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A Diagnostic Model for Dye Plume Meandering in Oceanic Waters
海洋에서의 染料 플럼의 蛇行에 대한 모델

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Abstract This study is concerned with the meandering of plume axis in oceanic waters. The process is understood that it is a consequence of the differential contribution by the multiple harmonics of local velocity field to variances of center of mass of crossplume as a function of distance from the source point. A diagnostic model is proposed which is aimed to delineate the eddying motions and furthermore the amplified meandering of plumeaxis. From the data base of dye plumes, wave lengths of meandering eddies are estimated to range between 5.5 to 60.3 (m) in coastal surface waters. A numerical simulation is conducted to predict the concentration field of meandering plume.

要 旨: 본 研究는 海洋에서의 染料 플럼축의 蛇行에 관한 것이다. 濃度 擴散場에서의 이러한 過程은 橫斷 染料 濃度場의 무게 중심의 分散에 대하여 速度場의 여러 週期 成分이 原點으로부터의 거리의 函數로서 差別的으로 寄與하기 때문에 일어나는 것으로 이해된다. 提示된 診斷 모델은 渦動場을 보여줄 뿐만 아니라 플럼축이 增幅되어 蛇行하는 現象까지도 설명할 수 있다. 染料 플럼의 航空寫眞을 이용한 자료로부터 蛇行 渦動의 波長을 推定한 바 沿岸 表層水에서 5.5에서 60.3 미터까지 分布하였다. 또한 数値 시뮬레이션을 통하여 蛇行 플럼 濃度場을 豫測하였다.

1. INTRODUCTION

Plumes resulted from continuous release of any material such as dye or smoke into water bodies or air often exhibit large scale structures, one of which is the meandering of plumeaxis. Multiple dynamical processes are concurrently occurring in the surface layer of the ocean, several of which are interacting with local diffusion process to produce surface features in concentration distribution. Those features possess very peculiar and salient structures and are distinctively related to the spectrum of local eddies. It is of interests to understand this behavior not only from scientific point of view but from the practical stand point particularly in predicting the concentration field of health-related hazardous materials. Without taking considerations of such a behavior, prediction will inevitably undergo through errors leading to unrealistic concentration field.

Literatures (Kenney, 1968; Csanady, 1971; Murthy, 1972, 1976) have reported pronounced surface features in that the trajectory of the center of mass of individual patch emitted from the source is complicated, not following the straight line as is expected in a slender plume model. These features show a great resemblance to the aspect of fluctuating smoke plumes in the atmosphere, although two systems are occurring in two different planes, horizontal and vertical respectively. This phenomenon is traditionally called meandering of plume. As has been qualitatively described, the center of mass of plume experiences lateral bodily displacements as it travels downstream from a source point. Local eddies whose scales are larger than the width of plume are acting to displace the plume. The extent of the displacement seems to be correlated to the increase in size of the

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acting eddies with the travelling distance of plume, and thus the amplification of the displacement takes place.

So far, theoretical approach to quantify the meandering feature is scarce (Kenney and Jones, 1972), while numerous dye studies have been carried out, mostly concerned with describing visual phenomenological features of plume meandering. To the author's knowledge, Gifford's fluctuating plume model(1959) seems to be the only theoretical attempt.

Thus, this study will examine the meandering of plumeaxis from the view point of the statistical characteristics of local velocity field. Furthermore, a diagnostic model will be developed to elucidate the underlying mechanism of meandering phenomena and will be interpreted in an analogy to a harmonic oscillator problem. Using the data base of dye plumes, the wave lengths of meandering eddies will be analyzed. To demonstrate the usefulness of the proposed diagnostic model, numerical simulations are conducted and provide the concentration field of meandering plume.

2. STOCHASTIC CHARACTERISTICS OF PLUME MEANDERING

In idealizing plume model, conventional assumptions are usually made in that constant unidirectional velocity is convecting the material which is balanced by the lateral diffusion by local eddies to result a slender plume. However, in reality such a condition is rarely met. In fact, the velocity field in ocean water would not show constant at all, yet fluctuate in time and space, i.e., composed of multiple harmonics in frequencies and wave numbers. Non-constant low frequency motions will thus have other implications beyond the lateral diffusion of plume width. In this study, attempt is made to relate the low-frequency components to one of most distinctive surface structures which reveal several deterministic natures due to its slowly varying properties.

To understand the statistical characteristics of meandering of plume, a line of thought analogous to the lateral diffusion process proposed by Taylor (1921) is followed.

Consider an ensemble of realizations of individual patches emitted from a source, each moving with a mean velocity, U in the downstream direction and fluctuating with a random velocity, v ' in the lateral direction. According to Taylor, the lateral variance of the center of mass of the patches is given by

$$\overline{y'^2}=2\overline{v'^2}\int_0^t dt' \int_0^{t'} R_L(\tau)\ d\tau \tag{1}$$

where $R_L(\tau)$ is Lagrangian autocorrelation function. The increase of the variance, $\overline{y'^2}$ has two regimes in which

$$\overline{y'^2}=\overline{v'^2}t^2 \quad \text{for t very small}$$
$$\overline{y'^2}=2\overline{v'^2}Tt \quad \text{for t very large} \tag{2}$$

where T is a Lagrangian intergral time scale.

As a result, the envelope of the trajectory of the center of mass will expand as the patch travels downstream in a Lagrangian sense. This implies that the amplitude of meandering of an individual plume will increase as time, t or travel distance, x (= Ut) increases.

Next consider the effect on the variance of the various frequencies in the random motion of center of mass. Let

$$R_L(\tau)=\frac{1}{\overline{v'^2}}\int_{-\infty}^{\infty}E_L(f)\ \cos(2\pi f\tau)df \tag{3}$$

where E_L (f) is a Lagrangian energy frequency spectrum function. The use of (3) in (1) yields

$$\overline{y'^2}=2\int_0^{\infty}df\ E_L(f)\ \frac{1-\cos(2\pi ft)}{4\pi^2f^2} \tag{4}$$

For small values of t, (4) can be approximated as

$$\overline{y'^2}(t)\cong t^2\int_0^{\infty}E_L(f)\ df \tag{5}$$

This illustrates that for small diffusion time (in this case, small travel length, x), the variance of the center of mass is determined by the contribution from all frequency components. On the other hand, for very large values of t, the variance in given by

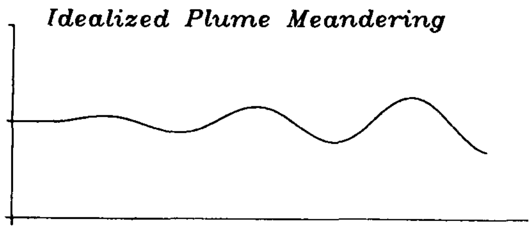

Fig. 1. Schematic picture of idealized meandering plume.

$$\overline{y'^2}=t\int_0^\infty 2t\frac{1-\cos(2\pi ft)}{(2\pi t)^2 f^2}E_L(f)\,df$$

$$=\frac{t}{\pi}\int_0^\infty d\nu\frac{1-\cos(\nu)}{\nu^2}E_L\left(\frac{\nu}{2\pi t}\right) \qquad (6)$$

where $\nu=2\pi ft$

The values of $(1-\cos(\nu)/\nu^2)$ for very large ν will be very small. Hence, we can expect that an appreciable contribution to the variance comes only from the values of the integrand corresponding to moderate or small values of ν.

Consequentially one may put

$$E(\nu/2t)=E_L(0)$$

and (6) is approximated by

$$\overline{y}^2=\frac{t}{\pi}E_L(0)\int_0^\infty d\nu\frac{1-\cos\nu}{\nu^2}=\frac{t\,E_L(0)}{2\pi} \qquad (7)$$

The arguments above-mentioned enable us to construct a simple scenario in the meandering behavior. The variance of meandering behaves differently according to the differential contribution of frequency components of motions. In the vicinity of source when travel time is small, the center of mass moves with the velocity dominated by the local mean velocity. High-frequency components of velocity only contribute its energy to the growth of plume width. Until this time, any meandering motion is not appreciable. As time lapses, the low frequency fluctuations of velocity field begin to dominate in the contribution to the variance of meandering as predicted by Equation (7), and the amplitude of meandering becomes comparable to that of local mean velocity. The low frequency components come from local big eddies.

Idealization of this meandering phenomenon is schematically sketched in Fig. 1. In the vicinity of origin (source), the plume does not show any meandering motion whereas somewhere in the downstream, it starts to fluctuate laterally and the amplitude of meandering is increasing.

3. A DIAGNOSTIC MODEL FOR PLUME MEANDERING

To build a diagnostic model of meandering, start with evolution equation for the trajectory of the center of mass of crossplume, R such as

$$\frac{\partial R(\underline{a},t)}{\partial t}=\underline{V}(\underline{x},t) \qquad (8)$$

where $\underline{a}$ and $\underline{x}$ represent the initial position vector and coordinate and the underlines are used to denote the vectorial quantities. By component notation, 8) reads

$$\frac{dx}{dt}=u=P(t,x,y)$$

$$\frac{dy}{dt}=v=Q(t,x,y) \qquad (9)$$

Equation (9) of two coupled first order differential equation represent a dynamical system of the second order. when the system is autonomous (Percival and Richards, 1982; Andronov *et al,* 1973), the time dependency of velocity field does not appear explicitly in the system equation such that

$$\frac{dy}{dx}=\frac{Q(x,y)}{P(x,y)} \qquad (10)$$

Introducing a stream function, Ψ for the velocity field, the system equation (9) can be expressed as

$$\frac{dx}{dt}=u=\frac{\partial\Psi}{\partial y}$$

$$\frac{dy}{dt}=v=-\frac{\partial\Psi}{\partial x} \qquad (11)$$

(11) constitutes the canonical equation of Hamiltonian in which the stream function, Ψ corresponds to the Hamiltonian. For an autonomous system, is a

Meandering Streamline

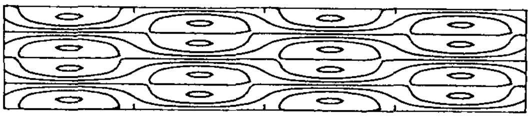

Fig. 2. Streamlines of array of eddies expressed in Equation (15). One tick mark in x-axis represents the wave length of meandering eddies.

Amplified Meandering Streamline

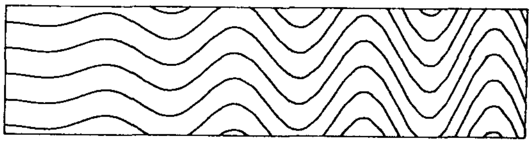

Fig. 3. Streamlines of amplified meandering eddies defined in Equation (17).

constant of the motion, and a particle is supposed to move along the curves of same values of ψ.

To determine a stream function uniquely with a given feature of meandering motion, it is needed to specify the function forms of P and Q. First of all, the equation of continuity is imposed on the functions, P and Q such as

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}=0 \tag{12}$$

The task of this problem is now to find function forms of P (x, y) and Q (x, y) under the constraint of the continuity condition.

One solution for P (x, y) and Q (x, y) is obtained in an analogy to Stommel (1949). This model represents an array of organized eddies convected by a constant mean velocity, U_o in the x direction;

$$P(x, y)=U_o-ak_2\sin(k_1x)\cos(k_2y)$$
$$Q(x, y)=\quad ak_1\cos(k_1x)\sin(k_2y) \tag{13}$$

The stream function for this system is given by

$$\Psi(x, y)=U_oy-a\ \sin(k_1x)\sin(k_2y) \tag{14}$$

Fig. 2 illustrates the streamlines of the eddying motions and the trajectory of center of mass defined by Equation (14). This model succeeds to reproduce a meandering plume, yet, it is not enough to delineate one of important features, the amplification of meandering described previously.

To make allowances of the more realistic behavior of meandering, a solution is sought for the functions, P (x, y) and Q (x, y) assuming that the flow is constant and uniform in the x direction. Consequentially, P becomes constant and Q must be a function of x only to satisfy the continuity condition in (12). A model is proposed such that

$$\frac{Q(x, y)}{P(x, y)}=A\sin(k\ x)+A\ k\ x\ \cos(k\ x)=\frac{dy}{dx} \tag{15}$$

(15) can easily be integrated to find

$$y=A\,x\sin(kx) \tag{16}$$

and the relevant stream function is

$$\Psi(x, y)=U_oy+Ax\ \sin(kx) \tag{17}$$

(16) represents a sine function whose amplitude increases linearly with x, thus the resulting stream lines show the amplification of meandering seen in Figure 3.

To cast the problem in a different way, differentiate Equation (10) once more with respect to x and by using (16), it results

$$\frac{d^2y}{dx^2}+k^2y=2A\ k\ \cos(kx) \tag{18}$$

(18) is the equation of a harmonic oscillator with a sinusoidal forcing function. It is interesting to compare this equation to the harmonic oscillator problem in general (Crawford, 1968) such as

$$\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=F(x) \tag{19}$$

or forced Helmholtz wave equation such as

$$\frac{d^2n}{dx^2}+k^2n=F(x)=\sum_i B(k_i)\cos(k_ix) \tag{20}$$

It is obvious that (18) is a special case of (20) with p (x) = 0 and q (x) = constant. (18) corresponds to the Helmholtz equation in which the forcing function is a single mode, $k_l = k$, i.e, the resonance mode. The speculation of the meandering process in relation to the spectrum of eddy field will postulate that the meandering of the center of mass is a consequence of harmonic motion in response to the forcing of local eddy whose length scale will be 1/k. A wide spectrum of local eddy field will be more realistic to represent the forcing function in (18) such as

$$F(x) = \sum_i A_i \sin(k_i x) + A_i k_i x \cos(k_i x) \qquad (21)$$

Although the meandering process is expressed in linear equation in (18), the reality would be in most cases non-linear as is in the problems of turbulence. It will be hence very informative to estimate the wave length of local eddy responsible for the meandering process using the data base from the Plates I-VI.

4. ESTIMATION OF WAVE LENGTH OF MEANDERING EDDY

Plates I-VI out of 295 cuts are chosen to represent the typical features of dye plumes resulted from a continuous point source in coastal waters. These photographs of Rhodamine WT dye plumes were taken during the dye diffusion experiments in South Shore off Long Island, U.S. from the altitude ranging from 850 to 1500 feet using Cessna aircraft. Aerial mapping camera with special optical filter sets was used to get the enhanced contrast between Rhodamine dye field and ambient waters. The negative films of dye plumes were then scanned to get the optical densities by use of micro-photodensitometer. The optical density field were then calibrated into dye concentration field. These concentration fields constitute the data base for this study. In Table 1, scanning resolution and actual sampling rate are listed. The scanning micro-photodensitommetry provided quite unusual opportunity to obtain very detailed concentration field of dye plumes, the spatial sampling rate of which is less than 1 meter. The detailed technical informations of aerial photography and subsequent processing can be referred to Okube *et al.* (1983) and Ro (1985).

Three features are quite visible as follows; 1) slender plumes with straight plumeaxis in Plate III and IV are apparent, 2) meandering of plumeaxis with varying wave lengths exist in Plates I, II, III, V, and VI, 3)in every Plates, striations perpendicular to plumeaxis are visible which are suspected to be related to the local Langmuir cell circulations. These features usually interact each other to make dye plumes more complicated than assumed or expected from theoretical point of view.

Table 1 summarizes the estimates of the wave lengths of meandering eddies through the regression

Table 1. Information for the dye plumes with oceanographic conditions. The estimates of wave length of plume meandering are listed.

Plate	Date	dx	dy	Alt	DX	DY	U	k
I	22	100	100	1500	30	30	3.3	2.5
II	24	100	100	1000	20	20	25.3	6.4
III	25	100	100	880	18	18	17.3	15.7
IV	25	100	100	940	19	19	17.9	13.2
V	25	500	50	850	85	9	9.5	60.3
VI	26	100	200	1000	20	40	24.0	5.5

Plate : Dye Plume ID.
Date : the day when the photograph of dye plume was taken in August, 1980.
dx, dy : scanning unit by micro-photodensitometer, (μm)
Alt : the altitude where the phoptographs were taken (ft),
DX, DY: Sampling resolution in real ocean (cm).
U : mean current speed to the plumeaxis, (cm/sec)
k : wave length of plume meandering (m).

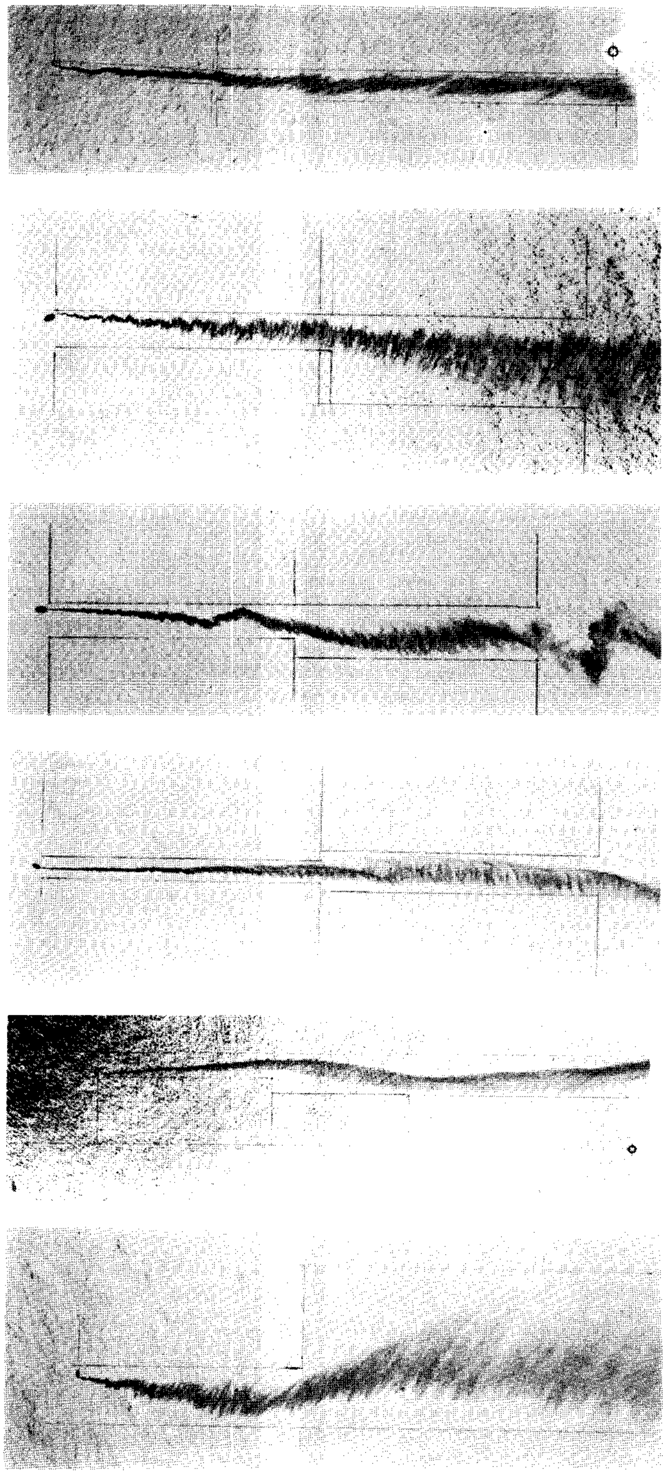

Plate I-VI. Aerial photographs of dye plumes of Rhodamine WT from a contiunous point source. These were taken during the dye diffusion experiments in the coastal surface waters of South Shore off Long Island, USA in August, 1980.

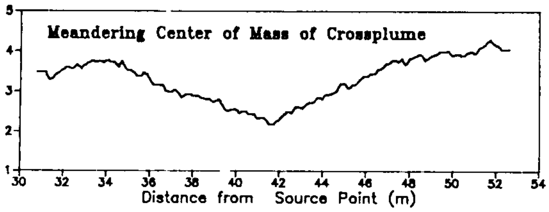

Fig. 4. Typical example of the trajectory of plumeaxis obtained from the Plate III by using the microphotodensitommetry. X-axis represents only a small portion of the plume.

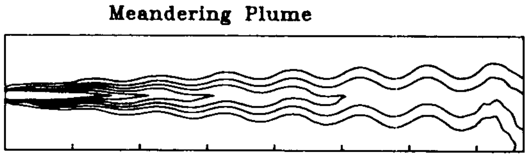

Fig. 5. Typical example of the concentration field of meandering plume reproduced by the numerical simulation. One tick mark in x-axis represents the wave length of meandering eddies. Spatial difference grids were chosen as 10% of the wave length. The amplitude of the lateral oscillatory velocity was chosen as 1% of mean convecting velocity. Contour lines ranges from 0.5 to 3 with contour level of 0.5 concentration unit. Source intensity is one concentration unit per time step.

analysis. In estimating the wave lengths, first set up the data base of the coordinates, (y_i, x_i) of peak concentrations of any particular dye fields. And then, (y_i, x_i) are regressed to the equation of $y = A\,x\sin(kx)$ in Equation (16). Since this involves nonlinear regression with regard to k, special cares were paid. In this study, IMSL and BMDP library were useful. The wave lengths of meandering eddies ranges from 5.5 to 60.3 meters which represent the diffusion field of the length scales, i.e., up to 3 (Km) to plumeaxis and 100 (m) to the lateral directions. Of course, these estimates will vary according to the local conditions of eddy field and to the domain of diffusion field.

5. NUMERICAL SIMULATION OF PLUME MEANDERING

The horizontal eddy diffusion equation for the dye concentration, C,

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}+v(x,y)\frac{\partial C}{\partial y}=K\frac{\partial^2 C}{\partial y^2}+S(x,y) \qquad (22)$$

where U is constant, while v (x, y) is given by the equation (18), and continuous source is defined at initial point with constant intensity, is numerically integrated. For numerical integration, the finite difference explicit schemes of forward time, alternating direction scheme (Saulyev, 1957) for the second derivative, and centered space for the first derivative are used. The schemes used yield the accuracy of first order in time and second order in space domain. Iterations are made until it yields near steady state. Figure 4 shows the numerically obtained concentration field which is successful in simulating realistic meandering plume. Such a concentration field would not be predicted without taking considerations of the eddying motion to the lateral direction.

6. SUMMARY AND CONCLUSION

This study relates the meandering of plumeaxis to non-constant, multiple harmonics of velocity field. The frequency components of velocity field behave differentially in contributing to the increase of variance of center of mass of plume. This line of thought leads to propose a diagnostic model of plume meandering in which amplified feature is reproduced under the constrain of continuity condition. The behavior of the amplified plume meandering is interpreted in an analogy to the forced harmonic oscillator in that local eddy field is acting as a forcing term to displace the center of plume laterally. Accordingly, the plumeaxis is oscillating and lateral displacement is increasing as the distance from the source point is increasing. The proposed diagnostic model is used in numerical simulation of dye concentration field to define the lateral velocity and the consequent plume is reproducing the realistic amplified feature successfully.

This study was motivated to exploit the statistical characteristics of meandering plume and to propose a diagnostic model for that. However, its arguments are initiated from the kinematic consideration of velocity field. In the future study, it should be further out from a dynamical consideration of velocity field. For example, it will be of interests and importance to include dynamical process such as local Langmuir cell circulation to assess their effects on the resulting concentration field.

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