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A Test for Distributivity Using Tables

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Let S be a finite set. We define two binary operations $(+)$, $(\cdot)$ on S. Then it is very tedious to know that the distributive laws hold on S. So we want to have a method for testing distributivity. Assume there are two binary operations $(+)$, $(\cdot)$ on $S = \{p, q, r, s\}$.

+	p	q	r	s
p	p	q	r	s
q	q	p	s	r
r	r	r	p	q
s	s	r	q	p

	p	q	r	s
p	p	p	p	p
q	p	q	r	s
r	p	r	s	q
s	p	s	q	r

We wonder, for example, whether r is left destributive on $(S, +, \cdot)$. First, we construct a table, called $r(x+y)$-table. From $(+)$-table we take out each row step by step, and reset those in row index of the following table and we fill the column index with r.

(1)

1st row in (+)				2nd row in (+)				3rd row in (+)				4th row in (+)			
p	q	r	s	q	p	s	r	r	r	p	p	s	r	q	p

In order to fill the blank of the table (1) we use the operation $(\cdot)$ on S. Then we get the following.

(2)

$\cdot$	p	q	r	s	q	p	s	r	r	r	p	q	s	r	q	p
r	p	r	s	q	r	p	q	s	s	s	p	q	q	s	r	p

In table (2) we divide the results into 4 parts, i.e., $prsq, rpqs, sspr, qsrp$. We rearrange this of the following form:

(3)

p	r	s	q
r	p	q	s
s	s	p	r
q	s	r	p

Table (3) is the desirde one, i.e., $r(x+y)$-table. Next, we construct so called $(rx+ry)$-table. From $(\cdot)$-table we take out a row corresponding to r, i.e., $prsq$. And we regard the chosen row as a column index and a row index of the following new table.

(4)

	p	r	s	q
p				
r				
s				
q				

To fill the blank of (4) we use the operation $(+)$ on S, and then we can get the following table.

(5)

$+$	p	r	s	q
p	p	r	s	q
r	r	p	q	r
s	s	q	p	r
q	q	s	r	p

Table (5) is the desired $(rx+ry)$-table. Comparing table (3) with table (5) we can see that two tables are not identical and hence r is not left distributive on $(S,+,\cdot)$. With same method we can construct two tables about q.

p	q	r	s
q	p	s	r
r	r	p	q
s	r	q	p

$q(x+y)$-table

p	q	r	s
q	p	s	r
r	r	p	q
s	r	q	p

$(qx+qy)$-table

From this we know that q is left distributive on $(S,+,\cdot)$. We can test left distributivity for another elements of S. Moreover, we can easily apply this method for testing the right distributivity.