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A NOTE ON REAL HYPERSURFACES OF TYPE B

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Introduction

A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a complex space form, which is denoted by $M^n(c)$. Let J be its complex structure. The complete and simply connected complex space form consists of a complex projective space CP^n, a complex Euclidean space C^n or a complex hyperbolic space CH^n according as $c > 0$, $c = 0$ or $c < 0$.

In study of real hypersurfaces of a complex projective space CP^n, Takagi [9] classified all homogeneous real hypersurfaces of CP^n. He showed also that real hypersurfaces of CP^n with 2 or 3 distinct constant principal curvatures are homogeous.

On the other hand, Cecil and Ryan [2] studied pseudo-Einstein real hypersurfaces of CP^n on which $\xi = -J\mathbf{C}$ is principal, where $\mathbf{C}$ is the unit normal vector field on M. They showed that if ξ is principal, then M lies on a tube over a Kaehler submanifold. The structure vector ξ is said to be *principal* if $A\xi = \alpha\xi$, where A is the shape operator in the direction of $\mathbf{C}$. By making use of this notion and the results of Takagi's classification, Kimura [4] proved the following.

THEOREM A. *Let M be a connected real hypersurface of CP^n. Then M has constant principal curvature and ξ is principal if and only if M is locally congruent to one of the following:*

(A_1) *a tube over a hyperplane CP^{n-1}.*
(A_2) *a tube over a tolly geodesic $CP^k (1 \le k \le n-2)$.*
(B) *a tube over a complex quadric Q_{n-1}.*
(C) *a tube over $CP^1 \times CP^{(n-1)/2}$ and $n(\ge 5)$ is odd.*
(D) *a tube over a complex Grassmann $G_{2,5}(C)$ and $n = 9$.*

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(E) *a tube over a Hermitian symmetric space* $SO(10)/U(5)$ *and* $n = 15$.

According to Takagi's classification [9], the principal curvatures and their multiplicities of the above homogeneous real hypersurfaces are given.

On the other hand, real hypersurfaces of a complex hyperbolic space CH^n have been investigated by Berndt [1], Montiel [6], Montiel and Romero [7]. In particular, by using the notion of the tube in Cecil and Ryan [2], Montiel [6] classified the real hypersurface of complex hyperbolic space with at most two distinct principal curvatures.

Recently, Berndt [1] classified all real hypersurfaces with constant principal curvature of CH^n under the condition such that ξ is principal. Namely he proved the following.

THEOREM B. *Let* M *be a connected real hypersurface of* $CH^n (n \geq 2)$. *Then* M *has constant principal curvatures and* ξ *is principal if and only if* M *is locally congruent to one of the following:*

(A_0) *a horosphere in* CH^n.
(A_1) *a tube over a complex hyperbolic hyperplane* CH^{n-1}.
(A_2) *a tube over a totally geodesic submanifold* $CH^k (1 \leq k \leq n-2)$.
(B) *a tube over a totally real hyperbolic space* RH^n.

In the present paper, one obtains the tensorian representation about real hypersurfaces of type B in $M^n(c)$ $(c \neq 0)$ which characterizes them.

The purpose of this paper is to prove the following:

THEOREM. *Let* M *be a real hypersurface of type* B *in* $M^n(c)$, $c \neq 0$. *Then* $\nabla_X A(Y) = a[2\eta(X)(A\Phi - \Phi A)Y + \eta(Y)(A\Phi - \Phi A)X + g((A\Phi - 3\Phi A)X, Y)\xi]$ *if and only if* M *is locally congruent to a real hypersurface of type* B, *where* $a \in \mathbf{R}$.

1. Preliminaries

Let M be a real hypersurface of $M^n(c)$ and let $\mathbf{C}$ be its unit normal vector field on a neighborhood of a point x in M. For arbitrary vector fields X and Y on M we define a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η on M by $g(\phi X,Y) = G(JX,Y)$ and $g(\xi,X) = \eta(X) = g(JX,C)$, that is, M has an almost contact metric structure induced from the almost complex structure J on $M^n(c)$, where g denotes the Riemannian metric of M induced from the Riemannian metric G of $M^n(c)$. Then we have

$$(1.1) \qquad \phi^2 X = -X + \eta(X),\ \phi\xi = 0,\ \eta(\phi X) = 0,\ \eta(\xi) = 1$$

for any tangent vector field X on M.

Futhermore, the covariant derivatives of the structure tensors are obtained:

$$(1.2) \qquad \nabla_X\phi(Y) = \eta(Y)AX - g(AX,Y)\xi, \quad \nabla_X\xi = \phi AX,$$

where ∇ is the induced Riemannian connection of g. Since the ambient space $M^n(c)$ is of constant holomorphic sectional curvature c, the equation of Gauss and Codazzi are respectively given as follows:

(1.3)

$$R(X,Y)Z = \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y - 2g(\phi X,Y)\phi Z\} + g(AY,Z)AX - g(AX,Z)AY,$$

$$(1.4) \quad \nabla_X A(Y) - \nabla_Y A(X) = \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X,Y)\xi\},$$

where R denotes the Riemannian curvature tensor of M. Using (1.3), the Ricci tensor S of M is acquired:

$$(1.5) \qquad S = \frac{c}{4}\{(2n+1)I - 3\eta\otimes\xi\} + hA - A^2,$$

where $h = \text{trace } A$ and I being the identity transformation.

Recently, in order to give an another characterization of homogeneous hypersurfaces of type A_1, A_2 and B in CP^n, Kimura and Maeda [5] introduced the notion of a η-parallel second fundamental form, which was defined by $g(\nabla_X A(Y), Z) = 0$ for any tangent vector fields X, Y and Z orthogonal to ξ. Now, we prepare without proof the followings:

THEOREM C ([5]). *Let M be a real hypersurface of CP^n. Then the second fundamental form is η–parallel and ξ is principal if and only if M is locally congruent to one of the homogeneous real hypersurfaces of type A_1, A_2 or B.*

THEOREM D ([8]). *Let M be a real hypersurface of CH^n. Then the second fundamental form is η–parallel and ξ is principal if and only if M is locally congruent to one of type A_0, A_1, A_2 or B.*

2. Tensorian representation of type B

Let M be a real hypersurface of type B in a complex space form $M^n(c), c \neq 0$ $(n \geq 3)$. Then the structure vector ξ is principal, that is,

$$A\xi = \alpha\xi \tag{2.1}$$

and

$$A\phi + \phi A = k\phi, \text{ where } k = -c/\alpha. \tag{2.2}$$

In this case, it is seen that α is given by $\sqrt{c}$ cot 2θ, $0 < \theta < \pi/2$ when $c > 0$ and $\sqrt{-c}$ tanh 2θ, $0 < \theta < \infty$ when $c < 0$. So the covariant derivative of (2.1) gives

$$\nabla_X A(\xi) = \alpha\phi AX - A\phi AX,$$

where we have used the second equation of (1.2). Thus it follows from (1.4) that

$$2A\phi AX = \alpha(\phi A + A\phi)X + \frac{c}{2}\phi X,$$

which coupled with (2.2) leads to

$$A\phi A = -\frac{c}{4}\phi. \tag{2.3}$$

Moreover, using (2.2) and (2.3), we obtain

$$(A^2 - kA - \frac{c}{4})\phi = 0,$$

which together with (1.1) and (2.1) implies

$$A^2 - kA - \frac{c}{4} = \beta\xi \otimes \eta, \tag{2.4}$$

where $\beta = \alpha^2 - k\alpha - \frac{c}{4} \in \mathbf{R}$.

Taking the covariant derivative of (2.4) along M and using the second formula of (1.2), we get

$$\nabla_X A(AY) + A\nabla_X A(Y) - k\nabla_X A(Y) - \beta\{\eta(Y)\phi AX + g(\phi AX, Y)\xi\} = 0 \tag{2.5}$$

for any tangent vector fields X and Y.

Replacing X by AX into the last equation and making use of (2.4), it is clear that

$$\begin{aligned} &\{\nabla_{AX} A(AY) + A\nabla_{AX} A(Y) - k\nabla_{AX} A(Y)\} \\ &\quad - \beta[\eta(Y)(k\phi A + \frac{c}{4}\phi)(X) + \{kg(\phi AX, Y) + \frac{c}{4}g(\phi X, Y)\}\xi] = 0 \end{aligned} \tag{2.6}$$

for any tangent vector fields X and Y. Since $g(\nabla_{AX} A(AY), Z) = g(\nabla_{AX} A(Z), AY)$, we make use of (1.4), (2.1), (2.3), (2.4) and (2.5), we then have

$$\begin{aligned} &g(\nabla_{AX} A(AY), Z) \\ &= -\frac{c}{4}g(\nabla_Z A(X), Y) - \frac{c}{4}\beta\{\eta(Y)g(\phi Z, X) + \eta(X)g(\phi Z, Y)\} \\ &\quad + \frac{c}{4}\{\alpha\eta(X)g(\phi Z, AY) + 2\alpha\eta(Y)g(AX, \phi Z) + \frac{c}{4}\eta(Z)g(\phi X, Y)\}, \end{aligned} \tag{2.7}$$

which connected with (2.6) and (1.4) gives rise to

$$\begin{aligned} &g(\nabla_{AX} A(AY) + A\nabla_{AX} A(Y) - k\nabla_{AX} A(Y), Z) \\ &= -\frac{c}{4}\{g(\nabla_Z A(X), Y) + g(\nabla_Y A(X), Z)\} - kg(\nabla_Y A(Z), AX) \\ &\quad + \eta(X)[\frac{c}{4}\alpha\{g(AY, \phi Z) + g(AZ, \phi Y)\} - \frac{c}{4}k\alpha g(\phi Y, Z)] \\ &\quad + \eta(Y)[-\frac{c}{4}\beta g(X,\phi Z) + \frac{c}{2}\alpha g(AX,\phi Z) + (\frac{c}{4})^2 g(\phi X,Z) + \frac{c}{4}kg(\phi AX,Z)] \\ &\quad + \eta(Z)[(\frac{c}{4})^2 g(\phi X,Y) - \frac{c}{4}\beta g(X,\phi Y) + \frac{c}{2}\alpha g(AX,\phi Y) + \frac{c}{2}kg(\phi AX,Y)]. \end{aligned}$$

And, from (2.6), the last equation yields

$$\begin{aligned}&\frac{c}{4}\{g(\nabla_Z A(Y),X)+g(\nabla_Y A(Z),X)\}+kg(\nabla_Y A(Z),AX)\\ =\ &\eta(X)[\frac{c}{4}\alpha\{g(AY,\phi Z)+g(AZ,\phi Y)\}+\frac{c^2}{4}g(\phi Y,Z)]\\ &+\eta(Y)[(\frac{c}{4})^2 g(\phi X,Z)+g(AX,\phi Z)(\beta k+\frac{c}{2}\alpha-\frac{c}{4}k)]\\ &+\eta(Z)[(\frac{c}{4})^2 g(\phi X,Y)+g(AX,\phi Y)(\beta k+\frac{c}{2}\alpha-\frac{c}{2}k)],\end{aligned}$$

which linked with the definition of β leads to

(2.8)
$$\begin{aligned}&\frac{c}{4}\{g(\nabla_Z A(Y),X)+g(\nabla_Y A(Z),X)\}+kg(\nabla_Y A(Z),AX)\\ =\ &\frac{c}{4}\eta(X)[\alpha\{g(AY,\phi Z)+g(AZ,\phi Y)\}+cg(\phi Y,Z)]\\ &+\frac{c}{4}\eta(Y)[\frac{c}{4}g(\phi X,Z)+2(k-\alpha)g(AX,\phi Z)]\\ &+\frac{c}{4}\eta(Z)[\frac{c}{4}g(\phi X,Y)+(k-2\alpha)g(AX,\phi Y)].\end{aligned}$$

On the other hand, from (2.5), we win

(2.9)
$$\begin{aligned}&g(\nabla_X A(Y),AZ)+g(\nabla_X A(Z),AY)\\ &=kg(\nabla_X A(Y),Z)-\beta\{\eta(Y)g(AX,\phi Z)+\eta(Z)g(AX,\phi Y)\}\end{aligned}$$

and taking the symmetric part of (2.8) with respect to X and Z, we get

$$\begin{aligned}&\{g(\nabla_X A(Y),Z)+2g(\nabla_Y A(Z),X)+g(\nabla_Z A(X),Y)\}\\ &+\frac{4k}{c}\{g(\nabla_Y A(Z),AX)+g(\nabla_Y A(X),AZ)\}\\ =\ &\eta(X)\{\alpha g(AY,\phi Z)+(k-\alpha)g(AZ,\phi Y)+\frac{3}{4}cg(\phi Y,Z)\}\\ &+2(k-\alpha)\eta(Y)\{g(AX,\phi Z)+g(\phi X,AZ)\}\\ &+\eta(Z)\{\alpha g(\phi X,AY)+(k-\alpha)g(AX,\phi Y)+\frac{3}{4}cg(X,\phi Y)\},\end{aligned}$$

which combined with (2.9) and (1.4) gives forth

$$\begin{aligned}\frac{4}{c}(k^2+c)g(\nabla_Y A(Z),X) = (k-\alpha)[\eta(X)\{3g(AY,\phi Z)+g(AZ,\phi Y)\}\\ +2\eta(Y)\{g(AX,\phi Z)+g(\phi X,AZ)\}\\ +\eta(Z)\{g(AX,\phi Y)+3g(\phi X,AY)\}].\end{aligned}$$

From the definition of k, we have $k-\alpha = -\frac{1}{\alpha}(\alpha^2+c)$ and $k^2+c = \frac{c}{\alpha^2}(\alpha^2+c)$, and hence $\alpha^2+c \neq 0$ because M is of type B. Thus we attain

(2.10)
$$\begin{aligned}\nabla_X A(Y) = -\frac{\alpha}{4}[2\eta(X)(A\phi-\phi A)Y+\eta(Y)(A\phi-3\phi A)X\\ +g((A\phi-3\phi A)X,Y)\xi]\end{aligned}$$

for any tangent vector fields X and Y.

REMARK 2.1. *Let M be a real hypersurface of type B in $M^n(c)$, $c\neq 0$ $(n\geq 3)$. Then equation (2.10) is equivalent to*

(2.11)
$$\begin{aligned}\nabla_X A(Y) = -\frac{c}{4}\{\eta(Y)\phi X+g(\phi X,Y)\xi\}\\ +\frac{\alpha}{2}\{\eta(X)(\phi A-A\phi)Y+\eta(Y)(\phi A-A\phi)X+g((\phi A-A\phi)X,Y)\xi\}.\end{aligned}$$

Indeed, making use of (2.2), we get $A\phi-3\phi A = 2(A\phi-\phi A)-k\phi$, which connected with (2.10) implies

$$\begin{aligned}\nabla_X A(Y) =&\frac{\alpha k}{4}[\eta(Y)\phi X+g(\phi X,Y)\xi]\\ &+\frac{\alpha}{2}[\eta(X)(\phi A-A\phi)Y+\eta(Y)(\phi A-A\phi)X\\ &+g((\phi A-A\phi)X,Y)\xi].\end{aligned}$$

Therefore, from the definition of k, the assertion is true.

REMARK 2.2. *It was proved that if real hypersurfaces of $M^n(c)$ satisfy (2.10), then ξ is principal [3].*

From the above remark, we get a tensorian representation of real hypersurfaces of type B in $M^n(c)$. Namely we have

THEOREM 2.1. *Let M be a real hypersurface of $M^n(c)$, $c \neq 0$. Then* $\nabla_X A(Y) = a[2\eta(X)(A\phi - \phi A)Y + \eta(Y)(A\phi - \phi A)X + g((A\phi - 3\phi A)X, Y)\xi]$ *if and only if M is locally congruent to a real hypersurface of type B, where $a \in \mathbf{R}$.*

Proof. It is enough to show the "only if" part is true.
Let M satisfies (2.10), then ξ is principal and M is η-parallel. So, by using the Theorem C and D, the proof is completed since the equation (2.12) is not realized for real hypersurfaces of type A (type A means A_1 or A_2 when $c > 0$ and A_0, A_1 or A_2 when $c < 0$).

REMARK 2.3. *The tensorian representation in above theorem is meaningful because we obtain the results which coincide with Takagi's and Berndt's table by the different proof from theirs. In fact, let M be a real hypersurface of type B in $M^n(c)$, then M satisfies the equation (2.11). Putting $Y = \xi$ in (2.11) and using (1.1), we get*

$$(2.12)\quad \nabla_X A(\xi) = -\frac{c}{4}\Phi X + \frac{\alpha}{2}\{\eta(X)\Phi A\xi + (\Phi A - A\Phi)X + g(X, \Phi A\xi)\xi\},$$

which joined with the second formula of (1.2) implies

$$(2.13)\qquad \nabla_\xi A(\xi) = \alpha \nabla_\xi \xi.$$

For any point x on M we can choose an orthonormal basis $\{E_1, \ldots, E_{2n-1}\}$ for the tangent space $T_x M$ such that $\nabla_{E_i} E_j = 0$ $(i, j, \ldots, 2n-1)$. Then differentiating (2.11) covariantly along M and making use of (1.2), we have

(2.14)

$$\begin{aligned}
\nabla_W \nabla_X A(Y) = &-\frac{c}{4}\{g(\phi AW, Y)\phi X + g(\phi X, Y)\phi AW \\
&+ \eta(X)\eta(Y)AW + \eta(X)g(AW, Y)\xi - 2\eta(Y)g(AW, X)\xi\} \\
&+ \frac{\alpha}{2}[g(\phi AW, X)(\phi A - A\phi)Y + g(\phi AW, Y)(\phi A - A\phi)X \\
&+ g((\phi A - A\phi)X, Y)\phi AW + \eta(X)\{\eta(AY)AW + g(AW, Y)A\xi \\
&- 2g(A^2 W, Y)\xi + \phi\nabla_W A(Y) - \nabla_W A(\phi Y)\} + \eta(Y)\{\eta(AX)AW \\
&+ g(AW, X)A\xi - 2\eta(X)A^2 W + \phi\nabla_W A(X) - \nabla_W A(\phi X)\} \\
&+ \{\eta(AX)g(AW, Y) + \eta(AY)g(AW, X) - 2\eta(Y)g(A^2 W, X) \\
&- g(\nabla_W A(X), \phi Y) - g(\nabla_W A(\phi X), Y)\}\xi],
\end{aligned}$$

which combined with the Ricci formula for the shape operator A gives forth

(2.15)

$$\begin{aligned}
&R(W,X)AY - A(R(W,X)Y)\\
&= -\frac{c}{4}\{g(\phi AW,Y)\phi X - g(\phi AX,Y)\phi W + g(\phi X,Y)\phi AW\\
&\quad - g(\phi W,Y)\phi AX + \eta(X)\eta(Y)AW - \eta(W)\eta(Y)AX\\
&\qquad\qquad + \eta(X)g(AW,Y)\xi\\
&\quad - \eta(W)g(AX,Y)\xi\} + \frac{\alpha}{2}[g((\phi A + A\phi)W,X)(\phi A - A\phi)Y\\
&\quad + g(\phi AW,Y)(\phi A - A\phi)X - g(\phi AX,Y)(\phi A - A\phi)W\\
&\quad + g((\phi A - A\phi)X,Y)\phi AW - g((\phi A - A\phi)W,Y)\phi AX\\
&\quad + \eta(X)\{\eta(AY)AW + g(AW,Y)A\xi - 2g(A^2W,Y)\xi + \phi\nabla_W A(Y)\\
&\qquad\qquad - \nabla_W A(\phi Y)\}\\
&\quad - \eta(W)\{\eta(AY)AX + g(AX,Y)A\xi - 2g(A^2X,Y)\xi\\
&\qquad\qquad + \phi\nabla_X A(Y) - \nabla_X A(\phi Y)\}\\
&\quad + \eta(Y)\{\eta(AX)AW - \eta(AW)AX - 2\eta(X)A^2W\\
&\qquad\qquad + 2\eta(W)A^2X + \phi\nabla_W A(X)\\
&\quad - \phi\nabla_X A(W) - \nabla_W A(\phi X) + \nabla_X A(\phi W)\} + \{\eta(AX)g(AW,Y)\\
&\qquad\qquad - \eta(AW)g(AX,Y)\\
&\quad - g(\nabla_W A(X),\phi Y) + g(\nabla_X A(W),\phi Y) - g(\nabla_W A(\phi X),Y)\\
&\qquad\qquad + g(\nabla_X A(\phi W),Y)\}\xi].
\end{aligned}$$

If we put $W = E_i$ in (2.15), taking the inner product of this result and E_i and summing up with respect to i ($i = 1, \ldots, 2n-1$), we find

(2.16)

$$\begin{aligned}
&hA^2X + \{\frac{c}{2}(n+1) - h_2 + \alpha^2\}AX + \frac{c}{4}(\alpha - h)X\\
&= -c\Phi A\Phi X + \frac{1}{2}(c - \alpha h)\eta(X)A\xi + (\frac{c}{2} - \frac{\alpha}{2}h + \alpha^2)\eta(AX)\xi\\
&\quad - [\frac{c}{4}h + \frac{\alpha}{2}\{\frac{(2n-1)}{2}c + 2\alpha^2 - 2h_2\}]\eta(X)\xi
\end{aligned}$$

$$+\frac{\alpha}{2}\{A\Phi A\Phi X-\Phi A\Phi AX-2\Phi A^2\Phi X+2\eta(AX)A\xi$$
$$-\eta(X)A^2\xi+\eta(A^2X)\xi\},$$

where we put $h_2 = \text{trace } A^2$ and used (1.1), (1.4), (1.5) and (2.11). Now, if we use (2.13), then $\phi A\phi A = A\phi A\phi$, which connected with (2.16) gives

$$hA^2+\{\frac{c}{2}(n+1)-h_2+\alpha^2\}A+\frac{c}{4}(\alpha-h)I$$
$$-\{c(\alpha-\frac{h}{4})+\frac{(2n-1)}{2}c+2\alpha^2+2\alpha h-2h_2\}\xi\otimes\xi=-\alpha\phi A^2\phi-c\phi A\phi,$$

where we have used (2.1).
Since M is of type B, the shape operator A of M has three distinct principal curvatures (say α, λ, μ) such that $AX = \lambda X$ and $A\phi X = \mu\phi X$ for any tangent vector field X orthogonal to ξ. The last equation yields

$$h\lambda^2+\{\frac{c}{2}(n+1)-h_2+\alpha^2\}\lambda+\frac{c}{4}(\alpha-h)=-\alpha\mu^2-c\mu$$

and similarly we get

$$h\mu^2+\{\frac{c}{2}(n+1)-h_2+\alpha^2\}\mu+\frac{c}{4}(\alpha-h)=-\alpha\lambda^2-c\lambda.$$

Combining the last two equations, we find $(h-\alpha)(\lambda+\mu)+\frac{c}{2}(n-1)+\alpha^2-h_2=0$, where we have used the fact $\lambda-\mu\neq 0$. Since we have $h-\alpha=(n-1)(\lambda+\mu)$ and $h_2=\alpha^2+(n-1)(\lambda^2+\mu^2)$ because λ and μ have multiplicity $n-1$ respectively, it follows that

$$\lambda\mu=-\frac{c}{4}. \tag{2.17}$$

On the other hand, it is, using (2.1), seen that $(\lambda-\frac{\alpha}{2})\mu=\frac{\alpha\lambda}{2}+\frac{c}{4}$, which joined with (2.17) yields $\alpha\lambda^2+c\lambda-\frac{c}{4}\alpha=0$ and hence we see that

$$\alpha=\sqrt{c}\cot 2\theta,\ \lambda=\frac{\sqrt{c}}{2}\cot(\theta-\frac{\pi}{4})\text{ or }-\frac{\sqrt{c}}{2}\tan(\theta-\frac{\pi}{4})$$

when $c>0$ and

$$\alpha=\sqrt{-c}\tan h\,2\theta,\ \lambda=\frac{\sqrt{-c}}{2}\cot h\theta\text{ or }\frac{\sqrt{-c}}{2}\tan h\theta$$

when $c < 0$.
This results coincide with Takagi's table Berndt's one according to $c > 0$ and $c < 0$, respectively.

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