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Location Equilibria in Spatial Competition under Price Control*

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1. Introduction

In his seminal paper, Hotelling provided the framework for the basic model of spatial competiton which two firms are competing to sell a homogeneous product to customers spread evenly along a linear market. In equilibrium the duopolists are located at the center of the market rather than being in the locations that would minimize transport costs (Hotelling, 1929). Hotelling originally suggested that his model explained a wide variety of social phenomena. However, it is well known that the Hotelling model with three firms admits no equilibrium solution. Furthermore, Hotelling type model with four, five and six firms respectedly have restricted pairs of equilibrium solutions only in some limiting cases (Eaton and Lipsey, 1975).

These non-existence results are associated with the assumption that customers patronize the nearest firm. Empirical evidence supports the idea that consumers do not necessarily choose to buy from the closest firm, since they also take variables other than distance into account. As these variables are often unobservable, firms cease to be fully informed about consumers' motivations. Consequently, they can at best determine the shopping behavior of a particular customer up to a probability distribution (de Palma, et al., 1987).

In some industries firms do not exert any control over their price level because of either cartel agreements or price control by government. Such constraints on prices drive competition among firms to alternative paths. In location theory, it is usual to consider that, instead of reducing price in order to attract customers, firms compete in locating their sales outlets so as to guarantee for themselves the largest possible sales (Gabszewicz, et al., 1986). Thus, the equilibrium solutions could be different from those of the case that firms can choose freely their price and location.

The purpose of this paper is to consider a location equilibrium within a probabilistic framework when a strategy for a firm is defined as the choice of a location on price control. Firms are free to choose their locations but prices are bounded by price agreements or regulation. The market share of a firm is the measure of the set of consumers who are located closer to that firm, but with unobservable consumers' behaviors. For formulating consumers' probabilistic behavior, I retain the multinomial logit which is the most widely used. This modelling approach agrees with recent advances in discrete choice theory which aims at describing individuals' behavior facing mutually exclusive alternatives (Ben-Akiva and Lerman, 1985).

The remaining of this paper is organized as follows. In section 2, I describe the model in detail. The results obtained are given in section 3. Conclusions are drawn in section 4.

2. The Model

We assume there is a uniform distribution of consumers over a linear market normalized, without loss of generality, to [0, 1]. There are n (≥ 2) firms, each with a single outlet. Their locations are denoted $x_i \in [0, 1]$; i = 1, 2,..., n. We re-index the firms so as $x_1 < x_2 < ... < x_n$. Each consumer is assumed to purchase

*This paper is based on de Palma, Hong and Thisse (1988). I would like to thank de Palma and Thisse for their valuable insight and ideas.

one unit of product. Each firm produces with constant and identical marginal costs, which we can therefore set equal to zero. Transport costs per unit shipped are assumed linear in distance and invariant to volume. Also, each firm charges a fixed price to all consumers and all transport costs are passed on to consumers. Given the form of their preferences, consumers choose to purchase from the firm with the nearest locations. When several firms are located at the same point, they share equally the market of the individuals choices. Also, it is assumed that each firm maximizes its market share, denoted by S_i ($i = 1, 2, \ldots, n$).

A Nash equilibrium is defined as a location vector $(x^*_1, x^*_2, \ldots, x^*_n) \in [0, 1]$ such that no firm can increase its market share by changing unilaterally its location:

$$S_i\ (x^*_1, \ldots, x^*_j, \ldots, x^*_n) > S_i\ (x^*_1, \ldots, x_j, \ldots, x^*_n)$$

for all $x_i \in [0, 1]$ and $i = 1, 2, \ldots, n$.

The following proposition summarizes the state-of-the-art results for the location problem considered (de Palma, et al., 1988). 1) If $n = 2$, then there exists a unique Nash equilibrium and $x^*_1 = x^*_2 = 1/2$. 2) If $n = 3$, then no Nash equilibrium exists. 3) If $n = 4$, there exists a unique Nash equilibrium with $x^*_1 = x^*_2 = 1/4$ and $x^*_3 = x^*_4 = 3/4$. 4) If $n = 5$, there is a unique Nash equilibrium with $x^*_1 = x^*_2 = 1/6$, $x^*_3 = 1/2$ and $x^*_4 = x^*_5 = 5/6$. 5) If $n \geq 6$, there is a continuum of equilibria characterized by the following conditions: (a) no more than two firms have the same location; (b) peripheral firms are paired with their neighbors; (c) paired firms have equal market shares; (d) isolated firms have market shares which are at least as large as those of paired firms but not more than twice as great.

The above proposition hinges crucially on the assumption that consumers purchase the product from the nearest firm. However, as mentioned in the introduction, this assumption may not be realistic. Extraneous considerations unobservable to the firms do influence consumers' decisions. Most sellers are inherently differentiated by a multitude of factors which are valued differently by different consumers. For example, in addition to the difference in spatial locations of retailers, consumers may have a preference for one over another because of some difference in service or quality. Given that individual consumer tastes over the many attributes of sellers are typically unobservable, the best firms can do is to make estimates of them. Hence firms look at the probability that a given consumer will choose its product. We shall use the terms consumer taste heterogeneity and retailer heterogeneity interchangeable throughout the paper: retailers are only differentiated from each other because consumers view them as such.

Assuming that these considerations are independent of the consumers' view, firms are led to model consumers' behavior by means of a random term. More precisely, the utility of consumer x purchasing from firm i is described as

$$U_i(x) = -P_i(x) + e_i(x); \quad i = 1, 2, \ldots, n, \quad x \in [0, 1]. \qquad (2\text{-}1)$$

where $P_i(x)$ is the fixed price changed by firm i at location $x \in [0, 1]$, and $e_j(x)$ is the consumer-specific evaluation of the seller i by the consumer at x. If a tie occures ($U_1(x) = U_2(x)$), the consumer is assumed to purchase from each firm with probability one half. Whenever $e_i(x) = 0$ for all $x \in [0, 1]$, the model reverts to the standard in the literature on spatial pricing. Here we consider the case where $e_i(x)$ is not constrained to be zero. In accord with discrete choice theory, the precise value of $e_i(x)$ is assumed not to be observed by the firm, so that the firm must form an estimate of the probability that the consumer at x prefers to do business with it. In particular, we shall assume that $e_i(x)$ is distributed in the consumer population according to:

$$e_i(x) = \mu\, \varepsilon_i,\ \mu \geq 0;\ i = 1, 2, \ldots, n. \qquad (2\text{-}2)$$

where the ε_i are independent random variables, with zero mean and unit variance, which are identically distributed according to the double exponential distribution. The terms

$\mu \varepsilon_i$ therefore reflect idiosyncratic tastes, and the parameter μ conveys the degree of dispersion of these tastes across consumers.

The probability of an consumer at x purchasing the product from firm i is given by the multinomial logit model as

$$p_i(x) = \frac{\exp(-p_i(x)/\mu)}{\Sigma^n_{j=1}\exp(-p_j(x)/\mu)},$$
$$i = 1, 2, \ldots, n \qquad (2\text{-}3)$$

When $\mu \to 0$, $p_i(x) = 1$ for the firm associated with the highest measured utility, i.e., the nearest firm, and 0 otherwise. When $\mu \to \infty$, all firms have the same probability $1/n$ to be selected by consumer x. The market share for the firm i is defined by

$$\bar{S}_i(x_1, \ldots, x_n) = \int_0^1 p_i(x)dx \qquad (2\text{-}4)$$

For any value of n, the explicit form of $\bar{S}_i$ can be determined. For example, when n = 2 and $x_1 < x_2$ we have

$$\bar{S}_1 = \frac{x_1}{1 + \exp(-|x_1 - x_2|/\mu)} + \frac{|x_1 - x_2|}{2} + \frac{1 - x_2}{1 + \exp(|x_1 - x_2|/\mu)} \qquad (2\text{-}5)$$

For larger values of n, however, the form of $\bar{S}_i$ becomes much more intricate and, for this reason, is not given here.

3. The Results

One of the main features of the model defined in this paper is the multiplicity of equilibria. The existence of an agglomerated equilibrium, i.e., a market configuration in which all firms choose the same location in terms of the measured utility, is investigated by de Palma et al. (1985). Summarizing the results are:

If $\mu > 1 - 2/n$, then $x^*_1 = \ldots = x^*_n = 1/2$ is a Nash equilibrium for the n firms. Furthermore, when $\mu < 1/2 - 1/n$, there exists no agglomerated equilibrium. In words, this result means that the firms select a location corresponding to the median consumer behavior under sufficient variations.

We now consider the case of dispersed equilibria, i.e., a market configuration in which not all firms select the same location along [0, 1]. Specifically, we want to know whether such equilibria exist and to determine the impact of the value of μ on the corresponding configuration. The complexity of the problem has made it impossible for us to find dispersed Nash equilibria analytically. We have therefore decided to resort to numerical computation. For simplicity, we focus on symmetric equilibria and restrict the analyses to the values n = 2, 3, 4 and 5. These cases cover all the possibilities in terms of existence of a Nash equilibrium for $\mu = 0$. The computation procedure is based on a complete enumeration of all possible symmetric configurations for a tolerance equal to 10^{-2}.

The following general results have been obtained. First, when an equilibrium exists for $\mu = 0$, this property carries over to any positive value of μ. Such is the case here for n = 2, 4 and 5. However, the type of configuration may change drastically according to the particular value of μ. Second, some qualitatively new solutions appear for strictly positive values of μ. Third, dispersed equilibria prevail for small values of μ and agglomerated equilibria for large values of μ. Fourth, in the four cases analyzed, an agglomerated equilibrium arises for $\mu \geq 1/2 - 1/n$.

The equilibrium patterns corresponding to n = 2, 3, 4 and 5 are now discussed in some detail.

(1) n = 2: There always exists a unique equilibrium for which two firms are located at the center ($x^*_1 = x^*_2 = 1/2$).

(2) n = 3: A dispersed equilibrium as depicted on Figure 1 exists for $0.16 \leq \mu < 0.26$. Furthermore, over the range [0.16, 0.26] both agglomerated and dispersed equilibrium exist. For $\mu > 0.26$, only the agglomerated equilibrium remains.

(3) n = 4: There are two types of dispersed equilibrium. In the first, as μ increases from 0, firms 1 and 2 (3 and 4) paired and their location converges continuously to

the center which is reached at $\mu = 0.04$. The second exists from $\mu \geq 0.2$. In this configuration, firms 2 and 3 select the center while firms 1 and 4 choose symmetrically dispersed locations. As μ increases, the locations of these firms converges gradually to the center which is again reached when $\mu = 0.42$. Over the range [0.2, 0.4], a glance at Figure 2 and 3 reveals immediately that then the paired firms {1, 2} and {3, 4} in the first equilibrium are less seperated than the isolated firms 1 and 4 in the second equilibrium. This can be explained as follows. In the second configuration, the center is more crowded than in the first one, thus making more peripheral locations attractive for the remaining firms. Finally, it is apparent from Figures 2 and 3 that the second equilibrium, when it exists, permits a better covering of the market share.

(4) n = 5: The situation is rather comparable to that obtained with four firms. We have two types of equilibrium. The first exists for all $\mu \geq 0$ and is such that one firm (3) is at the center, while the others {1, 2} and {4, 5} are paired and located symmetrically. As μ increase, the corresponding configuration evolves continuously toward the agglomerated one, a situation which occurs for $\mu = 0.45$. The second equilibrium exists for $\mu \geq 0.27$ and involves these firms at the center (2, 3 and 4) together with two isolated forms 1 and 5. As in the preceding case, for a given value of μ these two firms are more distant than the two pairs of clustered firms in the first equilibrium. The reason is the same as before. However, the clustering of locations is obtained for the same value of μ in the 4-party case, ie., $\mu = 0.45$.

4. Conclusions

A few general principles emerge from the above analysis. (1) When an equilibrium exists for $\mu = 0$, there is an equilibrium for any μ positive. (2) When no equilibrium exists for $\mu = 0$, one can always find a strictly positive, finite value of μ from which an equilibrium exists. (3) For some $\mu > 0$, there may be multiple equilibria even when the equilibrium is unique for $\mu = 0$. (4) For some $\mu > 0$, a higher number of firms does not favor a better covering of the market share. (5) When the number of firms is large enough, competition may lead to very unstable situations in the sense that a slight increase in variation yields very significant changes in the market share. (6) For n = 4 and 5, there is always an equilibrium in which peripheral firms are paired. And when peripheral firms are isolated, at least two firms are clustered at the center.

Against the approach described in this paper, it could be argued that the multinomial logit suffers from some severe drawbacks due to the property of independence from irrelevant alternatives implied by this model (Ben-Akiva and Lerman, 1985). The significance of the results obtained in this paper would therefore be weak. To that, we would answer that several qualitative results obtained under the logit still hold under more general probabilistic choice models. In addition, the analytical difficulty of the problem has forced us to resort to numerical analysis.

References

Ben-Akiva, M. and S. Lerman. 1985. Discrete Choice Analysis, MIT Press, Cambridge, MA.

de Palma, A., V. Ginsburgh, Y. Papageorgiou and J. Thisse. 1985. "The Principle of Minimum Differentiation Holds under Sufficient Heterogeneity", Econometrica, 53, pp. 767-781.

de Palma, A., V. Ginsburgh and J. Thisse. 1987. "On Existence of Location Equilibria in the 3-Firm Hotelling Problem." CEME, Discussion paper No. 8702, Universite Libre de Bruxelles.

de Palma, A., G. S. Hong and J. Thisse. 1988. "Equilibria in Multi-party Competition under Uncertainty." CORE, Discussion paper No. 8839, Universite Catholique de Louvain.

Eaton, B. and R. Lipsey. 1975. "The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition." Review of Economic Studies, 42,

pp. 27-49.
Gabszewicz, J., J. Thisse, M. Fujita and U. Schweizer. 1986. Location Theory, Harwood Academic Publishers, N.Y.
Hotelling, H. 1929. "Stability in Competition," Economic Journal, 39, pp. 41-57.