements $D^{(k)}_{sis'j}$, $k = 1, 2$, respectively, where s and s' denote spin indices. Let P_Λ be the projection operator from $l^2(\mathbf{Z}^\nu)$ onto $l^2(\Lambda)$. We write the restriction of any operator B to Λ byB_Λ, i.e.,

$$B_\Lambda = P_\Lambda B P_\Lambda \tag{1.6}$$

The local Hamiltonian acting on $\mathcal{H}_\Lambda$ is given by

$$H_\Lambda = H_{\Lambda,B} + H_{\Lambda,F} \tag{1.7}$$

with

$$H_{\Lambda,B} = \frac{1}{2}\Big[\sum_{i\in\Lambda}(-\frac{\partial^2}{\partial x_i^2}) + \sum_{i,j\in\Lambda} x_i((M^2)_\Lambda)_{ij}x_j\Big] \tag{1.8}$$

$$\begin{aligned} H_{\Lambda,B} = &\sum_{i,j\in\Lambda}\sum_{s,s'=\pm\frac{1}{2}} [C^*_{s,i}(D^{(1)}_\Lambda)_{sis'j}C_{s',j} + h.c.] \\ &+ \sum_{i,j\in\Lambda}\sum_{s,s'=\pm\frac{1}{2}} [C^*_{s,i}(D^{(2)}_\Lambda)_{sis'j}C_{s',j} + h.c.](A_\Lambda x)_i \end{aligned}$$

where $h.c.$ denotes the hermitian conjugate, and $(Ax)_i = \sum_j (A)_{ij}x_j$. The operators $C^*_{s,i}$ and $C_{s,i}, i \in \Lambda, s = \pm\frac{1}{2}$ satisfy the CAR in (1.3).

Before proceeding further, let us give some examples:

Example 1. Quantum polyacetylene model with Dirichlet boundary conditions. Let Δ be the discrete Laplacian on $l^2(\mathbf{Z}^\nu)$, and let ∂ be the operator defined by

$$(\partial)_{ij} = \begin{cases} 1, & \text{if } j = i+1 \\ -1, & \text{if } i = j \\ 0, & \text{otherwise .} \end{cases} \tag{1.9}$$

For the model we choose

$$M^2 = -\omega^2\Delta/2 \quad \textit{and} \quad A = \partial,$$

and $D^{(1)}$ and $D^{(2)}$ with

$$(D^{(1)}_\Lambda)_{sis'j} = \begin{cases} t\delta_{ss'}, & \text{if } j = i+1 \\ 0, & \text{otherwise.} \end{cases}$$

$$(D^{(2)}_\Lambda)_{sis'j} = \begin{cases} -g\delta_{ss'}, & \text{if } j = i+1 \\ 0, & \text{otherwise.} \end{cases}$$

Then the local Hamiltonian H_Λ becomes that of the quantum polyacetylene model with Dirichlet boundary conditions.

Example 2. The Yukawa Interaction in $\mathbf{Z}^3$ [4]. We choose

$$M^2 = -\Delta + m^2, \qquad A = 1$$

where m is a positive constant. Let $e_\mu, \mu = 1,2,3$ be the unit vectors parallel to the coordinate axes in $\mathbf{Z}^3$, and let σ_μ, $\mu = 1,2,3$ be the Pauli matrices and let

$$Q = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Define the operator D_μ and M_0 by

$$(D_\mu f)(j) = f(j + e_\mu) - f(j - e_\mu)$$
$$(M_0 f)(j) = (-1)^{|j|_1} m_0 f(j)$$

where $|j|_1$ is the l^1- norm of j and m_0 a positive constant. We write

$$\sigma \cdot D = \sum_{\mu=1}^{3} \sigma_\mu \otimes D_\mu$$

For the local Hamiltonian in (1.8) we choose

$$(D^{(1)})_{sis'j} = \frac{1}{2}\Big[i \sum_{\mu=1}^{3} (\sigma_\mu)_{ss'} (D_\mu)_{ij} + \delta_{ss'} (M_0)_{ij}\Big]$$
$$(D^{(2)})_{sisj'} = \frac{1}{2} Q_{ss'} \delta_{ij}$$

Then the Hamiltonian becomes $H_\Lambda = H_{\Lambda,B} + H_{\Lambda,F}$ with

(1.10)

$$H_{\Lambda,B} = \frac{1}{2}\Big[\sum_{i\in\Lambda}(-\frac{\partial^2}{\partial x_i^2}) + \sum_{i\in\Lambda} x_i((-\Delta_\Lambda + m^2)x)_i\Big]$$
$$H_{\Lambda,F} = \sum_{j\in\Lambda}\sum_s [C^*_{s,j}((i\sigma\cdot D + M_0)C)_{s,j}]$$
$$+ \sum_{j\in\Lambda}\Big(\sum_s 2s C^*_{s,j} C_{s,j}\Big) x_j$$

The above is the local Hamiltonian for the Yukawa interactions in $\mathbf{Z}^3$ [4].

We list our assumptions on the matrices in the local Hamiltonians defined in (1.7) and (1.8). For given operators M^2 and A in (1.8), we write

$$\overline{M}_\Lambda \equiv ((M^2)_\Lambda)^{1/2}$$

$$\overline{M}_\Lambda(\beta) \equiv (1 - e^{-\beta\overline{M}_\Lambda})^{-1} e^{-\beta\overline{M}_\Lambda} \tag{1.11}$$

$$\widehat{M}_{\Lambda,\sigma}(s) \equiv A_\Lambda \overline{M}_\lambda^{-1/2} (1 - e^{-\sigma\beta\overline{M}_\Lambda})^{-1} e^{-\sigma\beta(1-s)\overline{M}_\Lambda}$$

where $\beta > 1$ and $\sigma = \pm 1$. For a given matrix B on $l^2(\mathbf{Z}^3)$, we say that B is translational invariant if $(B)_{ij}$ depends only on $|i-j|$, and that B is of finite range if $(B)_{ij} = 0$ if $|i-j| \geq r$ for some $r > 0$. The infimum of such r is called the *effective radius* of B. For a bounded region Λ , let Λ_r be the r-interior of Λ defined by

$$\Lambda_r = \{j \in \Lambda : \operatorname{dist}(j, \partial\Lambda) \geq r\} \tag{1.12}$$

Throughout the rest of this paper we impose the following conditions :

ASSUMPTION 1.1. *The matrices $M^2, A, D^{(1)}$ and $D^{(2)}$ satisfy the following conditions:*

(a) *$M^2, A, D^{(1)}$ and $D^{(2)}$ are translational invariant and of finite range. The effective radii are equal or less than $r \geq 0$.*

(b) *There exists a constant c_1 independent of Λ such that the bound*

$$\|\overline{M}_\Lambda^{-1} A_\Lambda\|_2 + \|\overline{M}_\Lambda^{-1} A_\Lambda^*\|_2 \leq c_1$$

holds uniformly in Λ where $\|B\|_p$ is the l^p -norm of B, and $\overline{M}_\Lambda$ has been defined in (1.11).

(c) *There exists a constant c_2 independent of Λ such that the bound*

$$\sup_{\substack{s\in[0,1]\\ \sigma=\pm1}} \left\{ \|\widehat{M}_\sigma(s)\overline{M}_\Lambda^{1/2} A_\Lambda^*\|_1 + \|\widehat{M}_\sigma(s)\overline{M}_\Lambda^{1/2} A_\Lambda^*\|_2 \right\} \leq c_2$$

holds uniformly in Λ .

REMARK 1.1.

(a) *It follows from Assumption (1.1) that for $p = 1, 2$ the l^p -norms of $M^2, A, D^{(1)}$ and $D^{(2)}$ are finite. As a consequence there is a constant c_3 independent of Λ such that the bound*

$$\sum_{p=1}^{2}\Big(\|\overline{M}_\Lambda\|_p + \|D_\Lambda^{(1)}\|_p + \|D_\Lambda^{(2)}\|_p + \|A_\Lambda\|_p\Big) \le c_3 \tag{1.13}$$

holds uniformly in $\Lambda \in \mathbf{Z}^\nu$.

(b) *Since M^2 is positive,$\overline{M}_\Lambda$ is strictly positive for any bounded region Λ and so $\overline{M}_\Lambda^{-1}$ exists.*

(c) *In order to show the bound in Assumption 1.1(c) for a specific model,it suffices to show that $\|A_\Lambda \overline{M}_\Lambda^{-1}\|_1 \le const$ uniformly in Λ . See the appendix for details.*

(d) *In appendix we will prove that all conditions in Assumption 1.1 are satiafied for the quantum polyacetylene model and the Yukawa interaction defined in Example 1 and Example 2 respectively.*

Next, we introduce the algebras of local observables and the local Gibbs states. For notational simplifications we denote that

$$\begin{aligned}(Ax)_j &= \sum_i A_{ji}x_i \\ (A\frac{\partial}{\partial x})_j &= \sum_i A_{ji}\frac{\partial}{\partial x_i} \\ x(f) &= \sum_i x_i f(i) \\ \frac{\partial}{\partial x}(g) &= \sum_i \frac{\partial}{\partial x_i} g(i)\end{aligned} \tag{1.14}$$

Let $\mathcal{A}_{\Lambda,B}$ be the C^* -algebra generated by $\exp[i\{i\frac{\partial}{\partial x}(A^*g) + x(A^*f)\}]$, $f, g \in l^2(\Lambda_r)$, where Λ_r is the r-interior of Λ defined in (1.12). Since the effective radius of A is not greater than r by Assumption 1.1(a), it follows that $\mathcal{A}_{\Lambda,B} \subset \mathcal{L}(L^2(\mathbf{R}^{|\Lambda|}))$. Let $\mathcal{A}_{\Lambda,F}$ be the C^* -algebra generated by

$C_{s,i}, C^*_{s,i}$, $s = \pm 1/2$, $i \in \Lambda$. The local algebra of observables and the quasi-local algebra of observables are defined by

$$\mathcal{A}_\Lambda = \mathcal{A}_{\Lambda,B} \otimes \mathcal{A}_{\Lambda,F} \tag{1.15}$$
$$\mathcal{A} = \overline{(\bigcup_{\Lambda \subset \mathbf{Z}^\nu} \mathcal{A}_\Lambda)}$$

respectively. The finite volume Gibbs state and the partition function are defined by

$$\rho_\Lambda(B) = \mathbf{Z}_\Lambda^{-1} Tr_{\mathcal{H}_\Lambda}(Be^{-\beta H_\Lambda})) \tag{1.16}$$
$$\mathbf{Z}_\Lambda = Tr_{\mathcal{H}_\Lambda}(e^{-\beta H_\Lambda})$$

where $B \in \mathcal{A}_\Lambda, \beta > 0$.

In order to construct infinite volume limit equilibrium states on $\mathcal{A}$,we introduce Green's functions [1]. Let α^Λ be the time evolution automorphism given by

$$\alpha_t^\Lambda(B) = e^{itH_\Lambda} B e^{-itH_\Lambda}, \quad B \in \mathcal{A} \tag{1.17}$$

The finite volume Green's functions are defined by

$$G_\Lambda(B, C; t) = \rho_\Lambda(B\alpha_t^\Lambda(C)) \tag{1.18}$$

Although ρ_Λ is defined as a state on $\mathcal{A}_\Lambda$, it has an extension to a state on $\mathcal{A}$ by the Hahn-Banach theorem, which we denote again by ρ_Λ. The bounds

$$\|G_\Lambda(B, C; t)\| \leq \|B\|\|C\|$$

imply that there exists a subnet $\{\Lambda_\alpha\}$ such that the limits

$$G(B, C; t) = \lim_{\Lambda_\alpha \to \mathbf{Z}^\nu} G_{\Lambda_\alpha}(B, C; t) \tag{1.19}$$

exist for all $B, C \in \mathcal{A}$ and all $t \in \mathbf{R}$. This is a consequence of a Tychonoff theorem. Clearly the values

$$\rho(B) \equiv G(B, 1; 0) \tag{1.20}$$

determine a state ρ over the quasi-local algebra $\mathcal{A}$ [chapter 6 of [1]].

We now state our results :

THEOREM 1.1. *Assume that the matrices M,A,$D^{(1)}$ and $D^{(2)}$ satisfy all conditions in Assumption 1.1. Let $(\mathcal{H}_\rho, \pi_\rho, \Omega_\rho)$ be the cyclic representation of $\mathcal{A}$ with respect to the state ρ defined in (1.20). Then there exists a Hilbert space $\mathcal{H}$ containing $\mathcal{H}_\rho$, a strongly continuous representation U of $\mathbf{R}$ such that*

(i) $\mathcal{H} = \bigvee_{t \in \mathbf{R}} U(t)\mathcal{H}_\rho$

(ii) $G(B, C; t) = (\pi_\rho(B)^*\Omega_\rho, U(t)\pi_\rho(C)\Omega_\rho), \quad B, C \in \mathcal{A}, t \in \mathbf{R}$

(iii) *ρ is a modular state on $\mathcal{A}$,i.e., Ω_ρ is separating $\pi_\rho(\mathcal{A})''$*

Furthermore (i) and (ii) determine $(\mathcal{H}, U)$ uniquely up to unitary equivalence.

The contents of this paper are as follows : In section 2, we propose some estimates (Proposition 2.1) for local Gibbs states. Using the uniform estimates and the Green function method employed in chapter 6 of Ref.1, we prove theorem 1.1. Section 3 is devoted to prove uniform estimates (Proposition 2.1). In appendix we show that all conditions in Assumption 1.1 are satisfied for the quantum polyacetylene model and Yukawa interaction defined in Example 1 and Example 2 respectively.

2. Construction of Infinite Volume Limit Theories

In this section we employ the Green's function method (Chapter 6 of Ref.[1]) to construct infinite volume limit theories for the models described in Introduction. We first state a uniform estimate for the local Gibbs states and then use the uniform estimate together with the Green's function method to prove Theorem 1.1.

For any $h \in l^1(\Lambda_r) \cap l^2(\Lambda_r)$ let $\frac{\partial}{\partial x}(A^*h)$ and $x(A^*h)$ be defined as in (1.14):

$$(2.1) \qquad \frac{\partial}{\partial x}(A^*h) = \sum_j \frac{\partial}{\partial x_j}\Big(\sum_i A^*_{ji}h(i)\Big)$$

$$x(A^*h) = \sum_j x_j\Big(\sum_i A^*_{ji}h(i)\Big)$$

where Λ_r is the r-interior of Λ.

PROPOSITION 2.1. *Under Assumption 1.1, there exists a constant c independent of Λ such that the bounds*

$$\left|\rho_\Lambda\left(exp\{\frac{\partial}{\partial x}(A^*h_1)+x(A^*h_2)\}\right)\right| \leq exp\{c\sum_{i=1}^{2}(\|h_i\|_1+\|h_i\|_2)\}$$

hold uniformly in $\Lambda \in \mathbf{Z}^\nu$ for any $h_1, h_2 \in l^1(\Lambda_r)\cap l^2(\Lambda_r)$.

COROLLARY 2.2. *Under the condition same as in Proposition 2.1, there exists a constant c independent of Λ such that the bounds*

$$\left|\rho_\Lambda\left(\prod_{j=1}^{n}\{\frac{\partial}{\partial x}(A^*f_j)+x(A^*g_j)\}\right)\right|$$
$$\leq c^n(n!)^{1/2}\left(\prod_{j=1}^{n}\{\|f_j\|_1+\|f_j\|_2+\|g_j\|_1+\|g_j\|_2\}\right)$$

hold for any $f_j, g_j \in l^p(\Lambda_r), p=1,2$ and $j=1,2,\ldots,n$.

Proof. The proof follows from Proposition 2.1 and the method used in the proof of Lemma 2.2 of Ref.[6].

Let $G_\Lambda(B,C;t)$ be the local Green's function defined as in (1.18). Then we have the following bounds:

LEMMA 2.3. *For any $B, C \in \mathcal{A}$, there exists a constant $M_{B,C}$ independent of Λ such that the bound*

$$\left|\frac{d}{dt}G_\Lambda(B,C;t)\right| \leq M_{B,C}$$

holds uniformly in t.

Proof. Since $\cup\mathcal{A}_\Lambda$ is dense in $\mathcal{A}$, it suffices to show the lemma for C of the form $C=\exp\{i(i\frac{\partial}{\partial x}(A^*f)+x(A^*g))\}\widehat{C}(h_1)\ldots\widehat{C}(h_m)$ for real valued functions $f, g, h_i \in l^2(\Lambda), i = 1, \ldots, m$, where $\widehat{C}(h)$ represents either $C^*(h)$ or else $C(h)$. Notice that

(2.2)
$$\begin{aligned}
|\frac{d}{dt} G_\Lambda(B, C; t)| &= |\frac{d}{dt} \rho_\Lambda(B \alpha_t^\Lambda(C))| \\
&= |\rho_\Lambda(B \exp\{itH_\Lambda\}[H_\Lambda, C] \exp\{-itH_\Lambda\})| \\
&= |\rho_\Lambda(\alpha_{-t}^\Lambda(B)[H_\Lambda, C])| \\
&\leq \|B\| \rho_\Lambda([H_\Lambda, C]^*[H_\Lambda, C])^{1/2}
\end{aligned}$$

Here we have used the Schwarz inequality for ρ_Λ. Using CCR and CAR, we obtain

(2.3)
$$\begin{aligned}
[H_\Lambda, \frac{\partial}{\partial x}(A^* f)] &= x(M_\Lambda^2 A^* f) \\
&\quad + \sum_i \sum_s [C_{s,i}^* (D_\Lambda^{(2)} C)_{s,i} + h.c.](A_\Lambda A_\Lambda^* f)_i \\
[H_\Lambda, C^*(h)] &= \sum_{i,s} [C_{s,i}^* (D_\Lambda^{(1)} h)_{s,i} + h.c.] \\
&\quad + \sum_{i,s} [C_{s,i}^* (D_\Lambda^{(2)} h)_{s,i} + h.c.](A_\Lambda x)_i
\end{aligned}$$

and similar expressions for $[H_\Lambda, x(A^* g)]$ and $[H_\Lambda, C(h)]$. Since $\|\widehat{C}(h)\| = \|h\|_1$, it follows from Corollary 2.2 that

(2.4)
$$\begin{aligned}
\|[H_\Lambda, \widehat{C}(h)]\| &\leq const \sum_{i=1}^{2} \|D_\Lambda^{(i)} h\|_1 \\
&\leq const \|h\|_1
\end{aligned}$$

Denote that

$$W(f, g; t) = \exp\Big\{ it\big(i \frac{\partial}{\partial x}(A^* f) + x(A^* g)\big)\Big\}$$

then a direct calculation gives

(2.5)
$$[H_\Lambda, W(f,g;t)] \\ = i\int_0^t W(f,g;s)\Big[H_\Lambda, \big(i\frac{\partial}{\partial x}(A^*f) + x(A^*g)\big)\Big]W(f,g;1-s)ds$$

(2.6)
$$\big[x(M_\Lambda^2 A^* f), W(f,g;t)\big] = \Big(\sum_{i\in\Lambda}(M_\Lambda^2 A^* f)(i)\Big)W(f,g;t)$$

Next let us estimate $\rho_\Lambda\big([H_\Lambda, W(f,g;1)]^*[H_\Lambda, W(f,g;t)]\big)$. We first use (2.5) and (2.3), and then use (2.6) to move all $x_j's$ to the right.Then using the Schwarz inequality for ρ_Λ and Corollary 2.2, we obtain that

(2.7)
$$\begin{aligned}&\rho_\Lambda\big([H_\Lambda, W(f,g;1)]^*[H_\Lambda, W(f,g;1)]\big)\\ &\le \big(4\|D^{(2)}\|_1\|A_\Lambda A^* f\|_1 + 2\|M_\Lambda^2 A^* f\|_1\big)^2\\ &\le const\end{aligned}$$

uniformly in Λ. Note that $[H_\Lambda, A_1, A_2, \dots, A_m] = \sum_{j=1}^m A_1 \dots A_{j-1} \cdot [H_\Lambda, A_j] \dots A_m$. The proof of Lemma 2.3 now follows from (2.2),(2.4) and (2.7).

We finally produce the proof of our main result:

Proof of Theorem 1.1. Let $G(B,C;t)$ be the infinite volume limit Green's functions defined as in (1.19). We assert that $G: \mathcal{A}\times\mathcal{A}\times\mathbf{R} \to \mathbf{C}$ satisfies the following properties:

(1) $B, C \longrightarrow G(B,C;t)$ is bilinear for all $t \in \mathbf{R}$.
(2) $t \longrightarrow G(B,C;t)$ is continuous for all $A, B \in \mathcal{A}$.
(3) $G(B, DC; 0) = G(BD, C; 0)$ for all $B, C, D \in \mathcal{A}$.
(4) $G(1,1;0) = 1$.
(5) $\sum_{i,j=1}^n G(B_i^*, B_j; t_j - t_i) \ge 0$ for any finite sequence $\{B_i\}_{i=1}^n$ in $\mathcal{A}$ and $\{t_i\}_{i=1}^n$ in $\mathbf{R}$.
(6) Weak KMS conditions. For all $B, C \in \mathcal{A}$ and for all $\hat{f} \in \mathcal{D}$, $\int dt f(t)G(B,C;t) = \int dt f(t+i\beta)G(C,B;-t)$.

Since the local Green's functions $G_\Lambda(B,C;t)$ satisfy the above properties,it follows that $G(B,C;t)$ satisfy the properties (1) and (3) - (6). The property (2) is a consequence of Lemma 2.3 and Lemma 6.3.23 of Ref.[1]. Now our main result follows from Theorem 6.3.27 and Theorem 6.3.28 of Ref.[1].

3. Uniform Bounds: Proof of Proposition 2.1

We now turn to prove Proposition 2.1 and so complete the proof of Theorem 1.1. Throughout this section, we suppress Λ from the operators (matrices)$\overline{M}_\Lambda, A_\Lambda, D_\Lambda^{(1)}$ and $D_\Lambda^{(2)}$ if there is no confusion involved. Let us rewrite the local Hamiltonians in more convenient forms. Define the annihilation and creation operators by

$$\begin{aligned} a_j &= 2^{-1/2}\Big\{(\overline{M}^{1/2}x)_j + (\overline{M}^{-1/2}\frac{\partial}{\partial x})_j\Big\} \\ a_j^* &= 2^{-1/2}\Big\{(\overline{M}^{1/2}x)_j - (\overline{M}^{-1/2}\frac{\partial}{\partial x})_j\Big\} \end{aligned} \tag{3.1}$$

Then these operators satisfy the CCR:

$$[a_i, a_j] = 0, \quad [a_i, a_j^*] = \delta_{ij} \tag{3.2}$$

The local Hamiltonians can be written as
(3.3)

$$\begin{aligned} H_\Lambda &= H_{\Lambda,B} + H_{\Lambda,F} \\ H_{\Lambda,B} &= \sum_{j,k\in\Lambda} a_j^*\overline{M}_{jk}a_k + Tr(\overline{M}) \\ H_{\Lambda,F} &= \sum_{j\in\Lambda}\sum_s\{C_{s,j}^*(D^{(1)}C)_{s,j} + h.c.\} \\ &\quad + 2^{-1/2}\sum_{j\in\Lambda}\Big\{\sum_s\{C_{s,j}^*(D^{(2)}C)_{s,j} + h.c.\}\Big\}(A\overline{M}^{-1/2}(a^*+a))_j \end{aligned}$$

where $(DC)_{s,j} = \sum_{s',k} D_{sjs'k}C_{s'k}$ and $(Ba)_j = \sum_k B_{jk}a_k$,etc.

We next derive some commutator formulas which we will use later. It follows from (3.2) and (3.3) that

$$[a_i, H_{\Lambda,B}] = (\overline{M}a)_i \tag{3.4}$$
$$[a_i, H_{\Lambda,F}] = \sum_{j\in\Lambda} E_j(A\overline{M}^{-1/2})_{ji}$$

where

$$E_j \equiv 2^{-1/2}\sum_s \{C^*_{s,j}(D^{(2)}C)_{s,j} + h.c.\} \tag{3.5}$$

Thus, for any given real matix B, one obtains that

$$\big[(Ba)_i, H_{\Lambda,B}\big] = (BMa)_i \tag{3.6}$$
$$\big[(Ba)_i, H_{\Lambda,F}\big] = \sum_{j\in\Lambda} E_j(A\overline{M}^{-1/2}B^*)_{ji}$$
$$\equiv (E, A\overline{M}^{-1/2}B^*)_i \tag{3.7}$$

Next, we use (3.6) and (3.7) to get

$$\begin{aligned}
&\big[(Ba)_i, e^{-\beta H_\Lambda}\big]\\
&= -\beta\int_0^1 e^{-\beta(1-s)H_\Lambda}[(Ba)_i, H_\Lambda]e^{-\beta s H_\Lambda}ds\\
&= -\beta\int_0^1 e^{-\beta(1-s)H_\Lambda}(B\overline{M}a)_i e^{-\beta s H_\Lambda}ds\\
&\quad -\beta\int_0^1 e^{-\beta(1-s)H_\Lambda}(E, A\overline{M}^{-1/2}B^*)_i e^{-\beta s H_\Lambda}ds\\
&= -\beta e^{-\beta H_\Lambda}(Ba)_i\\
&\quad +\beta^2\int_0^1 ds\int_0^s ds_1 e^{-\beta(1-s_1)H_\Lambda}(B\overline{M}^2 a)_i e^{-\beta s_1 H_\Lambda}\\
&\quad +\beta^2\int_0^1 ds\int_0^s ds_1 e^{-\beta(1-s_1)H_\Lambda}(E, A\overline{M}^{-1/2}MB^*)_i e^{-\beta s_1 H_\Lambda}\\
&\quad -\beta\int_0^1 ds e^{-\beta(1-s)H_\Lambda}(E, A\overline{M}^{-1/2}B^*)_i e^{-\beta s H_\Lambda}
\end{aligned}$$

Notice that $\int_0^1 ds \int_0^s ds_1 g(s_1) = \int_0^1 ds_1(1-s_1)g(s_1)$. After iterating n-steps we arrive at the following relation:

$$
\begin{aligned}
&[(Ba)_i, e^{-\beta H_\Lambda}] \\
&= e^{-\beta H_\Lambda}\Big(\sum_{j=1}^{n-1}\frac{1}{j!}(-\beta)^j(B\overline{M}^j a)_j\Big) \\
&+ (-\beta)^n \int_0^1 ds_1 \int_0^{s_1} ds_2 \dots \int_0^{s_{n-1}} ds_n e^{-\beta(1-s_n)H_\Lambda}(B\overline{M}^n a)_i e^{-\beta s_n H_\Lambda} \\
&- \beta \sum_{j=1}^{n} \frac{1}{(j-1)!}\int_0^1 dse^{-\beta(1-s)H_\Lambda}(-\beta(1-s))^{j-1}(E, A\overline{M}^{-1/2}M^{j-1}B^*)_i \\
&\qquad \cdot e^{-\beta s H_\Lambda}
\end{aligned}
$$

By Assumption 1.1(a) (Remark 1.1(a)), it is easy to show that the second term in the right hand side of the above expression converges to zero in the norm topology. By taking n to infinity we obtain

(3.8)

$$
\begin{aligned}
(Ba)_i e^{-\beta H_\Lambda} =& e^{-\beta H_\Lambda}(Be^{-\beta\overline{M}}a)_i \\
&- \beta \int_0^1 dse^{-\beta(1-s)H_\Lambda}(E, A\overline{M}^{-1/2}e^{-\beta(1-s)\overline{M}}B^*)_i e^{-\beta s H_\Lambda}
\end{aligned}
$$

for any real matrix B. Taking adjoint form of (3.8) and changing the integration variables, we also obtain

$$
\begin{aligned}
&e^{-\beta H_\Lambda}(Ba^*)_i \\
=&(Be^{-\beta\overline{M}}a^*)_i e^{-\beta H_\Lambda} \\
&- \beta \int_0^1 dse^{-\beta(1-s)}(E, A\overline{M}^{-1/2}e^{-\beta s\overline{M}}B^*)_i e^{-\beta s H_\Lambda}
\end{aligned} \tag{3.9}
$$

Setting $B = B'e^{\beta\overline{M}}$ in (3.9) we get

(3.10)

$$
\begin{aligned}
&(Ba^*)_i e^{-\beta H_\Lambda} \\
=&e^{-\beta H_\Lambda}(Be^{\beta M}a^*)_i \\
&+ \beta \int_0^1 dse^{-\beta(1-s)H_\Lambda}(E, A\overline{M}^{-1/2}e^{\beta(1-s)\overline{M}}B^*)_i e^{-\beta s H_\Lambda}
\end{aligned}
$$

We call the identies in (3.8) - (3.10) the *pull through formulas*. For any $f, g \in l^1(\Lambda)$, let us write

$$a(f) = \sum_{i\in\Lambda} a_i \overline{f(i)}, \quad a^*(f) = \sum_{i\in\Lambda} a_i^* f(i) \tag{3.11}$$

Then it follows that

$$[a(f), a^*(g)] = (f, g)1, \quad [a(f), a(h)] = 0 \tag{3.12}$$

The following is the main result in this section:

PROPOSITION 3.1. *For any $f, g \in l^1(\Lambda)$ the bound*

$$\begin{aligned}\Big|\rho_\Lambda\big(\exp\{a^*(f) + a(g)\}\big)\Big| \le \exp\Big[\{\tfrac{1}{2}|(g, f)| + |(f, \widehat{M}(\beta)g)|\} \\ + \sup_{s\in[0,1]} \{2\|D^{(2)}\|_1 \|\widehat{M}_{-1}(s)g\|_1\} \\ + \sup_{s\in[0,1]} \{2\|D^{(2)}\|_1 \|\widehat{M}_1(s)\overline{f}\|_1\}\Big]\end{aligned}$$

holds, where $\widehat{M}_\sigma(s)$ has been defined in (1.11).

The proof of the above result is postponed to the end of this section. Using the above result we now prove Proposition 2.1.

Proof of Proposition 2.1. Take f and g as follows:

$$\begin{aligned}f &= 2^{-1/2}\{-i\overline{M}^{1/2} A^* h_1 + \overline{M}^{-1/2} A^* h_2\} \\ g &= 2^{-1/2}\{i\overline{M}^{1/2} h_1 + \overline{M}^{-1/2} A^* h_2\}\end{aligned}$$

Then from (3.1) and (3.11) it follows that

$$a^*(f) + a(f) = i\frac{\partial}{\partial x}(A^* h_1) + x(A^* h_2)$$

Under Assumption 1.1, Proposition 2.1 follows from Proposition 3.1, the above relation and the fact that $\|\widehat{M}(\beta)A^*\|_2 \le const\|\overline{M}^{-1}A^*\|_2$

The remainder of this section is devoted to the proof of Proposition 3.1. The following result is provably known in literatures. But to make this paper as much as self contained, we will produce the proof in Appendix.

LEMMA 3.2. *Let $A, B_1, B_2, \ldots, B_n$ be bounded operators in a separable Hilbert space and let $A > 0$. Assume that $A^{1/n}$ is of trace class. Then, for $\alpha_1 \ge 0, \alpha_2 \ge 0, \ldots, \alpha_n \ge 0$ with $\alpha_1 + \alpha_2 + \ldots + \alpha_n = 1$, the bound*

$$\left|Tr(B_1 A^{\alpha_1} B_2 A^{\alpha_2} \ldots B_n A^{\alpha_n})\right| \le \left(\prod_{i=1}^{n} \|B_i\|\right) Tr(A)$$

holds.

We now produce the proof of Prpposition 3.1.

Proof of Proposition 3.1. From the CCR in (3.2) it follows that for any $g, f_j, g_l \in l^2(\Lambda), j = 1, 2, \ldots, m, l = 1, 2, \ldots, n$,

(3.13)

$$\begin{aligned} a(g)\Big(\prod_{j=1}^{m} a^*(f_j)\Big)\Big(\prod_{l=1}^{n} a(g_l)\Big) &= \Big(\prod_{j=1}^{m} a^*(f_j)\Big)\Big(\prod_{l=1}^{n} a(g_l)\Big)a(g) \\ &\quad + \Big(\sum_{j=1}^{m}(g, f_j)\prod_{\substack{k=1\\k\neq j}}^{m} a^*(f_k)\Big)\Big(\prod_{l=1}^{n} a(g_l)\Big) \end{aligned}$$

For $\sigma = \pm 1$ and $s \in [0, 1]$, let $\widehat{M}_\sigma(s)$ be defined as in (1.11), and let

$$\begin{aligned} (F_\sigma(s))_i &\equiv \sum_{j\in\Lambda} E_j(\widehat{M}_\sigma(s))_{ji} \\ (F_\sigma'(s)B)_i &\equiv \sum_{j\in\Lambda} E_j(A\overline{M}^{-1/2} e^{-\sigma\beta(1-s)\overline{M}} B)_{ji} \end{aligned}$$

where E_j has been defined in (3.5) and $B \in \mathcal{L}(l^2(\Lambda))$.
For any $G_1(s), \dots, G_n(s) \in \mathcal{L}(\mathcal{H}_\Lambda), s \in [0,1]$, which commute with a_j^* and $a_j, j \in \Lambda$, we define

(3.14)
$$G^{(0)} = e^{-\beta H_\Lambda}$$
$$G^{(1)}(G_1) = \int_0^1 ds_1 e^{-\beta(1-s_1)H_\Lambda} G_1(s) e^{-\beta s_1 H_\Lambda}$$
$$\dots$$
$$G^{(n)}(G_1, G_2, \dots, G_n) = \int_0^1 ds_1 \int_0^{s_1} ds_2 \dots \int_0^{s_{n-1}} ds_n \Big\{ e^{-\beta(1-s_1)H_\Lambda} \cdot G_1(s_1) e^{-\beta(s_1-s_2)H_\Lambda} G_2(s_2) e^{-\beta(s_2-s_3)H_\Lambda} \dots G_n(s_n) e^{-\beta s_n H_\Lambda} \Big\}$$

Then it follows from the pull through formula (3.8) that

(3.15)
$$\begin{aligned} &(Ba)_i G^{(n)}(G_1, G_2, \dots, G_n) \\ &= G^{(n)}(G_1, G_2, \dots, G_n)(Be^{-\beta \overline{M}} a)_i \\ &- \sum_{l=1}^{n+1} G^{(n+1)}(G_1, \dots, G_{l-1}, (F_1^{'} B^*)_i, G_1, \dots, G_n) \end{aligned}$$

We next use (3.15) and (3.13) to obtain

$$\begin{aligned} &Tr\big(a^*(g)^m a(f)^n (Ba)_i G^{(p)}(G_1, G_2, \dots, G_p)\big) \\ &= -\sum_{l=1}^{p+1} Tr\Big(a^*(g)^m a(f)^n G^{(p+1)}(G_1, \dots, G_{l-1}, (F_1^{\prime} B^*)_i, G_l, \dots, G_p)\Big) \\ &+ m(Be^{-\beta\overline{M}} g)(i) Tr\big(a^*(g)^{n-1} a(f)^n G^{(p)}(G_1, \dots, G_p)\big) \\ &+ Tr\big(a^*(g)^m a(f)^n (Be^{-\beta\overline{M}} a)_i G^{(p)}(G_1, \dots, G_p)\big) \end{aligned}$$

We subtract the last term from both sides, choose $B = (1 - e^{-\beta\overline{M}})^{-1}$, product $\overline{f(i)}$ to the both sides and then sum over $i \in \Lambda$ to obtain

(3.16)
$$\begin{aligned}
&Tr\big(a^*(g)^m a(f)^{n+1} G^p(G_1, \dots, G_n)\big) \\
&= -\sum_{l=1}^{p+1} Tr\Big(a^*(g)^m a(f)^n G^{(p+1)}(G_1, \dots, G_{l-1}, F_1(\overline{f}), \dots, G_p)\Big) \\
&\quad + m(f, \widehat{M}(\beta)g) Tr\big(a^*(g)^{m-1} a^{(n)}(f) G^{(p)}(G_1, \dots, G_p)\big)
\end{aligned}$$

where $\widehat{M}(\beta)$ has been defined in (1.11). From the pull through formula (3.10) it also follows that

$$\begin{aligned}
&Tr\big(a^*(g)^m G^{(n)}(G_1, \dots, G_n)\big) \\
&= \sum_{l=1}^{n+1} Tr\Big(a^*(g)^{m-1} G^{(n+1)}(G_1, \dots, G_{l-1}, F_{-1}(g), \dots, G_n)\Big)
\end{aligned}$$

Notice that, since $\|C_{i,\sigma}\| = 1$,

$$\|F_\sigma(s) f\|_{\mathcal{H}_\Lambda} \le \sqrt{2}\, \|D^{(2)}\|_1 \, \|\widehat{M}_\sigma(s) f\|_1 \tag{3.18}$$

For notational simplifications, let us write

$$Q_\sigma(f) \equiv \sup_{s \in [0,1]} \{\sqrt{2}\, \|D^{(2)}\|_1 \, \|\widehat{M}_\sigma(s) f\|_1\} \tag{3.19}$$

Iterating the procedure in (3.17) m-times, and using Lemma 3.2, (3.12) and the fact that $\int_0^1 ds_1 \int_0^{s_1} ds_2 \dots \int_0^{s_{l-1}} ds_l = (l!)^{-1}$, we obtain that

$$\begin{aligned}
&\Big|Tr\Big(a^*(g)^m G^{(m)}(F_1(\overline{f}), F_1(\overline{f}), \dots, F_1(\overline{f}))\Big)\Big| \\
&\le (n!)^{-1} \big(Q_{-1}(g)\big)^m \big(Q_1(\overline{f})^n Tr(e^{-\beta H_\Lambda})\big)
\end{aligned} \tag{3.20}$$

Iterating (3.16) n-times and using (3.20) it is easy to show that for $m \geq n$

(3.21)
$$
\begin{aligned}
&\left|Tr(a^*(g)^m a(f)^n G^{(0)})\right| \\
&= \sum_{r=1}^{n} \binom{n}{r}(n-r)!\binom{m}{r} r! \left|(f, \widehat{M}(\beta)g)\right|^r \\
&\quad \cdot \left|Tr\left(a^*(g)^{n-r} G^{(n-r)}(F_1(\overline{f}), \dots, F_1(\overline{f}))\right)\right| \\
&\leq \sum_{r=1}^{n} n!m!((n-r)!(m-r)!r!)^{-1} \left|(f, \widehat{M}(\beta)g)\right|^r \\
&\quad \cdot (Q_{-1}(g))^{m-r} (Q_1(\overline{f}))^{n-r} Tr(e^{-\beta H_\Lambda})
\end{aligned}
$$

We now use (3.21) and the adjoint form of (3.21) for $m \leq n$ to conclude that

(3.22)
$$
\begin{aligned}
&\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m!n!)^{-1} \left|Tr\left(a^*(g)^m a(f)^n G^{(0)}\right)\right| \\
&\leq \exp\left\{\left|(f, \widehat{M}(\beta)g)\right| + Q_{-1}(g) + Q(\overline{f})\right\} Tr(e^{-\beta H_\Lambda})
\end{aligned}
$$

We note that

(3.23)
$$
e^{a^*(g)+a(f)} = e^{(f,g)/2} e^{a^*(g)} e^{a(f)}
$$

The proof of Proposition 3.1 now follows from (3.23) and (3.22). This completes the proof.

Appendix

A.1. Quantum Polyacetylene Model andYukawa Interactions.

As stated in Remark 1.1(d) we show that the quantum polyacetylene model and the Yukawa Interactions defined in Introduction satisfy all the conditions in Assumption 1.1.

a)**Yukawa Interactions.** Let $\Delta_\Lambda^{(D)}$ be the discrete Laplacian with Dirichlet boundary condition. It is well-known that [7]

$$\left((-\Delta_\Lambda^{(D)} + m^2)^{-1}\right)_{ij} \le const \exp\{-m|i-j|\}$$

uniformly in $\Lambda \subset \mathbf{Z}^\nu$. Thus for $p = 1, 2$

$$\|\overline{M}_\Lambda^{-1}\|_p \le \|M_\Lambda^{-2}\|_p \|\overline{M}_\Lambda\|_p \le const$$

uniformly in Λ. Since

$$\|\widehat{M}_\Lambda(\beta)\|_2 \le \|\overline{M}_\Lambda^{-1}\|_2$$
$$\|\widehat{M}_{\Lambda,\sigma}(s)\|_1 \le \|\overline{M}_\Lambda^{-1/2}\overline{M}_\Lambda^{-1}\|_1$$

it is almost trivial to show that all conditions in Assumption 1.1 are satisfied.

b)**Quantum Polyacetylene Model.** Let $\Lambda = \{-n, -n+1, \dots, n\} \subset \mathbf{Z}$. Remember that $(M^2)_\Lambda = -\Delta_\Lambda^{(D)}$ and $A = \partial$. We write

$$\begin{aligned} \mu(k)^2 &= (2 - 2\cos k) \\ &= 4(\sin(k/2))^2 \\ T_\Lambda^D &= \frac{\pi}{n} Z \cap [-\pi, \pi] \\ f_k(j) &= \frac{1}{n}\begin{cases} \sin(kj), & \text{if } nk/\pi \text{ is even} \\ \cos(kj), & \text{if } nk/\pi \text{ is odd} \end{cases} \\ g_s(k) &= (1 - e^{-\beta\mu(k)})^{-1} e^{-\beta(1-s)\mu(k)} \end{aligned} \tag{A.1}$$

Then it is easy to show that [7]

$$4\pi^{-2}k^2 \le \mu(k)^2 \le 4k^2, \quad k \in [-\pi, \pi] \tag{A.2}$$

and by the completeness of the eigenvectors of $\Delta_\Lambda^{(D)}$

$$\sum_{k \in T_\Lambda^{(D)}} f_k(i)\overline{f}_k(j) = \delta_{ij} \tag{A.3}$$

The matrix elements of $(-\Delta_\Lambda^{(D)})$ are given by [7]

$$(-\Delta_\Lambda^{(D)})_{jl} = \frac{2}{|\Lambda|} \sum_{k \in T_\Lambda^D} \mu(k)^2 e_{jl} \tag{A.4}$$

$$e_{jl} \equiv [e^{-ik(j-l)} - e^{-ik(j-(2n-l))}]$$

Let us again suppress Λ from the matrix notation. Thus for $M^2 = -\Delta_\Lambda^{(D)}$,

$$(M^{-2})_{jl} = \frac{1}{n} \sum_{k \in T_\Lambda^D}{}' \mu(k)^{-2} e_{jl} \tag{A.5}$$

and

$$\left(\overline{M}^{-q} e^{-s\beta\overline{M}}(1 - e^{-\beta\overline{M}})^{-1}\right)_{jl} = \frac{1}{n} \sum_{k \in T_\Lambda^D}{}' \mu(k)^{-q} g_s(k) e_{jl} \tag{A.6}$$

where $\sum'$ means that the term for $k = 0$ is omitted in the summation, and $g_s(k)$ has been defined in (A.1). Thus from (1.9) and (A.5) it follows that

$$(\partial M^{-2} \partial^*)_{jl} = \begin{cases} \delta_{jl}, & \text{if } l \ne n\ ,\ j \ne n \\ 0, & \text{otherwise} \end{cases}$$

Therefore the bounds in Assumption 1.1(c) follow from the above identity. We next consider the bounds in Assumption 1.1(d). From the definition of $\widehat{M}_\sigma(s)$ in (1.11) it follows that
(A.7)

$$(\widehat{M}_1(s)\overline{M}^{-1/2}\partial^*)_{jl} = \begin{cases} \frac{1}{n}\sum_{k\in T_\Lambda^D}{}' \mu(k)g_s(k)e_{jl}, & \text{if } l \neq n \text{ and } j \neq n \\ 0, & \text{otherwise} \end{cases}$$

We investigate the decay property of the above expression. Notice that

$$\mu(k)g_s(k) = \frac{\beta e^{-\beta(1-s)\mu(k)}}{\int_0^1 ds' e^{-\beta s'\mu(k)}} \tag{A.8}$$

We write
(A.9)

$$(\widehat{M}_1(s)\overline{M}^{-1/2}\partial^*)_{jl} = \begin{cases} F_s^{(1)}(j,l) - F_s^{(2)}(j,l), & \text{if } j \neq n \text{ and } l \neq n \\ 0, & \text{otherwise} \end{cases}$$

where $F_s^{(1)}(j,l)$ and $F_s^{(2)}(j,l)$ were obtained from (A.7) by replacing e_{jl} by $\exp\{-ik(j-l)\}$ and $\exp\{-k(j-(2n-l))\}$ respectively. Define

$$\begin{aligned} (D_\Lambda f)(k) &\equiv \frac{2n}{\pi}\left[f(k+\frac{\pi}{2n}) - f(k-\frac{\pi}{2n})\right] \\ \|x\|_D^2 &\equiv (\frac{2n}{\pi})^2\left[2 - 2\cos(\frac{\pi}{n}x)\right] \end{aligned} \tag{A.10}$$

Then by (A.2) there exist constants c_1 and c_2 independent of n such that

$$c_1|x|^2 \leq \|x\|_D^2 \leq c_2|x|^2, \quad |x| \leq n \tag{A.11}$$

We note that

$$\|j\|_D^2 \exp\{i(kj)\} = -D_\Lambda^2 \exp\{i(kj)\} \tag{A.12}$$

We use (A.11) and the discrete version of the integration by parts twice to conclude that

(A.13)

$$\begin{aligned}&\left|\|(j-l)\|_D^2\ F_s^{(1)}(j,l)\right|\\ \leq&\frac{1}{n}\sum_{\substack{k\in T_\Lambda^D\\ |k|\neq 0,1,n-1,n}}\left|D_\Lambda^2\left(\mu(k)g_s(k)\right)\right|\\ &+\frac{1}{n}\sup\{2\mu(k)f_s(k): s\in[0,1], k=0,1,n-1,n\}\end{aligned}$$

Since

$$\sup_{|k|\in(0,\pi)}\{|\mu'(k)|+|\mu''(k)|\}<\infty,$$

it follows from (A.8) and (A.10) that for any $k\in T_\Lambda^D$ with $|k|\neq 0,1,n-1,n$

(A.14)

$$\begin{aligned}&\left|(D_\Lambda^2\mu g_s(k)\right|\\ =&(\frac{2n}{\pi})^2\int_{-\pi/2n}^{\pi/2n}\int_{-\pi/2n}^{pi/2n}|(\frac{d}{dk}\mu g_s)|(k+x+x')dxdx'\\ <&\infty\end{aligned}$$

The bounds in (A.13) and (A.14) imply that

(A.15) $$\left|\|j-l\|_D^2F_s^{(1)}(j,l)\right|\leq const$$

By the method used above we conclude that

(A.16) $$\left|\|(j-(2n-l))\|_D^2F_s^{(2)}(j,l)\right|\leq const$$

since $|j-l|\leq|j-(2n-l)|$ for any $j,l\in\Lambda$, it follows from (A.9), (A.11), (A.15) and (A.16) that

$$\sup_{s\in[0,1]}\left|\left(\widehat{M}_1(s)\overline{M}^{-1/2}\partial^*\right)_{jl}\right|\leq c\left(1+|j-l|^2\right)^{-1}$$

uniformly in Λ, and so

$$\sup_{s\in[0,1]} \|\widehat{M}_1(s)\overline{M}^{-1/2}\partial^*\|_1 \leq c.$$

Other bounds in Assumption 1.1(d) can be proved by the method used above.

A.2. Proof of Lemma 3.2.

Let $f(z_1, z_2, \dots, z_n) \equiv Tr(B_1 A^{z_1} B_2 A^{z_2-z_1} \dots B_n A^{z_n-z_{n-1}})$ on $D = \{z \in \mathbf{C}^n : 0 \leq Rez_1 < Rez_2 < \dots < Rez_n \leq 1\}$. Then f is holomorphic in D and bounded on its clousure,$\overline{D}$. By the repeated application of three line theorem (Ref.[1]),the maximum value of f is attained at the one of the $n+1$ points

$$t_1 = t_2 = \dots = t_j = 0, \quad t_{j+1} = t_{j+2} = \dots = t_n = 1,$$

where $t_i = Rez_i$. But at such a point

$$\begin{aligned}
&\left|f(is_1, \dots, is_j, 1+is_{j+1}, \dots, 1+is_n)\right| \\
=&\left|Tr\left(B_1 A^{is_1} \dots B_j A^{i(s_j-s_{j-1})} B_{j+1} A^{1+i(s_{j+1}-s_j)} \dots B_n A^{i(s_n-s_{n-1})}\right)\right| \\
\leq&\left(\prod_{j=1}^{n} \|B_j\|\right) Tr(A).
\end{aligned}$$

This completes the proof.

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